A TREATISE ON SHIP-BUILDING AND NAVIGATION.

IN THREE PARTS

WHEREIN

The Theory, Practice, and Application of all the necessary Instruments are perspicuously handled.

WITH

The Construction and Use of a new invented Shipwright's Sector, for readily laying down and delineating Ships, whether of similar or dissimilar Forms.

ALSO

Tables of the Sun's Declination, of Meridional Parts, of difference of Latitude and Departure, of Logarithms, and of artificial Sines, Tangents and Secants.

By MUNGO MURRAY.
Shipwright, in his Majesty's Yard, DEPTFORD.

To which is added by way of Appendix,

An English Abridgment of another Treatise on Naval Architecture, lately published at Paris by M. DUHAMEL, Mem. of the R. Acad. of Sciences, Fellow of the Royal Society of London, and Surveyor General of the French Marine.

The whole illustrated with eighteen COPPER PLATES.

LONDON:

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M,DCC,LIV.

ADVERTISEMENT.

THE ſeveral Branches of Mathematicks treated of in this Book are expeditiouſly taught by the AUTHOR, at his Houſe in *Deptford*; where may be had all Sorts of Sliding Rules and Scales: As alſo Sectors for delineating Ships, Diagonal Scales, *&c.* on Braſs, Wood, or Paſte-board. Attendance from ſix to eight every Evening, except *Wedneſdays* and *Saturdays*.

To the right Honourable the

LORDS COMMISSIONERS,

For executing the Office of

Lord HIGH ADMIRAL

OF

GREAT BRITAIN and *IRELAND*,

And of his Majeſty's

PLANTATIONS ABROAD, &c.

THIS TREATISE

ON

SHIP-BUILDING AND NAVIGATION

Is with the utmoſt Submiſſion, inſcribed

By their Lordſhips

Moſt dutiful,

Moſt humble, and

Moſt obedient Servant

MUNGO MURRAY.

THE

PREFACE.

THOUGH the art of Ship-building is of the utmoſt conſequence to the trade and ſecurity of this nation, and a competent knowledge of the theory of it neceſſary for every Shipwright, yet I cannot think of a ſubject which has been ſo little treated of in our language.

This conſideration induced me to offer the following ſheets to the publick, which are calculated for the inſtruction and improvement of ſhipwrights in the art of delineating or drawing of ſhips, being fully perſuaded that any thing which may be conducive to this end muſt be of great ſervice to the publick in general. I therefore flatter myſelf that this attempt will meet with a favourable reception, though it were to be wiſhed it had been undertaken by ſome perſon of abilities greatly ſuperior to mine.

In order to make this treatiſe as uſeful as poſſible, I have briefly explained the nature of proportion, the principles of geometry, the invention of logarithms, and their uſe in the conſtruction of the line of numbers, by which, I preſume, any perſon that is acquainted with common arithmetick, may, with a little application, be able to conſtruct the lines himſelf, and have a clear idea of all the operations by the ſliding rule, in meaſuring ſurfaces and ſolids; a method ſo expedient and uſeful, that it is univerſally practiſed in meaſuring all the

plank

plank and timber received into his majeſty's yards, and uſed not only by ſhipwrights, but by many other artificers, as joiners, painters, &c.

The diviſions on the line of numbers, in plate No. II. are taken from the ſcale of equal parts in the ſame plate, which is divided in the exacteſt manner, and may be of great uſe to the reader, not only in comparing the diſtances meaſured on the ſcale with the table of logarithms, in order to examine how the lines have been graduated, but alſo in the conſtruction of geometrical figures, as any given or required diſtances may be meaſured by it to a very great accuracy.

Tho' it muſt be allowed that the principles of geometry, trigonometry, logarithms, and their various uſes, have been ſufficiently explained by many eminent authors who have wrote on theſe ſubjects, yet I thought it requiſite to treat conciſely of them here, as introductory to the main deſign, which is to inſtruct the Shipwright to form all his work by mathematical rules; and, as he is furniſhed with every thing that is neceſſary in this treatiſe, I have no occaſion to refer him to other books, with which perhaps he might not be furniſhed if I did.

In the ſecond part, I have endeavoured to explain the method of repreſenting ſolids upon a plane, with the application thereof to the delineating of ſhips, in which I have given definitions of all the terms and lines made uſe of in drawing; I have alſo ſhewn the methods that are generally practiſed, and the difficulties and inconveniences attending them, which I have endeavoured to facilitate by a ſector of my own invention, conſtructed for that purpoſe; a method entirely new, and, though deduced from mathematical principles, does not reſtrain the artiſt from diſplaying all the ſkill and judg-

judgment he is master of in varying the form of the curves so as to be most suitable for the service for which the ship is designed.

The third part contains land surveying, geography and navigation, with an analemma divided in so curious and accurate a manner, that by a nonius division the pole may be set to any latitude, even to three minutes, and all the problems that are usually solved by the globe may be performed with greater exactness by this instrument. I have also shewn the method of constructing the plain and *Mercator*'s charts, the manner of keeping a reckoning, and of finding the latitude and variation of the compass by cœlestial observation; to which I have added tables of the sun's declination, of difference of latitude and departure, of meridional parts, of logarithms, and of artificial sines, tangents, and secants.

Tho' it may be the opinion of many that this 3d part, with the tables, might have been omitted, as having no necessary connection with the theory of Ship-building, yet, as I was desirous of making this treatise universally useful, I thought it requisite it should contain them, though it would have been my interest to have done otherwise.

Some time after the first, and most of the second part had been in the press, a treatise in *French* on the same subject, by the ingenious M. *du Hamel du Monçeau*, Member of the Royal Academy of Sciences at *Paris*, and Fellow of the Royal Society at *London*, fell into my hands, and, as I imagined that every body would be desirous of seeing what has been wrote on Ship-building, by a foreign author of such distinction, I have added, by way of appendix, an abridgment of that work, which I doubt not will be agreeable to such as are not acquainted with the *French* language, or have no opportunity of perusing the original.

I flat-

I flatter myself, from the high repute Monf. *Du Hamel's* writings have every where juftly acquir'd, that the additional price of 2*s*. 6*d*. for the *Englifh* abridgment of his book on Ship-building, annexed to mine, will not be thought much of, when it is confidered that the original fells for 18*s*.

I cannot conclude this preface, without acknowledging the great obligation I am under to the principal officers and gentlemen in his majefty's fervice, not only in the yard where I have the happinefs to be employed, but in feveral others, as well as in the navy, for their kindnefs in encouraging this work, feveral of them perfons, whofe abilities are fuch, that it would be the greateft vanity in me to imagine they would countenance the undertaking on account of any information they could expect to derive from it themfelves; their true motives were doubtlefs a confcioufnefs of the important fervice of fuch a piece to young fhipwrights, and a generous difpofition to encourage induftry.

I believe it will appear very obvious to every body, that I have fpared no pains or coft in endeavouring to render this work as compleat as poffible, to which end I have fubmitted the mathematical part thereof to the perufal and amendment of a gentleman whofe abilities in thefe matters would be indifputable with the publick, were I permitted to name him.

The fuccefs of my labour I reft entirely on the judgment and candour of my readers, by which I muft ftand or fall; whatever may be the event, I fhall always have the fecret fatisfaction of reflecting, that I have fincerely aimed at what is ufeful, and very much wanted in the *Englifh* language.

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PART I.

CHAP. I. SECT. I.

Of Involution and Evolution of Quantities.

IF a number be multiplied by itſelf, and the product by the ſame number, and the new product again by the ſame, and ſo on, it is ſaid to be involved into itſelf ſo many times, and the ſeveral products are diſtinguiſh'd into powers of different denominations, of all which the original number is called the root; the firſt product being its ſecond power or ſquare; the ſecond the third power, or cube; the third the fourth power, or biquadrate, &c. Theſe powers are uſually denoted by a ſmall figure annexed above the laſt digit of the root. Thus 12^2 ſignifies the ſecond power or ſquare of the root 12, being equal to 144; 12^3 the third power, or cube thereof, equal to 1728, &c. and ſo the ſeveral powers may be orderly expreſſed thus, 12, 12^2, 12^3, 12^4, &c. that is, 12 root, $12\times12=144$ ſquare, $12\times12\times12=1728$ cube, $12\times12\times12\times12=20736$, biquadrate, &c.

Square and cube numbers have borrowed their denominations from geometrical figures or extenſions, the root being repreſented by a right line, which has but one dimenſion, *viz.* lengh; the ſquare by a plane, or right lined figure of two dimenſions, having equal length and breadth; and the cube by a right lined ſolid of three dimenſions, having equal length, breadth, and thickneſs.

The nature of ſpace admits of no other modes of extenſion, than length, breadth, and thickneſs, neither is it poſſible to conceive any body otherwiſe to exiſt than under theſe limitations.

That the reader may have a diſtinct idea of the method of extracting the

the ſquare and cube roots, we think it neceſſary to make the following remarks on multiplication, and to ſhew how any root, by being divided into two parts (or made a binomial) may be raiſed to the ſecond or third power, &c.

Since multiplication may be conſidered as a manifold addition, it is certain that if either or both multiplicand and multiplier be divided into parts, and all the parts of the one be multiplied by all the parts of the other, the ſum of all the products will be equal to the product of the whole multiplicand multiplied by the whole multiplier, and this method is what is in effect performed by the common rule, when either or both conſiſt of more than one ſignificant figure, as will appear by the following examples, *viz.*

EXAMPLE I.

Let the number 24 be divided into two parts, *viz.* 20 and 4, and multiplied by 4.

$$\begin{array}{rcr} 20 + 4 & = & 24 \\ 4 & & 4 \\ \hline 80 + 16 & = & 96 \end{array}$$

By the common rule

$$\begin{array}{rr} 4 \times 4 = & 16 \\ 4 \times 20 = & \underline{80} \\ & 96 \end{array}$$

EXAMPLE II.

Let the number 248 be divided into three parts, *viz.* 200, 40, and 8, and multiplied by 24 or 20+4.

$$\begin{array}{rcr} 200 + 40 + 8 & = & 248 \\ 20 + 4 & = & 24 \\ \hline 4 \times 200 + 4 \times 40 + 4 \times 8 & & 992 \\ 20 \times 200 + 20 \times 40 + 20 \times 8 & & 496 \\ \hline \end{array}$$

$$20 \times 200 + 20 \times 40 + 20 \times 8 + 4 \times 200 + 4 \times 40 + 4 \times 8 = 5952$$

By the common rule.

$$\begin{array}{rr} 4 \times 8 = & 32 \\ 4 \times 40 = & 160 \\ 4 \times 200 = & 800 \\ 20 \times 8 = & 160 \\ 20 \times 40 = & 800 \\ 20 \times 200 = & \underline{4000} \\ & 5952 \end{array}$$

Though there is no occaſion to ſet down the ſeveral products, as in the above examples, becauſe the exceſs above the tens may be retained in the memory, and added to the next place, which is always the method

thod obſerved in practice; yet this will very much aſſiſt us in perceiving the reaſon of the rules for extracting the ſquare and cube roots of any numbers, by carefully obſerving the ſteps by which any root is raiſed to thoſe powers, as in the following example, *viz.*

Let the root be $24 = 20 + 4$.

$$\begin{array}{r} 20 + 4 \\ 20 + 4 \\ \hline 4 \times 20 + 4 \times 4 \\ 20 \times 20 + 4 \times 20 \qquad\qquad \\ \hline \end{array}$$

Square, or ſecond power, $\left.\begin{array}{l} 20 \times 20 + 4 \times 20 + 4 \times 20 + 4 \times 4 \\ \text{or } 20^2 + 2 \times 4 \times 20 + 4^2 \end{array}\right\} = 576$

$$\begin{array}{rcl} & 24 & \\ & 24 & \\ \hline 4 \times 4 = 16 & & 4 \times 4 = 16 \\ \left.\begin{array}{r} 4 \times 20 = 80 \\ 4 \times 20 = 80 \end{array}\right\} & & 4 \times 40 = 160 \\ 20 \times 20 = 400 & & 20 \times 20 = 400 \\ \hline 576 & & 576 \end{array}$$

It is evident from this operation, that any ſquare number whoſe root is divided into two parts, is equal to the ſum of the ſquares of thoſe parts, and double their product added together.

This obſervation will hold equally true when the root conſiſts of more than two figures, by repeating the proceſs for every ſignificant figure, as in the following example, *viz.*

Let the root be 2435, or $2000 + 400 + 30 + 5$

1ſt, ſuppoſe the root	2400
binomial parts	$2000 + 400$
2d, root	2430
binomial parts	$2400 + 30$
3d, root	2435
binomial parts	$2430 + 5$

$$\begin{array}{rcr} 2000^2 & = & 4000000 \\ 2 \times 2000 \times 400 & = & 1600000 \\ 400^2 & = & 160000 \\ \hline 2400^2 & = & 5760000 \\ 2 \times 2400 \times 30 & = & 144000 \\ 30^2 & = & 900 \\ \hline 2430^2 & = & 5904900 \\ 2 \times 2430 \times 5 & = & 24300 \\ 5^2 & = & 25 \\ \hline 2435^2 & = & 5929225 \end{array}$$

By this method it is plain, that we find a ſquare for every figure in the root, and that in finding the ſquare of the firſt figures, we ſuppoſe all the reſt to be cyphers, and ſo on till the whole ſquare is compoſed by the addition of one ſignificant figure after each operation.

Note,

Note, That for every figure that is annexed to the first figure in the root, there will be two in the square; thus, 2^2 is 4, 20^2 is 400, 200^2 is 40000, 2000^2 is 4000000, &c.

In like manner the cube of any number may be found, by dividing the root into parts, as in the following example, *viz.*

Let the root be 24, or $20+4$, as before.

Then the square will be $20^2 + 2\times20\times4 + 4^2$

$$20 + 4 \text{ root}$$

$$20^2\times4 + 2\times20\times4^2 + 4^3$$

$$20^3 + 2\times20^2\times4 + 20\times4^2$$

Cube, or third power $20^3 + 3\times20^2\times4 + 3\times20\times4^2 + 4^3 =$ cube of $24 = 13824$

1st,	20^3	$= 8000$
2d,	$3\times20^2\times4$	$= 4800$
3d,	$3\times20\times4^2$	$= 960$
4th,	4^3	$= 64$
		13824

Hence it appears, that if any root be divided into two parts, the cube will be composed of the four following sums, *viz.*

1st, The cube of the first part.

2d, Three times the square of the first part multiplied, by the second part.

3d, Three times the first part, multiplied by the square of the second part.

4th, The cube of the second part.

If the root consists of more than two figures, the same method may be observed as in composing the square, by taking the two first figures for one part, and the next figure for the other part, and so on till all the significant figures are brought in. This is so plain, that we think it needless to give any example here; and as the only use we shall make of it will be to shew how to extract the cube root, we shall just observe under this head, that for every figure annexed to the first figure in the root, there will be three in the cube, besides the cube of the first figure; thus, 2^3 is 8, 20^3 is 8000, 200^3 is 8000000, &c.

SECT. II.

Evolution of Quantities.

EVolution is the unfolding, or resolving of any number into the parts of which it is composed, and is called the extraction of the root of any given power; by means of which, we find a number, that being multiplied by itself as many times less one, as the index of the power contains units, will produce the given number.

In

In order to this, the method already obſerved, and the ſteps taken in the involution of the binomial root muſt be carefully attended to, in which it will not be difficult to diſcern how each part of the root is concerned in the power.

To Extract the SQUARE ROOT.

As the ſquare was compoſed by multiplication and addition, the root muſt be found by diviſion and ſubtraction.

When the root of any number is required, the firſt thing to be done is to prepare it, by points ſet over ſuch places as the index of the power directs, always beginning at unity, and proceeding towards the left hand, if the given number (which we ſhall call the reſolvend) be integers, and towards the right hand, in decimal parts; now the index of the ſquare being 2, there muſt be a point over every ſecond fignre, as in the following example, *viz.*

Let the given number be $\dot{5}9\dot{2}9\dot{2}2\dot{5}$

Having pointed the given reſolvend as above, to find the firſt figure take the greateſt root that is contained in the firſt period, which in this caſe is 2000, the ſquare of which is 4000000. Subtract this from the reſolvend 5929225, and there will remain 1929225, which is called the firſt dividend.

Now this number contains double the product of the firſt figure multiplied by the ſecond figure, and the ſquare of the ſecond figure, as is evident from the method we uſed in compoſing the ſquare.

Divide this dividend, therefore, by double the firſt figure, and the quotient will be the ſecond figure, provided that when it is multiplied by double the firſt figure, and the product added to the ſquare of the ſecond figure, the ſum does not exceed the dividend. In this example 4000 is double the firſt figure in the root, and when this is made a diviſor to the dividend, the quotient will be 400.

In the next place, multiply this ſecond figure by double the firſt, or which is the ſame thing, by the diviſor, and add the ſquare thereof to the product, and their ſum will be 1760000; ſubtract this from the dividend, the remainder will be 169225, the ſecond dividend.

And now we have in effect ſubtracted the ſquare of the firſt two figures; for in the firſt ſtep we ſubtracted 2000^2, and in the next $2\times2000\times400+400^2$, all which make 2400^2. The ſecond dividend will therefore contain double the product of the firſt two figures multiplied by the third, and the ſquare of the ſame third figure. Therefore,

To find the third figure, double the firſt two figures for a diviſor to this

this dividend, and the quotient will be 30; multiply this by the divisor, and add the square of the quotient to the product; the sum will be 144900. When this is subtracted from the dividend, the remainder will be 24325, the third dividend; to which there must be a new divisor found by doubling the figures already found in the root, and the quotient will be 5, the fourth figure. And by observing the same method as before in the operation, there will be no remainder, so that 2435 will be the true root required.

Resolvend	5929225		
$2000^2 =$	4000000		2000 + 400 + 30 + 5 = 2435
1*st* divisor $2000 \times 2 = 4000$)	1929225	1*st* dividend	
$4000 \times 400 = 1600000$			
$400^2 = 160000$			
	1760000		
2*d* divisor $2400 \times 2 = 4800$)	169225	2*d* dividend	
$4800 \times 30 = 144000$			
$30^2 = 900$			
	144900		
3*d* divisor $2430 \times 2 = 4860$)	24325	3*d* dividend	
$4860 \times 5 = 24300$			
$5^2 = 25$			
	24325		

By comparing this operation with that by which the square number 5929225 was composed as above, the reader will observe, that the same numbers that were there added together to make up that sum, are hereby regularly subtracted from it. So that this method of discovering the root is only the reverse of that by which it was raised.

We shall conclude this head with observing, that there is no necessity in practice for annexing all the cyphers to the divisors and dividends, nor of subtracting the square of the first figure from the whole resolvend; but the several periods which it consists of, may be brought down one at a time, as in the following example, *viz.*

Re-

```
              Resolvend 179409850624(423568 root
                        16 = greatest square in 17
1st divisor   82)       194
               2        164 = 82 × 2
2d divisor    843)      3009
                3       2529 = 843 × 3
3d divisor    8465)     48085
                 5      42325 = 8465 × 5
4th divisor   84706)    576006
                  6     508236 = 84706 × 6
5th divisor   847128)   6777024
                  8     6777024 = 847128 × 8
```

Observe always, that after having doubled the root in the quotient for a divisor, we are to enquire how oft it may be had in the dividend; so as when the quotient figure is annexed to the divisor, and that increased divisor multiplied by the same quotient figure, the product may be the greatest number that can be had in the dividend: And so proceed from period to period till the whole is finished.

By pursuing this method in extracting the root of the square number 5929225, the reader will observe that the operation is exactly the same as before, omitting the cyphers.

To Extract the CUBE ROOT.

We shall observe the same method in extracting the cube root, as we have done already in the square root; that is, by considering it as divided into two parts in different operations, till we have discovered all the significant figures thereof.

Let the number, or resolvend, whose cube root is required be 13824.

When it is properly pointed as above, it appears that the root will consist of two figures.

The first figure in the root will be the greatest root that can be had in the first period 13000, which is 20; the cube of which 8000, must be subtracted from the resolvend, and the remainder will be 5824, for a dividend.

As we have already subtracted the cube of the first part 20, this number must contain three times the square of that first part multiplied by the second part, three times the first part multiplied by the square of the second, and the cube of the second part, added together.

ther. Therefore, if this dividend be divided by three times the ſquare of the firſt part = 1200, the quotient will be 4, the ſecond figure required: Then 3 times $20^2 \times 4$, 3 times 20×4^2, and 4^3 muſt be added together, and the ſum ſubtracted from the dividend, as in the following example, *viz.*

Reſolvend $\dot{1}3\dot{8}24$(24 root

$20^3 = 8000$

Diviſor $3 \times 20^2 = 1200$) 5824

$3 \times 20^2 \times 4 = 4800$

$3 \times 20 \times 4^2 = 960$

$4^3 = 64$

5824

....

By comparing this example with that which we have given before, where the binomial root 20 + 4 is cubed, the reader will obſerve, that the very ſame numbers which compoſe the cube 13824, and are there added together, are here regularly ſubtracted from it. This method of extracting the cube root, is, therefore, only the reverſe of that by which it was raiſed.

We ſhall give one example more, without annexing cyphers to the diviſors and dividends.

Let it be required to extract the cube root of 94818816

94818816(456

64

1 diviſor $4^2 \times 3 = 48$) 30818 dividend

$3 \times 4^2 \times 5 = 240..$

$3 \times 4 \times 5^2 = 300.$

$5^3 = 125$

27125

2 diviſor $45^2 \times 3 = 6075$) 3693816 dividend

$3 \times 45^2 \times 6 = 36450..$

$3 \times 45 \times 6^2 = 4860.$

$6^3 = 216$

3693816

....

It is here to be noted, that the two laſt figures in the dividend muſt be excluded, before we enquire how often the diviſor (which wants two cyphers) is contained in it, becauſe when we proceed to find the ſecond figure in the root, the firſt muſt be conſidered in the place of tens; and when

when we are to find the third figure, it muſt be conſidered in the place of hundreds, this will appear very plain by annexing the cyphers in the operation.

As the diviſor found by this rule, multiplied by the new figure in the root, is not all that is to be ſubſtracted from the dividend, but alſo three times the firſt part multiplied by the ſquare of the ſecond, and the cube of the ſecond, as in the foregoing example; it will ſometimes happen that the quotient muſt not be taken for the next figure in the root, thus, if the dividend 30818 be divided by 4800, the quotient will be 6, but $3 \times 40^2 \times 6 + 3 \times 40 \times 6^2 + 6^3 = 33336$ which exceeds the dividend, therefore it will be neceſſary ſometimes to try how much the number to be ſubſtracted will amount to by the foregoing rule, before we can determine upon the new figure in the root.

If, as it often happens, a number has not a root that can be expreſſed by a rational number, place as many pairs of cyphers in the ſquare, and ternaries of cyphers in the cube, on the right hand of the remainder, as you would have decimal places in the root, and work as before, diſtinguiſhing them from the integers by a comma between; and thus you may approach infinitely near the exact root.

Mathematicians have proceeded further in the involution and evolution of quantities, *viz.* to the 4th, 5th, 6th, 7th, 8th, and 9th powers, called biquadrat, ſurſolid, ſquare cubed, ſecond ſurſolid, and biquadrat ſquared, but as we ſhall not have occaſion to apply theſe in practice, it is ſufficient barely to mention them.

How to raiſe a given root to any power, or to extract the root out of any given power, by the help of logarithms, ſhall be ſhewn when we come to treat of logarithms, and their various uſes.

CHAP. II. SECT. I.

Of PROPORTION.

THE whole body of the mathematicks is chiefly concerned in comparing quantities one with another; their mutual relation is what is called proportion, which is either arithmetical or geometrical; and as quantities may be reprefented by numbers, or lines, we fhall firft confider proportion with refpect to numbers.

Arithmetical proportion is, when in comparing two or more numbers, the leffer is fubtracted from the greater, the remainder is called the difference; and when feveral differences are equal, thofe numbers are faid to be in an arithmetical proportion to one another. As if we compare 2 to 4, and 4 to 6, and 6 to 8; the difference between 2 and 4 is 2, equal to the difference between 4 and 6, and to that of 6 and 8; therefore, 2, 4, 6, 8, or 8, 6, 4, 2, or 2, 5, 8, 11, 14, or 14, 11, 8, 5, 2, are all ranks of numbers in arithmetical proportion, or in arithmetical progreffion continued. 2, 5, 7, 10, are alfo proportionals, for the difference between 5 and 2 is the fame as between 10 and 7, but then, becaufe it is not the fame with the difference betwixt 7 and 5, thefe four numbers are faid to be in a difcontinued, as the former ranks are faid to be in a continued arithmetical proportion. Hence the following inferences.

1. If three quantities are in arithmetical proportion continued, the fum of the extremes is equal to the double of the mean, as in this, 8, 10, 12, where 20, the fum of the extremes 8 and 12, is equal to double of the mean 10.

2. If four quantities are fo, the fum of the extremes is equal to the fum of the means.

as 3, 5, 7, 9, here, $3+9=12$ and $5+7=12$.

3. If never fo many quantities are fo proportional, the fum of the extremes is always equal to the double of the middle term, if the number of the terms be odd, or to the fum of any two terms equally diftant from the extremes, as in the following feries.

2, 4, 6, 8, 10, 12, 14.
$2+14=8\times 2=16$.
or $4+12=8\times 2=16$, &c.

And this muft always hold good, becaufe the laft term comprehends the

the firſt, together with the common difference ſuperadded, as often as the number of its place is diſtant from the firſt term: But the firſt term has no addition of the difference at all; and as the ſecond term has one difference or ratio more than the firſt; the third one more than the ſecond, &c. ſo the laſt but one has one leſs than the laſt of all; the laſt but two one leſs than the laſt but one, &c. whence the ſum of any two of theſe equally diſtant from the extremes muſt be equal to the ſum of the extremes, becauſe one increaſes as much as the other decreaſes.

Therefore, the ſum of any number of terms in ſuch a progreſſion may be had, by multiplying the ſum of the extremes by half the number of the terms.

To find the ſum of never ſo many quantities in this progreſſion, it is only neceſſary that the extremes and the number of terms be given: ſo that if by having the firſt term and the common exceſs you would find the laſt, it might be done with great diſpatch, by multiplying the number of terms, leſſened by unity, into the common exceſs, and then adding the firſt term to the product.

Thus, if the laſt term of a progreſſion of 73 places were required, and the common difference were 4, and the firſt term 3; you need only multiply 72 by 4, and to the product 288 add 3, and you have 291 for the laſt term in the progreſſion.

So that if the progreſſion begins with a cypher, which is the moſt natural and ſimple of all, then the ſum of all the terms will be equal to the ſum of the extremes multiplied by half the number of the terms. Thus ſuppoſe

0, 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39.

The laſt term 39, multiplied by 14, the whole number of terms, gives 546; the half of which 273, is the ſum of all the terms.

From whence it will follow, that the ſum of all the terms in any ſuch progreſſion beginning from 0, is half the ſum of ſo many terms, all equal to the greateſt.

SECT. II.

Of GEOMETRICAL PROPORTION.

GEometrical proportion is when in comparing two or more numbers, one is divided by the other, the quotient is called the ratio; and when ſeveral ratio's are equal, the numbers are ſaid to be in a geometrical proportion; thus, 2 : 6 : : 5 : 15 are proportionals; for 6 divided by 2 is 3, and 15 divided by 5 is 3, and ſo the ratio's are equal. When two numbers are compared, the former is called the antecedent, and the latter the conſequent; but when more than two are compared, they are called terms, of which the firſt and laſt are called extremes, and all the intermediate ones, means. In order to know whether numbers be proportionals, it is only finding the ratio of each pair, and here it will be indifferent whether the antecedents or conſequents of every pair be made diviſors, as in the preceding numbers 2 : 6 : : 5 : 15, if 2 the antecedent be divided by 6, the conſequent, the quotient is $\frac{1}{3}$, and if 5 be divided by 15, the quotient is $\frac{1}{3}$, ſo the ratio's are equal, as before when the antecedents were made diviſors.

When in a rank of numbers they increaſe in a geometrical proportion, the ratio will be a common multiplier, and is found by dividing any one of the conſequent terms by its antecedent, for ſo the quotient will be the ratio.

As { 3, 9, 27, 81, 243, &c. Here the common multiplier is 3.
  { 2, 4, 8, 16, 32, &c. Here the common multiplier is 2.

It is plain that if either of theſe antecedent terms be multiplied by the ratio, the product will be its conſequent.

When a in rank of numbers they decreaſe in a geometrical proportion, the ratio will be a common diviſor, and is found by dividing any one of the antecedent terms by its conſequent.

As { 243, 81, 27, 9, 3, &c. Here the common diviſor is 3.
  { 32, 16, 8, 4, 2, &c. Here the common diviſor is 2.

In like manner, if either of theſe terms is divided by the ratio, the quotient will be the next term in the progreſſion.

If in comparing ſeveral numbers together, we find the ratio of the firſt and ſecond, to be the ſame with the ratio of the ſecond and third, third and fourth, fourth and fifth, and ſo on; thoſe numbers are in a geometrical proportion continued.

If in comparing four numbers together, we find the ratio of the firſt and ſecond to be the ſame as the ratio of the third and fourth, but that the ratio of the ſecond and third is not the ſame; thoſe numbers are called diſcontinued proportionals, as 2 : 4 : : 6 : 12, for the progreſſion ſtops here at 4.

The manner of expreſſing continued proportionals is by ſeparating the terms by two points, as 2 : 4 : 8 : 16 : 32, &c. but in diſcontinued proportionals the terms where the progreſſion ſtops are ſeparated by four points, as

5 : 10 : : 6 : 12 : : 7 : 14, or 14 : 7 : : 12 : 6 : : 10 : 5.

If

PROPOSITION I.

If three numbers are in a geometrical proportion, the product of the extremes will be equal to the product of the middle term multiplied into itself, or which is the same thing, to the square of the middle term.

As in these numbers 2, 6, 18.
$2 \times 18 = 6 \times 6 = 36.$

PROPOSITION II.

If four numbers are in geometrical proportion, (whether continued or discontinued) the product of the extremes will be equal to the product of the 2 means.

Let the numbers be $5 : 10 :: 6 : 12$
$5 \times 12 = 10 \times 6 = 60$
or $5 : 10 : 20 : 40$
$5 \times 40 = 10 \times 20 = 200$

From these two propositions the following inferences may be drawn, *viz.*

1st, If the product of any two numbers is equal to the square of a third, those three numbers are in geometrical proportion continued.

2d, If the product of any two numbers is equal to the product of any other two, those four numbers are proportionals, and the numbers multiplied into each other will be either 2 means, or 2 extremes, as in the following examples, *viz.*

$2 \times 16 = 4 \times 8 = 32$
$16 : 4 :: 8 : 2$
$4 : 16 :: 2 : 8$, &c.
or $8 \times 12 = 6 \times 16 = 96$
$8 : 6 :: 16 : 12$
$6 : 12 :: 8 : 16$, &c.

PROBLEM I.

To find a mean proportional between any two given numbers.

Note, By a mean proportional we are to understand such a number as if multiplied by itself, the product will be equal to the product of the two given numbers.

Rule, Multiply the given numbers by one another, and extract the square root of the product, that root will be the mean required.

Ex-

Example, Let the given numbers be 3 and 27.

$3 \times 27 = 81$ the ſquare root of which is 9 the mean required.

PROBLEM II.

To find a fourth proportional to the three given numbers, ſo that the ratio of the third and fourth may be equal to the ratio of the firſt and ſecond.

Rule, Multiply the ſecond number by the third, and divide the product by the firſt, the quotient will be the fourth number required.

Example, Let the given numbers be 2 : 6 : : 7.

$6 \times 7 = 42$, and $42 \div 2 = 21$, the fourth number required.

The reaſon of both theſe rules is evident from the two foregoing propoſitions, for where there are three numbers given to find a fourth, though the fourth is not known, we know that the product of it when multiplied by the firſt will be equal to the product of the ſecond and third numbers, *(per prop.* II.*)* therefore, if that product is divided by the firſt term, the quotient will be the fourth number required. This is the foundation of the rule of three, which we ſuppoſe the reader acquainted with already.

In finding a fourth proportional where three numbers are given, the two firſt give the ratio, and the queſtion as to the fourth proportional concerns the third number.

Direct proportion is, when the greater the term is by which the queſtion is made, the fourth term will be alſo the greater; and the leſſer that term is, the fourth will alſo be the leſſer.

Reciprocal, or inverſe proportion is, when the greater the term is by which the queſtion is made, the fourth will be leſſer, and the leſſer that term is, the greater the fourth.

Continual proportion, thus expreſſed, ∺, is, when all the terms between the firſt and the laſt are both antecedents and conſequents in the ſame proportion.

Example, 8, 12, 18, 27, are ∺; for 8 : 12 : : 12 : 18 : : 18 : 27.

Wherefore in ſuch ſeries, the laſt term ſubtracted from the ſum of all the terms will give the ſum of all the antecedents, and the firſt term ſubtracted from the ſaid ſum will give the ſum of all the conſequents.

If four quantities be proportional, they will alſo be ſo alternately, inverſely, in compoſition, in diviſion, converſely, and mixtly.

Ex-

Example			A : B : : C : D
1, directly	A : B : : C : D	in numbers	12 : 9 : : 8 : 6
2, alternately	A : C : : B : D.		12 : 8 : : 9 : 6
3, inversely	B : C : : D : A.		9 : 8 : : 6 : 12
4, in compo-	A + B : B : : C + D : D.		21 : 9 : : 14 : 6
sition	A + C : C : : B + D : D.		20 : 8 : : 15 : 6
5, in divi-	A − B : B : : C − D : D.		3 : 9 : : 2 : 6
sion	A − C : C : : B − D : D.		4 : 8 : : 3 : 6
6, converse-	A : A + B : : C : C + D.		12 : 21 : : 8 : 14
	— —		12 : 3 : : 8 : 2
ly	A : A + C : : B : B + D.		12 : 20 : : 9 : 15
	— —		12 : 4 : : 9 : 3
7, mixt-	A + B : A − B : : C + D : C − D.		21 : 3 : : 14 : 2
ly	A + C : A − C : : B + D : B − D.		20 : 4 : : 15 : 3

All these are evidently proportionals, the product of the extremes being equal to that of the means, excepting the inverted, wherein the product of the first and second term is equal to that of the third and fourth, which is a property peculiar to that kind of proportion.

PROPOSITION III.

If the product of any two numbers is divided by a third, the quotient will be a fourth proportional, the divisor will be the first, and the numbers multiplied into each other the second and third terms in the progression.

Example, Let the given numbers be 5 and 8, and their product be divided by 10, a third number:

$$5 \times 8 = 40 \div 10 = 4.$$

Therefore, 10 : 5 : : 8 : 4 (by inference 2d) for the product of the means is equal to the product of the extremes.

PROPOSITION IV.

When two numbers are multiplied into one another, they may be made mean proportionals to other two numbers, of which 1 must be the first, and the product of the two numbers will be the last or fourth term.

Example, Let the given numbers be 6 and 8.

It will be 1 : 6 : : 8 : 48 $= 6 \times 8 = 1 \times 48$. Hence in multiplication,

As

As 1 is to the multiplier, ſo is the multiplicand to the product; and in diviſion, as the diviſor is to 1, ſo is the dividend to the quotient.

PROPOSITION. V.

If any two numbers be multiplied, or divided by any ſame third number, the products or quotients will be proportional to the numbers ſo multiplied or divided. Let the numbers be 2 and 4, each to be multiplied by 3; then $3 \times 2 = 6$, and $3 \times 4 = 12$, and $2 : 4 :: 6 : 12$; or if 18 and 15 be divided each by 3, the quotients will be 6 and 5, and $18 : 15 :: 6 : 5$.

The powers or roots of proportionals will likewiſe be proportionals.

$2 : 4 :: 3 : 6$ root
$4 : 16 :: 9 : 36$ ſquare
$8 : 64 :: 27 : 216$ cube

Hitherto we have conſidered the doctrine of proportion, only with reſpect to numbers; but as any quantity may be repreſented by numbers, all that has been ſaid with regard to them may likewiſe be applied to any thing that can be augmented or diminiſhed. A line of 2 feet long has the ſame proportion to a line of 6 feet long that the number 2 has to 6, but the method of finding the proportions of lines, &c. to one another requires the knowledge of the principles of geometry, which ſhall be the ſubject of the next chapter.

CHAP.

CHAP. III. SECT. I.

Of GEOMETRY.

AS in arithmetick we treat of numbers by comparing them to one another, without confidering their relation to any particular quantity, fo in geometry we fhew how to compare quantities to one another, and find their proportions without arithmetical calculation.

Geometry may therefore be fitly called, The fcience of extenfion abftractedly confidered, without any regard to matter.

Geometrical Definitions.

Def. 1. Quantity is any thing that can be augmented or diminifhed, and may comprehend extenfion, weight, motion, &c. for one may be taken as greater or leffer, heavier, or lighter, fwifter, or flower, in relation to another, of things of the fame kind; but there can be no comparifon between quantities of different kinds; as hours and miles; for an hour is neither greater nor lefs, heavier nor lighter, &c. than a mile.

Def. 2. All things that are capable of extenfion are to be confidered either as lines, furfaces, or folids.

Def. 3. A line is a quantity of one dimenfion, where the length only is confidered.

Def. 4. A furface is a quantity confidered under two dimenfions, *viz.* length and breadth.

Def. 5. A folid is that which has three dimenfions, *viz.* length, breadth, and thicknefs, thefe two laft are fometimes called height and depth.

Def. 6. A point, in the mathematical fenfe, and in refpect of continual quantity, is that wherein neither of the foregoing dimenfions are confidered. It therefore confifts of no parts; for then it would be a folid, furface, or line. It is analogous to an inftant in time, which partakes neither of the paft or the future. The centers of circles, &c. in diagrams are not mathematical points, but fenfible objects whereby the underftanding, confidering them abftractedly, is affifted in mathematical fpeculations.

PLATE I. Fig. 1. *Def.* 7. Parallel lines are ſuch as are every where equally diſtant from one another, as A B, and C D; for if the lines A C, and B D, are equal, and are the ſhorteſt that can be drawn from the points A and B, to the line C D, the right lines A B, and C D, if infinitely produced, would be always equidiſtant, and conſequently would never meet; but if B D were ſhorter or longer than A C, the lines if produced would meet.

Fig. 2. *Def.* 8. An angle is the inclination of two lines that meet in a point, ſo as not to conſtitute one ſtrait line, and this will be the caſe of all ſtrait lines that are not parallel, as the lines A B, and C D, if they are produced; they will meet in the point O, which is called the angular point; the lines that form the angle are called the legs of the angle.

The quantity of any angle is not determined by the length of the legs that form it, but by the diſtance of any two points, one of them in one leg, and the other in the other, both equally remote from the angular point; now the greater this diſtance is, the greater will be the angle, and if there are two angles, as A O C, and *a o c*, and if O B, O D, *o b*, *o d*, are all equal to one another, then the angle A O C will be greater than the angle *a o c*, becauſe the line *b d* is ſhorter than the line B D, and when theſe diſtances are equal, the angles will be equal.

All ſtrait lines are meaſured by a rule, ſtaff, or ſcale of equal parts, as inches, feet, yards, &c. but angles are meaſured by arches of circles.

Fig. 3. *Def.* 9. Angles that are form'd by any line drawn from any point obliquely to another line, are called oblique angles; and thoſe angles that are form'd by drawing a line from any point directly the ſhorteſt way till it meets any right line, are called right angles.

Fig. 4. *Def.* 10. A perpendicular line is the ſhorteſt that can be drawn from a given point to any given line, that is, when the right line A C, ſtanding upon the right one B D, leans unto neither part, and makes the angles A C D, A C B on both ſides equal; thoſe angles are called right ones, and the line A C is perpendicular to D B.

Hence it is evident, that if in the line D B any two points, *s*, *t*, be taken equally diſtant from C, and any point, *f*, in the line A C, be equally diſtant from theſe two points, *s*, *t*, then will the line A C be perpendicular to D B.

Def.

Def. 11. A circle is a plain surface contained within one continued curve line, called the periphery, or circumference; and is drawn by fixing the point of one leg of a pair of compasses in any assignable part of a surface, and carrying the point of the other leg round till it arrives again at the place from whence it began to move: The fixed point is called the center, and is equally distant from all points in the circumference described by the other leg. PLATE I. Fig. 5.

Def. 12. The radius or semi-diameter is a strait line drawn from the center to any part of the circumference, of which there may be an infinite number all equal to one another.

Def. 13. The diameter is a line drawn through the center till it meets the circumference both ways, and is therefore always double the radius; there may be an infinite number of diameters all equal, they all intersect one another in one point, which is the center; so that when the intersection of any two diameters is found, the center of the circle is likewise found. The diameter is the longest strait line that can be drawn within the circle.

Def. 14. A chord is a strait line less than a diameter, drawn within the circle from any one point to another, both in the circumference, and cuts the circle into two unequal parts, called segments.

Def. 15. An arch is any part of the circumference.

If a circle be cut by a diameter, the segments are equal, and are called semicircles; and if a radius be drawn perpendicular to a diameter, it will divide the semicircle into two equal parts, called quadrants, each containing one quarter part of the whole circle. The space contained between two radii and an arch, is called a sector.

What an arch wants of a semicircle is called the supplement of that arch, and what it wants of a quadrant is called the complement thereof.

Def. 16. When an arch is the 360th part of the circumference it is called a degree; the 60th part of a degree is called a minute; the 60th part of a minute is called a second; and the 60th part of a second is called a third, &c.

It is by these degrees and minutes that all angles or arches are measured, in order to which it must be observed, that the circumference of every circle, whether great or small, is supposed

PLATE I. Fig. 55. to be divided into 360 degrees; but the length of the degrees of a small circle will always be less than the degrees of a large one in proportion as the radius of the one is less than the radius of the other.

To illustrate this, draw four semicircles from the center C; divide the outer one into 180 equal parts; now it is very plain that if strait lines are drawn from each of these divisions to meet all at the center, they will pass through all the inner semicircles, and so divide them into the same number of degrees.

A circle may be conceived to be formed by the motion of a strait line, for if one end be fastened at the center, the other end carried round till it returns to the same point where the motion began will describe the circumference; and if several points are taken in the line, as at *a*, *b*, *d*, *f*, they will each describe circumferences of circles, and C *a*, C *b*, C *d*, C *f*, will be the several semidiameters: this line is represented by a thread fastened at the center; draw the diameter A C B, this thread by its motion from the point A to the point 180 will form all the various angles that can be made by two strait lines, and will always make two angles with the diameter; from this description the following inferences may be deduced, *viz.*

Inf. 1. The greatest angle that can be made by two right lines will be less than 180 degrees, for if the thread be drawn through the point A, it will then lie upon the diameter A B, and make no angle with it, but if carried to 20, the angle from A will be 20 degrees, and the angle from B will be so much less than 180 degrees, *viz*, 180 — 20 = 160; and as it is carried through the points 30, 40, 50, 60, 70, 80, the angle from A increases, and that from B decreases; when it comes to 90, both angles will be equal to 90 degrees, and in this position the thread will be perpendicular to the diameter; after it passes 90, the angle from A will still be increasing, and the other decreasing till the thread comes to the point B, where it again falls in with the diameter, and makes no angle; and as the arch from A to B is a semicirele, and contains 180 degrees, it is impossible to make an angle of 180 degrees by two strait lines.

Inf. 2. A right angle contains 90 degrees, and the two lines that form it are perpendicular to each other; if these lines are produced they will make 4 angles, each 90 degrees.

Inf. 3.

Inf. 3. If a right line ſtands upon another right line, and makes two angles with it, thoſe two angles will be equal to two right ones, or 180 degrees: For if the line is perpendicular, they will be equal, by the preceding; if the line is oblique, as from any point in the circumference to the center, except the point 90, ſuppoſe from 40; then from A to 40 is the meaſure of one angle, and from B to 40 is the meaſure of the other angle, but the ſum of theſe two is 180; the angle from A to 40 is as much leſs as that from B is more than 90 degrees; the firſt is called an acute, and the laſt an obtuſe angle. Plate I. Fig. 55.

Inf. 4. If one of the two angles formed by the meeting of two lines is known, the other may be found by ſubtracting the known angle from 180 degrees, the remainder being the other angle.

Inf. 5. If ſeveral right lines are drawn ſo as to meet in one point in another right line, the ſum of all the angles will be 180 degrees, and the ſum of all the angles that can be made round a point will be 360 degrees.

Inf. 6. If there are ſeveral equal chords in the ſame circle, the arches ſubtended by theſe chords will be equal.

Def. 17. A triangle is a ſpace contained within three lines, which by their meeting form three angles: When the three lines are ſtrait, it is called a rectilineal triangle; when they are arches of the ſame circle, it is called a ſpherical triangle; when the three ſides are equal, the triangle is called equilateral; when only two are equal, it is called iſoſceles; and when all the three ſides are unequal, it is called ſcalene. Fig. 6. 7. 8.

Def. 18. Quadrilateral, or four ſided figures are ſuch as are limited by four lines forming four angles. Such as are rectilineal are either parallelograms or trapezia.

Def. 19. A parallelogram is a figure whoſe oppoſite ſides are equal and parallel, of which there are four kinds, *viz.* a ſquare, a rectangle, a rhombus, and rhomboides.

Def. 20. A ſquare is a figure limited by four equal ſides, all perpendicular one to another, as A B C D; that is, a quadrilateral figure, whoſe ſides and angles are all equal, is called a geometrical ſquare. Fig. 9.

Def. 21. A rhombus is a figure that hath four equal ſides, but no right angle, the oppoſite angles being equal, *viz.* E G H = E F H, and G E F = G H F, but all oblique. Fig. 10.

Def. 22.

PLATE I. *Def.* 22. A rectangle, or a right angled parallelogram, hath four right angles, and its opposite sides equal and parallel, *viz.*

Fig. 11. I M = K L, and I K = M L; this figure is often called an oblong, or long square.

Fig. 12. *Def.* 23. A rhomboides is an oblique-angled parallelogram, and the sides that form the angles are unequal.

Fig. 13. *Def.* 24. Every quadrilateral figure that has neither opposite sides, nor opposite angles equal, is called a trapezium.

A right line drawn from any angle (as D) in a four sided figure to its opposite angle (B), is called a diagonal, and divides the figure into two triangles (A B D and B C D).

Def. 25. A polygon is a figure that hath more than four sides. and may be either regular or irregular.

Fig. 14. *Def.* 26. A regular polygon has all its sides and angles equal, and may be inscribed in a circle, and all the angular points (*a*, *b*, *c*, *d*, *e*, *f*) will touch the circumference.

Regular polygons derive their names from the number of their sides or angles at the center of the circle they are inscribed in; thus a polygon of 5 sides is called a pentagon, of 6 sides a hexagon, of 7 sides a heptagon, of 8 sides an octagon, &c.

Fig. 15. *Def.* 27. An irregular polygon has many unequal sides, standing at unequal angles, as A B C D E F G.

All irregular polygons may be reduced to regular figures, by drawing diagonal lines in them; thus the polygon A B C D E F G, the diagonals G B, B F, F C, and C E being drawn, will be reduced to five triangles, *viz.* A B G, G B F, B F C, F C E, and C D E.

S E C T. II.

GEOMETRICAL PROPOSITIONS.

PROPOSITION I. PROBLEM.

Fig. 16. AT a given point in a right line, (as *o*), to make an angle equal to a given one, (A B C.)

Open your compasses to any convenient distance, and from B as as a center, describe an arch, as $s\,t$; with the same extent describe the arch $f\,g$, from the center o; then take the chord $s\,t$ in your compasses, and set it off from f to k in the arch $f\,g$, and draw the line $o\,k$, and so the angles A B C, and $k\,o\,f$, will be equal. PLATE I.

Hence an angle of any number of degrees may be made; for if with the radius of any divided circle we describe an arch (as $l\,m$) from the point C, as center, in a right line, as C B, and then take the quantity of the given angle (suppose 20 degrees) from the divided circumference of that circle by whose radius the arch was described, and set that off upon the arch from l to m, and draw the line $C\,m$, $m\,C\,l$ will be the angle required; this in effect is making one angle equal to another given one, for any angle may be found in the divided circle by drawing two semidiameters to two points of division, including the measure thereof in the circumference.

If the angle is already made, describe an arch as before to meet both sides of the angle, the extent of this arch measured upon the divided circumference will give the quantity of the angle.

PROPOSITION II. Theorem.

If two right lines intersect or cut one another, the opposite angles will be equal; the contiguous angles, as a and c, taken together will make 180 degrees (by *Inf.* 3, *Def.* 16), but if instead of c we take d, the sum of the angles a and d will likewise be 180 degrees, therefore c and d must be equal; for the same reason, as the angles c or d, added to the angles a or b, will make 180 degrees, it will appear that those opposite angles are likewise equal. Fig. 17.

To illustrate this by numbers, let the angle a be 30 degrees, this subtracted from 180 will leave 150, for the angle c, and the angle d will likewise by the same method be found 150, then 150 subtracted from 180 will leave 30 for the angle b.

PROPOSITION III. Theorem.

If a right line G H intersects two parallels, A B and C D, the opposite angles G E B, C F H, will be equal. Fig. 18.

To prove this, as the lines A B, and C D, are parallel by construction, they may be considered as one broad line, crossed by the

PLATE I. the line G H, then the Angles G E B, and C F H, will be equal by the preceding, and if the angles are equal, the lines will be parallel; for supposing A B not parallel to C D, let the line L M be drawn, and supposed parallel to C D, then will the angle G E M be equal to C F H; but this was supposed equal to G E B, therefore G E M = G E B, which is impossible.

Fig. 19. *Inf.* 1. If a right line, G H, cuts several parallels, A B, C D, E F, I K, it will make all the inward angles on the same side of the line equal, that is, the angles 1, 2, 3, will all be equal to the angle G R B, and therefore will be equal to one another; this will likewise hold true with regard to the angles 4, 5, 6, for they will all be equal to the angle A R G.

Fig. 18. *Inf.* 2. The alternate opposite angles will likewise be equal, that is, the angle A E H will be equal to D F G, which may be thus proved, G E B is equal to C F H, by this proposition; it is also equal to A E H, by *prop.* II. therefore A E H. is equal to C F H, and C F H. equal to D F G.

PROPOSITION IV. THEOREM.

Fig. 20. If the diameter or radius of a circle cuts any chord into two equal parts, it will be perpendicular thereto, and a perpendicular cutting a chord into two equal parts will pass through the center.

Let *b* be the middle of the chord A B, through which draw the radius C D, draw also the dotted lines C A and C B, those lines (by *Def.* 12.) will be equal, the points A and B are equally remote from *b* by supposition, and therefore the line C *b* is perpendicular to A B (by *Def.* 10.)

Let *b* be the middle of the chord, and C *b* perpendicular to it; I say it must pass through the center, for suppose it passes through the point *f*, then the lines *f* A, and *f* B, would not be equal, therefore the line *f* B would not be perpendicular to A B, which is contrary to the supposition.

Inf. 1. The perpendicular that divides the chord into two equal parts, also divides the arch into two equal parts, for the line D *b* being supposed perpendicular to A B, the chords D A and D B will be equal, and therefore the arches will be equal.

Inf. 2. If two chords are bissected by two perpendiculars, those perpendiculars will intersect one another in the center of the circle.

PROP.

PROP. V. THEOREM.

If a right line A D be drawn to touch the circumference in the point B, and from that point the chord B F be drawn; the angle A B F, made by the chord and tangent A D, is measured by half the arch F E B, of which B F is the chord. PLATE I. Fig. 21.

To demonstrate this, draw the radius C E perpendicular to the chord B F; this will bisect the chord and arch (by the preceding); so B E is half the arch F E B, and is the measure of the angle B C E; all that remains to be proved then is, that the angles A B F and B C E are equal.

Draw the radius C B to the point B, which will be perpendicular to the line A D, because the radius is the shortest line that can be drawn from the center to the circumference; therefore the angle A B F, together with the angle F B C, will be 90 degrees: Draw the diameter L G parallel to the chord F B, which will be perpendicular to E C; therefore the angles E C B, and B C L, taken together, will also make 90; but the angles B C L and F B C are equal, being alternate to the parallels B F and G L; and if each be subtracted from 90 degrees, the remainders E C B and A B F will be equal.

To illustrate this by numbers, let the arch F E B be 80 degrees; then the arch E B will be 40 degrees = the angle at the center (*r*); and because the angle E C L is 90 degrees, the angle *a* must be 50 degrees; again, the lines F B and G L being parallels, and the line C B crossing them, the angles *a* and *d* are alternate; the angle *d* must therefore be 50 degrees = *a*; now as the angle A B C is 90 degrees, the angle A B F must be 40, which is half the arch F E B.

In like manner the angle F B D will be measured by half the arch F G H L B; for the line F B makes two angles with the line A D, therefore (*by Inf.* 3. *Def.* 16.) the angle F B D will be 140 degrees, which is half the arch F G H L B, for the arch F E B is 80 degrees by supposition, which subtracted from 360, the whole number of degrees in a circle, there remains 280 for the arch F G H L B, the half of which is 140.

This proposition ought to be well attended to, for the following propositions will be clearly demonstrated by it.

PROP. VI. THEOREM.

PLATE I. Fig. 22. An angle at the circumference of a circle is measured by half the opposite arch; that is, the angle B A C is measured by half the arch B D C.

To demonstrate this, draw the line G F to touch the circle at the point A, then the three angles G A B, B A C, C A F, taken together will make 180 degrees *(by Inf. 5. Def. 16.)*; the three lines B A, A C, C B, divide the whole circumference into three arches, B E A, A I C, C D B; therefore the halves of those arches added together will make 180 degrees; but B A G is measured by half the arch B E A, and the angle F A C is measured by half the arch A I C (by the preced.) and what remains to compleat the 180 degrees is half of the arch C D B, which must therefore be the measure of the angle B A C.

Inf. 1. The three angles of any rectilineal triangle added together will make 180 degrees; for being inscribed in a circle, the sides will be chords, and will cut the whole circumference into three arches, the halves of which arches are the measures of their opposite angles; the angle at A is measured by half the arch B D C, the angle at B by half the arch A I C, and the angle at C by half the arch A E B.

Inf. 2. The angle at the center is double the angle at the circumference, if they stand upon the same arch; the angle at the center O is measured by the whole arch B E A, and is therefore double the angle at the circumference at C, which is measured by half the same arch.

Fig. 23. *Inf.* 3. If several angles are made at different points of the circumference, and all stand upon the same arch, they will all be equal; and if the arch is a semicircle, they will all be right angles.

The angles at the points *q p m* are all equal, because they stand upon the same arch A E B D C; and the angles at the points 3, 4, 5, 6, are all 90 degrees, because they all stand upon the diameter A D, which divides the circumference into two equal parts, and therefore 180 degrees each.

Fig. 7. *Inf.* 4. In an Isosceles triangle, the angles at the base are equal, that is, the angle at B is equal to the angle at C.

N. B. By the base of an isosceles triangle is meant the side which joins the two equal sides.

Inf.

In an equilateral triangle all the three angles are equal. PLATE I.

Inf. 5. If in two triangles two sides of the one be equal to two sides of the other, each to each respectively; and if the angle formed by those two sides be also equal, their bases and the other two angles will likewise be equal. Fig. 24.

In the triangles A B C, and D E F, let the side A C be equal to D F, the side A B equal to D E, and the angle at A equal to the angle at D; then the angle at E will be equal to the angle at B, and the angle at F equal to the angle at C, and the side F E equal to the side C B; for let each be inscribed in a circle; because the angles at D and A are equal, the arches on which they stand will be equal, of which F E and C B are the chords, and therefore they must be equal; and the chords D F and A C, being equal, the angles E and B must be equal; and the same may be said of the angles at F and C.

This demonstration supposes the circles to be equal, which may be thus proved: The arch subtended by the chord B C, *and that subtended by the chord* E F, *contain the same number of degrees, because the angles at* A *and* D *are equal; the other two arches subtended by the chords* A B *and* A C, *will contain as many degrees as those subtended by the chords* D E *and* D F; *but these chords are equal; therefore, the arches, and of consequence the circles, are equal.*

Inf. 6. If in two triangles two angles of the one be equal to two angles of the other, each to each respectively, and a side of each opposite to the same angle be also equal, the triangles will be equal in all respects; and if all the respective sides of any two triangles are equal, their angles, and consequently all their parts, will be equal.

PROP. VII. THEOREM.

If between two parallel lines A B and C D, any two lines, as *l m*, *r s*, are parallel to each other, those lines will be equal, and the distances between the points of intersection *r l*, *s m*, will likewise be equal. Fig. 1.

To demonstrate this, draw the perpendiculars *l n*, *r t*, which (*by Def.* 7.) will be equal; the angles at *n* and *t* being both right, are likewise equal; the angles at *s* and *m* are also equal (*by Inf.* 1. *Prop.* 3.); therefore the triangles *r s t*, and *l m n*, will be equal in every respect, (*by Inf.* 6. *of the preceding*) the sides,

 there-

PLATE I. Fig. 25. therefore, *l m* and *r s* will be equal; and if to the equal sides *m n*, *s t*, be added the line *n s*, *s m* and *t n* will be equal; but *t n* is equal to *l r*; therefore *l r* and *s m* will be equal.

PROP. VIII. THEOREM.

In any triangle as A B C, if the side A B be produced to D, and A D is double of A B; if the line D E is drawn parallel to to B C, then will A E be equal to twice A C, and D E equal to twice B C.

To demonstrate this, draw the line C F parallel to A D; then C F will be equal to B D (*by Prop.* 7.) = A B by supposition; the angle C F E is equal to the angle A B C; for the angle at D is equal to either of them (*by Inf.* 1. *Prop.* 3.) and the angle at A is equal to that at C; therefore the triangles A B C and C F E will be equal in all respects, *viz.* C E = A C, F E = B C = D F, and C F = A B.

After the same manner it may be proved, that if A D contain A B any number of times, A E will contain A C, and D E will contain B C the same number of times.

Inf. 1. In any triangle, as A B C, if a line D E is drawn parallel to any side, as B C; then A B will be to A D as A C is to A E, and as B C to D E.

To demonstrate this, let the line A D be $\frac{3}{4}$ of A B, then D E will be $\frac{3}{4}$ of B C, and A E $\frac{3}{4}$ of A C.

Divide the line A B into four equal parts in the points 1, 2, D, through which draw lines parallel to A C to meet the line C B in the points *f e b*; through these draw lines parallel to A B to meet the line A C in the points E, 7, 8; these lines will form 10 equal triangles; for the lines E *f*, 7 *e*, 8 *b*, being parallels, the angles B *b* D, *b e c*, *e f a*, *f* C E, will be equal, and the angles C *f e*, *f e a*, *e b c*, *b* B D, will likewise be equal; the bases D B, *c b*, *a e*, E *f*, being between the same parallels, are also equal; therefore the triangles are equal in all respects, and C *f*, *f e*, *e b*, and *b* B, will be all equal; but C *f* is equal to E *a* = *a c* = *c* D; now C B contains C *f* four times, and D E contains its equal E *a* only three times; therefore D E is $\frac{3}{4}$ of B C; again, A C contains C E four times, and A E contains its equal E 7 three times; therefore A E is $\frac{3}{4}$ of A C.

To illustrate this by numbers, let the side A C be 40, the side A B

AB 32, and the ſide BC 52, and let the line DE be drawn parallel to BC; if any one of the ſides of the triangle EAD be given, the other two may be found by the rule of three. For AD (being $\frac{3}{4}$ of AB) = 24 by ſuppoſition, it will be Plate I. Fig. 25.

$$\begin{cases} AB : AD :: AC : AE \\ 32 : 24 :: 40 : 30 \end{cases}$$

$$\begin{cases} AB : AD :: BC : DE \\ 32 : 24 :: 52 : 39 \end{cases}$$

If the ſide DE (= $\frac{3}{4}$ BC =) 39 by ſuppoſition be given,

Then $\begin{cases} BC : BA :: DE : DA \\ 52 : 32 :: 39 : 24 \end{cases}$

And $\begin{cases} BC : CA :: DE : EA \\ 52 : 40 :: 39 : 30 \end{cases}$

If the ſide AE (= $\frac{3}{4}$ AC) = 30 by ſuppoſition be given

Then $\begin{cases} AC : AB :: AE : AD \\ 40 : 32 :: 30 : 24 \end{cases}$

And $\begin{cases} AC : CB :: AE : ED \\ 40 : 52 : 30 : 39 \end{cases}$

Inf. 2. As the radius AD is to the radius AB, ſo is the chord DE to the chord BC; for the lines DE and BC are parallel, which may be thus proved. Fig. 26.

In the triangle ABC let the angle at A be 40 degrees, then the angles at B and C will be 70 degrees each; and as the angle at A is common to both triangles, the angles at D and E will likewiſe be 70 degrees each; therefore the lines BC and DE will make equal angles with the line AB, and conſequently will be parallel.

Hence, as the radius of any circle is to the radius of another circle, ſo is a chord of the firſt circle to the chord of the ſame number of degrees of the ſecond circle; and if two triangles are ſimilar, that is, the angles of one equal to thoſe of the other reſpectively; then the ſides about or oppoſite to the equal angles will be proportional; for being inſcribed in a circle, their reſpective ſides will be chords of arches of an equal number of degrees.

PROP.

PROP. IX. PROBLEM.

PLATE I. Fig. 27. To make a triangle whoſe ſides ſhall be three given lines (of which any two put together muſt be greater than the third.)

Let the given lines be A B, C D, E F.

Make the line G H equal to any of the given lines, ſuppoſe E F; then take either of the other two lines, as C D, and from the point G, as a center with that extent, deſcribe an arch, and with the extent A B from the point H, as a center, deſcribe another arch to cut the former in L; draw the lines G L and H L and the triangle G H L will be the triangle required.

PROP. X. PROBLEM.

Fig. 6. To make an equilateral triangle upon any given line, as A B.

Take the line A B in the compaſſes, with that extent deſcribe an arch from the point A as a center, and with the ſame extent from the point B, as a center, deſcribe another arch to cut the former, in the point C; draw the lines A C, and B C; then will the triangle A B C be equilateral, and each ſide equal to the given line A B.

The demonſtration of both theſe propoſitions is manifeſt from the conſtruction of the triangles.

PROP. XI. PROBLEM.

Fig. 28. To raiſe a perpendicular from any given point D, of a given line A B.

Take any point C, at pleaſure; from this as a center deſcribe a circle to paſs through the point D, and to cut the line A B in E; through the center C, from E, draw a line to cut the arch in F, and E F, will be the diameter of the ſemicircle E D F; draw the line D F and it will be the perpendicular required.

PROP. XII. PROBLEM.

From any given point F to let fall a perpendicular to a given line, as A B.

Draw a line at pleaſure from F to cut the line A B any where, as at E; biſſect the line F E; upon C, the middle of the line, as a center, with the extent C E = C F, deſcribe an arch to cut the line

line A B in D; draw the line F D and it will be the perpendicular required. PLATE I.

This is only the reverse of *Prop.* 11.

All that is necessary to illustrate these two propositions is to prove that in the triangle E D F the angle at D is a right one; this is evident from *Inf.* 3. *Prop.* 6. for the line E F being a diameter, and the angle at D being at the circumference, it is therefore a right one.

In practice there is no occasion to describe the whole circle; it will suffice to intersect the line A B in E, and draw a small arch, as *l g*, opposite to the points E and C.

When the point D is not near the end of the line, the perpendicular may be raised thus; from D set off two points L M equally remote from D; from M as a center, and with any convenient extent of the compasses exceeding M D, describe an arch, and with the same extent from L, as a center, describe another arch to cut the former in F; the line F D being drawn, will be the perpendicular required.

But if the point F be given; from F, as a center, describe an arch to cut the line A B in L and M; from which points, as centers, describe two arches to intersect each other in N; draw the line F N, which will cut the line B A in D; then will F D be perpendicular to A B.

PROP. XIII. PROBLEM.

To divide any given right line (A B) into two equal parts. Fig. 29.

From the points A and B, as centers, describe two arches to intersect each other in D; and with the same extent describe two arches to intersect each other in C; the line D C being drawn will cut the line A B into two equal parts in G.

PROP. XIV. PROBLEM.

Thro' any three given points, as A, B, D, not lying in a right line, to draw a circle. Fig. 30.

Join the points A B, B D with right lines; divide those lines into two equal parts; draw the perpendiculars F *m*, G *n*, and let them be produced till they intersect each other; the point of intersection C will be the center of the circle required; for as the circumference is to pass thro' the points A, B, D, the lines A B, B D must

PLATE I. must be chords; and if any two chords are bissected by two perpendiculars, they will, if produced, meet at the center, by *Inf.* 2. *Prop.* 4.

By this problem it is evident, that if any three points are taken in the circumference of a given circle, the center may be easily found; and if the segment of any circle is given, the circle may be compleated, by taking any three points in the given segment, and then proceeding as before.

PROP. XV. PROBLEM.

Fig. 31. Thro' a given point F to draw a line parallel to a given line A B.

Draw the line F G at pleasure; at the point F make the angle D F G equal to F G B, the line D F being drawn, will be parallel to the line A B; because the alternate angles are equal, by *Prop.* 3.

This may be done without drawing the line F G, thus; open the compasses till with one foot fixed in F, the other may cut the line any where, as in B; then with the same extent, from any other point in the line, as G, describe an arch; and with the extent G B from F, as a center, describe an arch to cut the former in D; the line F D will be the parallel required.

DEMONSTRATION.

In the triangles D G F and B G F, the side G B is = D F, and F B = D G, and F G common to both. Therefore the triangles are similar, and the alternate angles G F B and F G D are equal; and therefore the lines G B and D F are parallels, by *Prop.* 3.

PROP. XVI. PROBLEM.

Fig. 32. To find a line which shall be a fourth proportional to three given lines as A B, C D, E F.

Upon the line L M take L *g* = A B, and draw the line *g b*, making any angle at pleasure with the line L M, and make *g b* = D C. Set off E F from L to N, and from the point N draw a line parallel to *b g*; thro' the point *b* draw the line L O, then will O N be the fourh proportional required.

For

For in the triangle L N O, $g\,h$ is parallel to N O by construction, therefore L g (= A B) : $g\,h$ (= C D) : : L N (= E F) : N O. (by *Inf.* 1. *Prop.* 8.). PLATE I.

If it was required to find a third proportional to two given lines (as A B and C D), proceed in the same manner, only take L I = $g\,h$, and draw the line I K parallel to $g\,h$, then it will be

L g : $g\,h$: : L I : I K the third proportional required.

PROP. XVII. PROBLEM.

To find a mean proportional betwixt two given lines as A B, C D. Fig. 33.

Upon the line L M set off L N = A B and N O = C D, bisect the line L O in C, upon which, as a center, with the extent C O = C L, describe a semicircle from the point N, &c.

From the point N erect a perpendicular, this will intersect the semicircle in the point E; the line E N being drawn, will be a proportional to the lines L N = A B, and N O = C D.

DEMONSTRATION.

The triangles L E O and L E N are similar, for the angle L E O is a right one (by *Inf.* 3. *Prop.* 6), the angle L N E is also right by construction; and the angle at L is common to both, therefore the angle L E N is equal to the angle at O, the triangles L E N and E N O are also similar, for they have each a right angle at N, and the angle at O in the one is equal to the angle L E N in the other; therefore the angle at L will be equal to the angle N E O; therefore L N : N E : : N E : N O by *Inf.* 2. *Prop.* 8.

PROP. XVIII. THEOREM.

The radius of any circle, is equal to the chord of 60 degrees of the same circle.

DEMONSTRATION.

Upon the radius A C make an equilateral triangle A B C, then (by *Inf.* 4. *Prop.* 6.) the angles at A, B, and C will be all equal, consequently 60 degrees each, but A B is a chord of the arch that mea- Fig. 34.

PLATE I. Fig. 35. measures the angle at C, which is 60 degrees; therefore the chord A B of 60 degrees, is equal to the radius A C or B C.

PROP. XIX. THEOREM.

Parallelograms or triangles, having the same or equal bases, and being between the same parallels, are equal.

DEMONSTRATION.

Fig. 35. Let the parallelograms be A B C D, and C E D F.

Because the sides C E and D F are equal, and parallel by construction; the angles A E C and B F D are equal (by *Prop.* 3.), and because A B and E F are also equal, if B E is added to each, then A E must be equal to B F, and as A C is equal to B D, therefore the triangles A E C and B F D are equal in every respect, and if the common triangle B E O is taken away from each, the planes A B C O and E F D O will remain equal; to each of which planes, if we add the triangle C O D, the parallelograms A B C D, and C E D F become equal.

Hence it appears that any two triangles (as C D A, C D E) standing upon the same base (C D), and between the same parallels (A F, C G) are equal; for they are halves of parallelograms of the same base and altitude (as A B C D and C D E F).

It is evident from this proposition, that we may make a right angled triangle equal to any given oblique one, for if we draw a line, as A F parallel, to either of the given sides, as C D, through the opposite angular point E, and erect a perpendicular at either end of the same side C or D, to meet the parallel in A or B; then will the right angled triangle C B D = C A D, be equal to the oblique one C D E.

Fig. 10. 12. It is likewise evident from hence, that the rhombus E F G H, is equal to the rectangle E F x z, and that the rhomboides N O P Q, is equal to the rectangle N n P p; and consequently that, by the same method we may make a rectangle equal to any given parallelogram.

PROP. XX. THEOREM.

Fig. 36. In a right angled triangle A B C, the square of the longest side A C (called the hypothenuse) is equal to the squares of both the

the other ſides A B, B C (called the baſe and perpendicular) taken together.

DEMONSTRATION.

The ſquare A C H I, whoſe ſide is the hypothenuſe A C, is equal to the rectangles A H F G, and F G I C taken together; therefore all that we have to prove is, that the ſquare B C D E is equal to the rectangle FGIC, and the ſquare A B K L to the rectangle A H F G.

Firſt it is evident, that the triangle B C I is one half of the rectangle F G I C, it is alſo equal to the triangle B C D, becauſe it ſtands upon the ſame baſe B C, and between the ſame parallels B C, E D; now the triangle B C D = B C I, is one half of the ſquare B C D E, therefore the whole ſquare B C D E, is equal to the whole rectangle F G I C, each being double the equal triangles B C I, B C D.

In like manner, the triangle A B H = half the rectangle A G H F, ſtanding upon the ſame baſe A B and between the ſame parallels A B, L H, is equal to the triangle A K B, which is one half of the ſquare A B K L; therefore the ſquare A B K L, and the rectangle A H F G being double of the equal triangles A B K, A B H, are equal betwixt themſelves.

PLATE I.

CHAP. IV.

Of the Construction and Mensuration of Geometrical Figures.

HAving explained the principles of geometry, we come now to ſhew their uſe in the conſtruction, and menſuration of all plain figures, which we ſhall reduce to three claſſes, *viz.*

1. Thoſe that are limited by three right lines, called triangles.
2. Thoſe that are limited by four right lines,called quadrilaterals.
3. Thoſe that are limited by many right lines, called polygons.

Mathematicians ſuppoſe all plain figures that incloſe any part of ſpace, to be comprehended under one of theſe denominations, for curvilineal figures are ſuppoſed to be limited by an infinite number of right lines; we ſhall begin with triangles, as no figure or ſpace bounded by right lines, can have fewer than three ſides.

SECT. I.

Rectilineal Trigonometry, or the Construction of Right lined Triangles.

TRigonometry is that ſcience by which we learn to meaſure the ſides and angles of triangles, and is one of the moſt uſeful branches of the mathematicks; for as every triangle conſiſts of ſix parts, *viz.* Three ſides and three angles, and it often happens that ſome of them are unknown, yet if any three of them, provided one be a ſide, be known, the other three may be found by this art, either geometrically by ſcale and compaſſes, or by an arithmetical calculation. It is either rectilineal or ſpherical, we ſhall only treat of the firſt.

Fig. 5. In order to delineate any triangle, it is abſolutely neceſſary to have a ſcale of equal parts to meaſure the ſides by, and a line of chords for meaſuring the angles, theſe and the lines of ſines, tangents, and ſecants, and ſeveral others, are laid down upon the plain ſcale, we ſhall firſt explain theſe terms, and then ſhew how to conſtruct the lines.

Plate I.

DEFINITIONS.

1. The RIGHT SINE of any arch, is a perpendicular line drawn from one end of it, to a radius or diameter drawn to the other end; A D is the right ſine of the arch A B, or it is half the chord of double that arch; for taking the arch B J equal to A B, A J will be the chord of the arch A B J; of which the ſine A D is one half. Fig. 37.

2. The SINE COMPLEMENT of an arch, is that part of the radius intercepted betwixt the right ſine and the center; thus, C D is the ſine-complement of the arch A B, for it is equal to F A; the right ſine of the arch E A, which is the complement of the arch A B.

3. The VERSED SINE, is that part of the radius, intercepted between the right ſine and the circumference; D B is the verſed ſine of the arch A B.

4. A TANGENT to a circle, is any right line ſo drawn as to touch the circumference in any point (B), and if to this point be drawn a radius, it will be perpendicular to the tangent; thus B M is a tangent to the circle in the point B, and the tangent of any arch, as A B, is that part of the line B M, intercepted between the point B; at that end of the arch to which the radius is drawn, and the point H, where a line drawn from the center C, thro' A at the other end of the arch, interſects the tangent line B M; therefore H B is the tangent of the arch A B.

5. The SECANT of an arch, is a ſtreight line drawn from the center thro' one end of the arch, produced till it meet a tangent drawn from the point B at the other end of the arch; ſo H C is the ſecant of the arch A B.

The ſine, tangent, and ſecant of the complement of an arch, are called the ſine-complement, tangent-complement, ſecant-complement, or co-ſine, co-tangent, co-ſecant; C R is the co-ſecant; E R is the co-tangent; A F the co-ſine of the arch A B, and the ſine, tangent, and ſecant of the ſupplement of an arch, are the ſame with thoſe of the arch, for being drawn according to the definitions, there reſults the ſame line.

Hence the ſine of 90 degrees is equal to the radius, for it is half the diameter by the definition of a ſine, and is the greateſt of all the ſines; the ſine of 30 degrees is half the radius, for it is half the chord of 60 degrees, which was proved to be equal to the

PLATE II. the radius; the tangent of 45 degrees is equal to the radius, for let the angle N C E, or N C K be 45 degrees, N K the tangent, and E C the radius, are parallel, being both perpendicular to the diameter K B; but N E and K C are alſo parallel, being both perpendicular to the diameter E C; and therefore the tangents N K and N E are equal to the radius.

Conſtruction of the Lines on the plain Scale.

1. THE LINE OF EQUAL PARTS.

Fig. 1. This may be made of any length, as A B, and divided firſt into ten equal parts deſtinguiſhed by the figures 1, 2, 3, *&c.* Then each of thoſe diviſions muſt be ſubdivided into 10 equal parts, ſo the whole line will be divided into 100 equal parts; the figures 1, 2, 3, *&c.* denoting 10, 20, 30, *&c.* of thoſe parts, there will be no neceſſity for numbering the ſmall diviſions betwixt the figures; but for the better illuſtration of the ſcale, we ſhall diſtinguiſh thoſe between A and 1, by the letters of the alphabet. Any number leſs than 100 may be readily found on this line, for if it be leſs than 10, count as many ſmall diviſions beyond the beginning of the line or point A, as there are units in the number, as, if 6 were required, it will be at the point *f*; if 60, at the figure 6; if 66, count 6 ſmall diviſions beyond the figure 6, and you will have the point required.

If the number exceed 100, the ſmall diviſions muſt be ſubdivided again each into 10, but the length of our line will not admit of theſe; we ſhall therefore make uſe of diagonals, by which means any number under 10,000 may be had with as great certainty, as if the line A B were actually divided into 10,000 equal parts.

To perform this, let there be 50 lines at equal diſtances from one another, drawn parallel to A B, and having compleated the parallelogram A B C D, let the lower line C D be divided exactly like the upper line A B, into 100 equal parts, and numbered 1, 2, 3, *&c.* and draw the diagonal A *a*.

Now the portions of the parallels, 10, 20, 30, *&c.* intercepted betwixt this diagonal and the perpendicular A C, will be 1 tenth, 2 tenths, 3 tenths, *&c.* of the line C *a*; for in the triangle A C *a*, let C *a* be the baſe, then the portions of the parallels 10, 20, *&c.* will be baſes of as many triangles, ſimilar to the triangle A C *a*; therefore A C : C *a* :: A 10 : the portion of the parallel 10, inter-

intercepted betwixt the diagonal A *a* and perpendicular A C, but Plate II. Fig. 1.
A 10 is the tenth part of A C by construction, therefore the portion of the parallel 10, is the tenth part of C *a*, or the thousandth part of the whole line C D or A B; the figures 1, 2, 3, *&c.* denote 100, 200, 300, *&c.* and *a*, *b*, *c*, 10, 20, 30, *&c.* of those parts.

Again, if there were 9 parallels drawn betwixt the line A B and the parallel 10, it is plain the portion of that next to the line A B, intercepted betwixt the diagonal C *a*, and perpendicular A C, would be the tenth part of the portion of the parallel 10, intercepted betwixt the same lines, or the 10000th part of the whole line C D or A B. In the plate we have only four intermediate parallels numbered 2, 4, 6, 8; so their portions intercepted betwixt the diagonal C *a*, and perpendicular A C, will be 2, 4, 6, 8, of those parts.

By these means the lines A B and C D, are in effect, divided into 10,000 equal parts; the figures 1, 2, 3, *&c.* denote 1000, 2000, 3000, *&c.* the small divisions on the lines A B and C D, betwixt those figures will be hundreds; the tens are not transfered to the lines A B and C D, but may be had at the intersections of the parallels 10, 20, 30, *&c.* with the diagonals. If the units Fig. 6, 7. 8, are even, they may be found at the intersection of the diagonals, and the intermediate parallels, but if they are odd, as 1, 3, 5, *&c.* half the distance in the diagonal, must be taken betwixt those parallels. To compleat the scale, there must be diagonals drawn from all the divisions in the line A B to the line C D, parallel to that drawn from the point A in the line A B, to the point *a* in the line C D.

It will be very easy to find any number under 10,000 upon this scale, as in the following example, *viz.*

For units, look for the parallel below the line A B, betwixt the point A and the parallel 10; thus, to find 6, look for the third parallel, its intersection with the diagonal A *a* (which is the first of all the diagonals) will be the point required; this parallel in Fig. 9. the plate is numbered 6, but the odd numbers must be found betwixt the parallels; for instance, the number 7 will be in the same diagonal A *a*, between the parallel 6 and the parallel 8.

If 10, 20, 30, *&c.* be required, look for the intersections of Fig. 10. the parallels so numbered, with the first diagonal A *a*, and you will have the point required. If the given number is 12, first

and

PLATE II. Fig. 1. find 10, and the interſection of the parallel next below that, with the ſame diagonal will be 12; the number 25 will be in the middle of that part of the diagonal betwixt the ſecond and third parallel below the parallel 20; ſo that if the number be leſs than 100, it will be in the firſt diagonal; if it exceeds 100, and is leſs than 200, it will be in the ſecond diagonal; and if leſs than 1000, it will be in ſome of the diagonals, drawn from the ſeveral points *a*, *b*, *c*, *d*, *&c.* between A and 1, in the line A B to the line C D, thus the number 600 will be the ſixth ſmall diviſion from A towards 1; but if 606 is required, it will be found at the interſection of the third parallel (numbered 6) with the diagonal *f g*; and where the ſame diagonal interſects the parallel, 60 will be the point for 660; and the middle of that part of the ſame diagonal, betwixt the third and fourth parallel below that of 60, will be the point for 667.

The number 6000 will be at Fig. 6. in the line A B; 6700 at the ſeventh ſmall diviſion beyond Fig. 6. in the ſame line; the number 6750 will be at the point where the diagonal drawn from the point 6700 in the line A B, interſects the parallel 50; and where the ſame diagonal cuts the third parallel below the parallel 50, will be the point for 6756.

The diagonals upon *Gunter*'s ſcales, conſiſt only of eleven parallels; the upper and lower are graduated, the one into inches, numbered 1, 2, 3, *&c.* to 9; the other into half inches, numbered 1, 2, 3, *&c.* to 18; ſo there are no ſmall diviſions betwixt the figures, but there is an inch before the beginning of one line, and half an inch before the beginning of the other line, divided into ten equal parts in the upper and lower lines; and diagonals drawn as in that, in the plate, which will divide the inch and half inch each into 100 equal parts; ſo that if the ſpaces betwixt the figures be 100, thoſe betwixt the diagonals will be tens, and the units will be at the interſection of the diagonals and intermediate parallels; one example will ſuffice to ſhew how to take off any number by this ſcale: ſuppoſe 748, firſt find the figure 7, and obſerve where the perpendicular, from that point, interſects the eighth parallel from it, there fix one foot of the compaſſes, extend the other to the interſection of that parallel with the fourth diagonal, and you will have the extent for 748.

There are ſeveral other lines of equal parts which are graduated, and numbered exactly as the upper or lower line of the diagonal

gonal ſcale, and the diviſions and ſubdiviſions are in tens; but as one inch is the twelfth part of a foot, the diagonal ſcales for feet and inches, conſiſt of ſeven parallels, the upper and lower lines are numbered as in the other diagonals; the ſpaces betwixt the figures are to repreſent feet, which ſuppoſe one inch in the upper line S T, and half an inch in the lower line N O: They are numbered 1, 2, *&c.* the one contrary to the other, as upon moſt of the ſhipwrights rules. Set off half an inch to the right from O, at the beginning of the lower line to *o*; and one inch to the left from S, at the beginning of the upper line to *m*; then draw two diagonals, to be diſtant from one another one inch on the upper line, and to meet in the lower line at the middle of the inch; and at the other end draw two diagonals, to be at the diſtance of half an inch from one another in the lower, and to meet at the middle of the half inch in the upper line. It is evident the interſections of theſe diagonals will repreſent inches. PLATE II. Fig. 4.

The other lines on the ſcale are taken from the equal diviſions of a circle, and are conſtructed in the following manner, *viz.* Fig. 5.

1. With the radius you intend for your ſcale at C as a center, deſcribe a ſemicircle, A D B; then upon the center C erect the perpendicular C S, which will divide the ſemicircle into two quadrants A D, D B, and draw the right lines A D, D B.

2. Divide the quadrant B D into 9 equal parts; each of thoſe diviſions will contain 10 degrees, as a quadrant or fourth part of a circle contains 90 degrees; place one foot of the compaſſes in B, and with the other transfer the ſeveral diviſions from the circumference to the right line B D, then will B D be a line of CHORDS.

3. Upon the point B erect the perpendicular B T; then draw lines from the center C through the ſeveral diviſions 10, 20, 30, *&c.* of the arch B D to meet the line B T, and number the ſeveral points of interſection 10, 20, 30, *&c.* the ſame as the arches; then will B T be a line of TANGENTS.

4. From the points 10, 20, 30, *&c.* in the arch B D, let fall perpendiculars to the radius C B; thoſe ſeveral perpendiculars will be the ſines of thoſe arches, and will divide the radius C B into a line of SINES, which muſt be numbered 10, 20, *&c.* from C towards B; for ſince the perpendicular let fall from the point 80, to the radius C B, is the ſine of 80 degrees; C 10 will be the ſine of 10 degrees (by *Def.* 2. of this.) Fig. 30.

Plate II. 5. If the same line be numbered from B towards C, it will become a line of VERSED SINES.

Fig. 5. 6. With one foot of the compasses in C, extend the other to the several divisions of the tangent line B T, and transfer those extents to the line C S; then will C S be a line of SECANTS.

As the shortest secant that can be drawn, is longer than the radius, and the longest sine is equal to the radius; the line of secants on the scale, is on the same line with the sines.

7. As the angle, which the diameter A B makes with any line drawn from the point A, to any point in the circumference, is one half of the angle, that the radius B C makes with a line drawn from the center C to the same point (by *Prop.* 6. *Chap.* 3.). Right lines drawn from the point A to the several points 10, 20, *&c.* in the arch B D, will divide the radius C D into a line of SEMI-TANGENTS, or tangents of half those arches.

8. Divide the quadrant A D into 8 equal parts, and with one foot of your compasses in A, transfer the several divisions to the right line A D, then will A D be a line of rumbs hereafter to be spoken of.

Fig. 6. All these lines are transferred from the semicircle to the scale where they are drawn parallel to one another, and distinguished by the initial letters of their proper names, being what is called a plain scale.

The line of equal parts, and the line of chords, will be sufficient for the construction of any rectilineal triangle, but we think it necessary here to shew the uses of the other lines.

PROB. I.

Plate I.

To make or measure any Angle by the Line of Chords

Fig. 39. 1. From the point C, let it be required to draw a line that shall make an angle of 40 degrees with the line C G.

With the extent of 60 degrees taken from the line of chords (which is always equal to the radius by *Prop.* 18. *Chap.* 3.) describe an arch; then take the extent of 40 degrees from the same line, and set it off upon the arch from A to D, and the angle A C D will be 40 degrees.

2. When an angle is given to be measured with the same extent, *viz.* 60 degrees of chords; from the angular point describe an arch as before, to intersect the lines that form the angle; the length

length of the chord of that arch intercepted between the lines, PLATE measured on the line of chords, will give the number of degrees I. that the angle contains.

If the given angle is greater than 90 degrees, it may be made or measured at twice. Thus, suppose the angle 130 degrees; first take the chord of 90, and set it off from K to E, then take the chord of 40 degrees, (the remainder) and set it off from E to A, then will the angle A C K be 130 degrees: Any other two chords whose sum is 130 degrees, would have answered the same purpose. But as every semicircle contains 180 degrees; when the given angle exceeds 90 degrees, it may be very readily made, by setting off its complement to 180 degrees, from the extremity of the diameter: Thus, if we set off 50 degrees of chords, from B to A, and draw the line A C, we have the angle A C K = 130 degrees as before. When such an angle is given to be measured, if that extent is taken, and substracted from 180 degrees, the remainder will be the quantity of the arch.

PROB. II.

To find the Sine, Tangent, or Secant of any Arch; the quantity of the Arch, and the length of the Radius being given. Fig. 38.

Let the angle be 40 degrees, and the radius 100.

Now as this is the length of the radius, by which those lines on the scale were constructed; take 40 from the line of sines, that extent measured on the line of equal parts, into which the radius is divided, will give the length in numbers, of the sine of 40 degrees; and by the same method, we have the tangents and secants of any arch; but when the radius is not the same, observe the following method.

Suppose the angle 40 degrees, as before, and the radius 88 inches.

Make D C A an angle of 40 degrees by the preceding, and produce the lines C D and C A; then take 88 from any scale of equal parts, and with that extent from the angular point C, describe an arch, as L B; from L let fall the perpendicular L M, which will be the sine; the perpendicular B T will be the tangent, and C T the secant of the arch L B; each of which measured, upon the same scale of equal parts as the radius was taken from, will give the sine, tangent, and secant of the arch in numbers.

PROB. III.

PLATE I. Fig. 38.

The Length of the Sine, Tangent, or Secant; and the Quantity of an Arch given to find the Length of the Radius.

Let the angle be 40 degrees as before.

If the ſine is given; at any point as M, of a right line, as C N; erect a perpendicular, and make M L equal to the given ſine; then at L make an angle of 50 degrees, the complement of the given angle, and draw the line L C, which will be the radius required.

If the tangent is given, make B T equal to it, and make the complement angle 50 at T; ſo the line B C will be the radius.

If the ſecant is given, make C T equal to it, and make an angle of 40 degrees at C, and one of 50 at T, the line T B will interſect the line C N in B; then will C B be the radius required.

PROB. IV.

The Length of the Sine, Tangent, or Secant; and the Length of the Radius given; to find the Quantity of an Arch.

Let the radius be 88.

If the ſine is given, at any point, as M, of a right line, as C N, erect a perpendicular, make M L equal to the given ſine; with the given radius 88, from L, as a center, deſcribe an arch to interſect the line C N in C, and draw the line C L; then will L C M be the angle required.

If the tangent is given, erect a perpendicular at any point, as B, of the line C N; make T B equal to the given tangent; from B, as a center with the given radius, deſcribe an arch to interſect the line C N in C; draw the line C T; then will T C B be the angle required.

If the ſecant is given, ſet off the given radius from C to B, at which point erect a perpendicular; then with the extent of the given ſecant, from C as a center, deſcribe an arch to interſect the perpendicular in T; draw the line C T; then will T C B be the angle required.

DEMONSTRATION.

Draw the perpendiculars D E and G A, the one will be the ſine,

sine, and the other the tangent of the arch A D, (by *Def.* 1 and 4. PLATE
of this.) and C G the secant of the same arch; for C A = C D I.
is the radius of the circle from which those lines are constructed. Fig. 38.

C D : D E :: C L : L M, C A : A G :: C B : T B, and
C A : C G :: C B : C T by *Prop.* 8. *Chap.* 3.

But C D and C A, are radii of the arch A D; and D E the sine, G A the tangent, and G C the secant of the same arch; therefore L M will be the sine, T B the tangent, and C T the secant of the arch B L, of which C B is the radius.

PROB. V.

To construct a right lined Triangle, three things being known, one of which must always be a Side.

This will admit of four different Cases.

1*st.* When the three sides are given, the triangle may be constructed, as already shewn *Chap.* 3. *Prop.* 9.

2*d.* Given two sides, (C A and C F) and the angle (C) included by those sides, to delineate a triangle (A F C).

First make the given angle at C; then set off C A and C F the Fig. 40.
given sides from the angular point, and draw the line A F.

3*d.* Given two sides A F and C F, and the angle (C) opposite to one of them.

Make the angle; as before, then from the angular point C set off one side C F, from F as a center; with the extent F A, the other given side, describe an arch to intersect the line C G, which will be in A or B, and C F A or C F B, will be the triangle required. So that as this case will admit of two solutions; we must know whether the angle opposite to the other given side is acute or obtuse, before the answer can be determined; for if obtuse, the least, if acute, the greatest will be the required triangle.

4*th.* Given one side (A C), and two angles (A and C).

Draw the given side A C; make one of the given angles at A, and the other at C, and produce the lines that form those angles, till they meet in F; then will A F C be the triangle required.

After the triangles are thus constructed, the unknown sides may be measured by the same line of equal parts, by which they were constructed, and the angles by the same line of chords; and although all triangles may be measured and delineated, as we have shewn in the solution of this problem, yet it is usual to divide them

PLATE I. them into two classes, *viz.* right and oblique angled triangles; from whence arises the division of trigonometry, into two parts, *viz.* right and oblique; the last of which contains only the foregoing four cases.

PROB. VI.

To construct a right angled Triangle.

Fig. 42. This will admit of six cases, occasioned by the different names, by which the sides are distinguished, *viz.* The hypothenuse, which is the side opposite to the right angle, as (A C). The perpendicular (C F) is one side, and the base (A F) the other side, meeting at the right angle. To distinguish one from the other, we shall draw the perpendicular up and down, and the base across the paper; the angle opposite to the base we shall call (simply) the angle, and that opposite to the perpendicular we shall call the complement angle.

Hence, as in a right angled triangle, one angle is always the same, being equal to 90 degrees; it may be said to consist of four variable parts, *viz.* the three sides, and an oblique angle; any two of which being given, by them the triangle may be constructed.

Fig. 41. *Case* 1. Given the hypothenuse A C, and the angle at C 50 degrees, to find the base and perpendicular.

Draw the line C G up and down the paper; then with 60 degrees of chords from the point C, describe an arch; make the angle at C 50 degrees, and set off the given hypothenuse from C to A; from the point A let fall the perpendicular A F, then will A F be the base, and C F the perpendicular of the triangle A F C, right angled at F.

Hence, if the hypothenuse of a right angled triangle be made the radius of a circle, by which the angles are to be measured, the base and perpendicular will become sines of their opposite angles; A F is the sine of the angle at C, and C F the sine of the angle at A, by the definition of a sine; and because the sine of an angle is the same with the sine of that arch, which is the measure of the angle; its length cannot be determined till the length of the radius is known; for which reason the same angle may have sines, differing infinitely in length from one another in proportion to the radii by which they are measured; so that this case is is the very same with *Prob.* 2. of this; for making the hypothenuse the radius of a circle, we can find the sine of any arch in the same manner as we have shewn in that problem. Plate I.

Case 2. Given the base A F, and the angle at C 40 degrees, required the perpendicular C F, and the hypothenuse A C. Fig. 42.

This may be done by *Prob.* 2. of this; for if the base of a right angled triangle be made the radius of a circle, the hypothenuse will be the secant, and the perpendicular the tangent of the angle at the base A, which being the complement of the given angle will be 50 degrees; this is also the same as *Case* 4. *Prob.* 5. of this for here is one side, and the angles given. Therefore draw the line A F across the paper, and make a right angle at F; from F to A set off the given base at A; make an angle of 50 degrees, and draw the line A C; then will A C F be the triangle required.

Case 3. Given the perpendicular C F, and the angle at C 50 degrees. Fig. 43.

This in effect, is the same with the preceding, and may be constructed as in *Prob.* 2. or *Case* 4. *Prob.* 5. of this; for if the perpendicular be made the radius of a circle, the base will be the tangent, and the hypothenuse the secant of the angle C. Therefore having set off the given perpendicular C F, make a right angle at F; at C make the given angle 50 degrees; draw the line A C; then will A C F be the triangle required.

Case 4. Given the base A F, and the hypothenuse A C, to find the perpendicular and the angles.

This and the two following cases may be done as in *Prob.* 4. of this; for either of the given sides may be made the radius of a circle, and the other sides will be sines, tangents, or secants; they may likewise be done as in *Case* 2, or 3. of *Prob.* 5. of this; for we have two sides, and either the included angle, or the angle opposite to one of the sides given; in this case the opposite angle is given. Therefore draw a line across the paper, and set off the given base from A to F; at the point F erect a perpendicular; then take the given hypothenuse A C with a pair of compasses, and from the point A, as a center, describe an arch to intersect the perpendicular in C; draw the line A C, and the triangle will be constructed.

Case 5. Given the perpendicular C F, and the hypothenuse A C, to find the base A F, and the angles.

This is exactly the same with the preceding, only calling what was then the base, now the perpendicular; for making the right angle

PLATE I. Fig. 43. angle at F as before, if we set off the perpendicular from F to C, and with the extent of the given hypothenuse A C, upon C as a center, describe an arch; it will intersect the line drawn perpendicular to C F in the point A, then will A C F be the triangle required.

Case 6. Given the base A F, and the perpendicular C F, to find the hypothenuse and the angles.

Here we have two sides, and the included angle given; therefore by *Case* 2. *Prob.* 5. make a right angle as at F; then set off the given base from F to A, and perpendicular from F to C, draw the line A C, and the triangle will be compleated.

It may now be presumed, that the reader, by a due attention to what has been said, may be able to construct any triangle when one side, and any other two parts are given; and it would be needless to give any more examples, as we shall have occasion to shew in another place, how to perform what has been done here geometrically, by arithmetical calculation: We shall therefore under this head only subjoin the following theorem, which should be well understood, and carefully attended to, *viz.*

THEOREM.

In a right angled triangle, any side may be made the radius of a circle, and the other sides may be found, by being considered as sines, tangents, or secants of the oblique angles.

Fig. 41. *Case* 1. If the hypothenuse A C is made radius, the base A F will be the sine, and the perpendicular C F the sine complement of the angle at C.

Fig. 42. *Case* 2. If the base A F is made radius, the hypothenuse will be the secant complement; and the perpendicular the tangent complement of the angle at C.

Fig. 43. *Case* 3. If the perpendicular C F is made radius, the hypothenuse will be the secant, and the base the tangent of the angle at C.

SECT.

SECT. II.

Construction of Quadrilateral Figures, &c.

PROB. I.

To make a Square whose Side shall be the given Line C D. PLATE I.

UPON one end of the line as D, erect a perpendicular; set off the given side C D from D to B; upon B and C, as centers; with the same extent, describe two arches, intersecting each other in A; and draw the lines A C, A B; then will A B C D be a geometrical square, all the sides and angles being equal. Fig. 9.

PROB. II.

To make a Rhombus whose Side shall be the given Line E F, and the Angle E 50 Degrees.

First make the angle at E 50 degrees; make E G and E F equal; then with the extent E F, describe an arch from the center F, and another from the center G, intersecting the former in H, and draw the lines G H, H F; then will E F H G be the rhombus required. Fig. 10.

PROB. III.

With two given Sides, as I K and K L, to make a Rectangle.

Erect a perpendicular at K, and set off the two given sides to I and L; from I, as a center, with the extent K L, describe an arch; and with the extent I K, from L as a center, describe another arch to intersect the former in M; draw the lines I M, L M, and the rectangle will be compleated. Fig. 11.

PROB. IV.

To Make a Rhomboides whose Sides N Q, O Q, and the Angle at O are given.

Make the angle at O, and set off the given sides to N and Q; Fig. 12.

 with

PLATE I. Fig. 12. with the radius O Q, from N as a center, describe an arch; then with the extent O N, from Q as a center, describe another arch to intersect the former in P; draw the lines N P, P Q, and the rhomboides will be made; and if from the points N and P, be let fall perpendiculars to the points *n* and P; the rectangle *N n P p* will be equal to the rhomboides N O P Q.

PROB. V.

To describe a regular Polygon, having the Length and Number of Sides given.

This is in effect, the same with *Case* 4. *Prob.* 5. of the preceding section, for every regular polygon may be inscribed in a circle, and if there be a radius drawn to every angle of the polygon, it will contain as many triangles as it has sides; so that if 360, the number of degrees in a circle, be divided by the number of sides, the quotient will be the angle at the center; and if that is subtracted from 180 degrees, the remainder will be the sum of both the angles at the base, which is the side of the polygon; and because the triangles have two of their sides equal, the angles at the base will also be equal; so that in every triangle we have one side, and all the angles given.

Fig. 14. Let *a b* be the side of a hexagon, then $360 \div 6 = 60$, and $180 - 60 = 120$, the sum of the angles at *a* and *b*, therefore each angle will be 60 degrees: Make therefore at *a* and at *b* an angle of 60 degrees, and produce the lines till they meet; from the point of their intersection describe a circle thro' *a* and *b*; the distance from *a* to *b* will divide the circumference into six equal parts, and when the chords are drawn to these points, we shall have the hexagon required. Here the angles at the base are the same with the angle at the center; consequently a polygon of 6 sides consists of as many equilateral triangles; but as all other polygons consist of as many isosceles triangles as they have sides; and the angles at the center, and circumference may be easily found by the foregoing method; there can be no difficulty in constructing them, as there is no more required, but to find the center of a circle which shall pass thro' the extremities of the given side.

PLATE I.

PROB. VI

To describe an Ellipsis.

An ellipsis is a figure contained under one curve line, called the periphery, but differs from a circle; for if there be two diameters drawn in a circle at right angles to one another, they must be of equal lengths, and all points in the circumference are equally distant from the center; but the ellipsis has two diameters of different lengths, at right angles to each other, T S the longest, is called the transverse diameter, and C E the shortest, the conjugate diameter; so that the circle is, as it were, flattened one way, and the radius T *x* reduced to *x* C = *x* E; and if all the sines in the semicircle are shortened in the same proportion, they will give the points through which the semi-periphery of the ellipsis must pass: In order to perform this, make the radius, or half the transverse diameter T *x* = *x* E, a side, and *x* E half the conjugate diameter = F G the base of a triangle *x* F G; from the circumference of the circumscribing circle, draw several lines, as *m*, *s*, *n*, *t*, perpendicular to the transverse diameter; transfer those sines to the side *x* F of the triangle, *viz.* make *x z* = *n t*, *x y* = *m s*, &c. draw *z b* and *y g* parallel to the base F G, transfer the extent *z b* from *n* to *p*, and the extent *y g* from *m* to *o*, then will *p* and *o* in the lines *n t*, *m s*, be two points thro' which the semi-periphery must pass; and if more lines are transferred from the semicircle to the side *x* F, we may by the foregoing method find a sufficient number of points, thro' which the semi-periphery may easily be drawn. Fig. 45.

If with the extent T *x* = S *x*, half the transverse diameter, and from the centers C or E, the extremities of the conjugate diameter, two arches are described, they will intersect the transverse diameter in the points F, *f*, each of which is called the focus of the ellipsis; and if from any point in the periphery a line be drawn to each of these points, the sum of those lines will always be equal to the transverse diameter, *viz.* F *a* + *f a* = F *b* + *f b* = F C + *f* C = F D + *f d* = F *e* + *f e* = T S; and this being the peculiar property of the ellipsis, any number of points may be found by the following method.

Having found the two focal points F, *f*, as before, with any extent of the compasses, so it be less than the transverse diameter,

 as

PLATE I. Fig. 45. as T i, from the focus F describe an arch; and with the extent S i, the remaining part of the transverse diameter, describe another arch to intersect the former in the point a, which will be in the periphery of the ellipsis; in the same manner you may find as many points as you please.

From what has been said it is plain, that if there be a thread equal in length to the transverse diameter, fastened to two pins one in each focus; and if a pencil, or any other convenient instrument be moved round within the thread, so as to keep it at its full extent, it will describe the true periphery of the ellipsis; and if both ends of the thread be fastened at the center, it will describe a circle; so that this figure may properly be said to have two centers, whereas a circle has but one.

PROB. VII.

To make a Circle equal to a given Ellipsis.

Find a mean proportional between the transverse, and conjugate diameters *by Prop.* 17. *Chap.* 3. and this will be the diameter of the circle required.

SECT. III.

Of Mensuration of plain Surfaces.

HAVNIG in the preceding sections shewn the construction of plain geometrical figures, and the method of reducing those that are oblique angled, to right angled ones of equal surfaces; we shall now briefly shew how to measure those surfaces, or the whole spaces contained within the circumscribing sides.

These spaces are always estimated, in squares of some assigned dimensions, as inches, feet, yards, &c. and the number of such squares contained in any figure, is called its area or superficial content.

Now as a square has four right angles it will necessarily follow, that all figures must be reduced to right angled ones before they can be measured geometrically, so that the whole of super-

superficial measure may be said to consist in finding the area of a square or rectangle; in order to which, let us suppose a rectangle as *a b c d*, to be one inch broad; it is plain it will contain as many square inches as it is inches in length, which in this case is 12, and the rectangle may be cut in 12 square inches, but if it were any broader, and of the same length, it would contain 12 times as many square inches as there are inches in breadth; so the square A B C D, of 12 inches long, and 12 inches broad, will contain 144 square inches, which is the area of a superficial foot, or square whose side is one foot; hence the following rules will appear very plain. PLATE I. Fig. 44.

PROB. I.

To find the Area of a Square or Rectangle, the Length and Breadth being given.

Rule. Multiply the length by the breadth, the product is the area; this will admit of two cases.

Case 1. If the length and breadth are taken by one kind of measure; as if there is a rectangle 48 inches long, and 24 inches broad; then $48 \times 24 = 1152$ the area in square inches. If the length and breadth are taken in feet, the area will likewise be in feet; the like may be said of yards, rods, *&c.*

Case 2. If the length is taken in one kind, and the breadth in another kind of measure; as suppose a plank 20 feet long, and 9 inches broad whose area is required.

Rule. Multiply the length by the breadth, and divide the product by 12; the quotient will be the area. $20 \times 9 = 180$; and $180 \div 12 = 15$ feet the area required.

The reason of dividing the product by 12 is obvious, for if the breadth were 9 feet, the area would be 180 feet; in this case the multiplier (9) is feet, whereas in the other it is but the 12th part of 9 feet, and of consequence the first product will be but the 12th part of the last, and therefore must be divided by 12, and the quotient will be the true product, which in this case will be less than the multiplicand, because the multiplier is a fraction, *viz.* $\frac{9}{12}$ and $20 \times \frac{9}{12} = \frac{180}{12} = 15$. We have been the longer upon this head, because we shall have occasion in another place to mention the measuring of plank; for it is always accounted as an oblong square, tho' it generally tapers, the breadth is taken in

PLATE I. in the middle, which will be exactly true when both edges of the plank are streight, as in *Fig* 11. *Plate* 1. the trapezium *n o p t*

Fig. 11. = the parallelogram I K M L, for the triangle *r n* M = triangle *r* I *o*; hence in measuring plank.

1 : length in feet	: :	breadth in feet	:	area in feet.
12 : length in feet	: :	breadth in inches	:	
144 : length in inches	: :	breadth in inches	:	

This may be easily applied to all artificers works, as wainscoting, paving, &c. 9 : length in feet : : breadth in feet : area in yards.

PROB. II.

To find the Area of a Triangle.

Rule. Multiply the base by half the perpendicular, (let fall

Fig. 35. from one one of the angles to the opposite side) the product will be the area required, as in the triangle D E F, let D F be 40 feet, and the perpendicular E M be 18 feet; the half of which is 9, the $40 \times 9 = 360$ the area in feet $= 20 \times 18$, and $360 \div 9 = 40$ the area in yards.

PROB. III

To find the Area of a Trapezium.

Fig. 45. 1. If the trapezium has two sides parallel; as *m n*, *s t*, let fall the perpendicular *t r*, and multiply it by half the sum of both the parallel sides; the product will be the area.

Note. We suppose the arch *s t* to be so small, that it may be accounted a right line; then the parallelogram *m n r t* + triangle *s r t* = trapezium *m n s t*, and the parallelogram's area = *r t* × (*n t*) half the sum of *n t* + *m r*, and the triangular area = *r t* ½ *r s*.

Fig. 13. 2. If none of the sides are parallels, as B C, D A, divide the given trapezium into two triangles, by drawing the diagonal B D, and draw the perpendiculars A *x*, and C *z*; then multiply the diagonal by half the sum of the two perpendiculars; the product will be the area required.

PROB. IV.

To find the Area of an irregular Polygon.

Fig. 15. *Rule.* Reduce the polygon (as A B C D E F G) into triangles (as

(as ABG, BGF, BFC, FCE, and CDE); find the areas of those triangles, and add them together; the sum will be the area of the whole trapezium.

PROB. V.

To find the Area of a regular Polygon.

It is evident that the area of every regular polygon is equal to the sum of the areas, of as many isosceles triangles as it contains sides, and that a perpendicular let fall from the center to any side, would be the radius of the inscrib'd circle; the area of the whole polygon may therefore be found by the following rule, *viz.*

Multiply the sum of all the sides by half the radius of the inscribed circle, or the radius by half the sum of the sides; the product will be the area required.

By this rule, it will be easy to find the area of any regular polygon, when the side, and the radius of the inscribed circle are both given: But as it sometimes happens that only the side, and sometimes only the radius of the circle can be practically measured, we shall lay down the proportions they bear to one another in most of the figures that occur in practice, which are calculated to a sufficient exactness in most cases, *viz.*

The Proportion that the Sides bear to the Radii of the inscribed Circles.

Equilateral triangle		as 1 to	0 . 288
Pentagon or polygon of	5 sides		0 . 688
Hexagon	6		0 . 866
Heptagon	7		1 . 038
Octagon	8		1 . 207
Nonagon	9		1 . 378
Decagon	10		1 . 538
Undecagon	11		1 . 703
Duodecagon	12		1 . 866

We shall give one example under this head, which we think will be sufficient.

Required the Area of an Octagon whose Side is 24.

As 1 : 1 . 207 : : 24 : 28, 968, the radius of the inscribed circle.
24 × 8 = 192 = sum of the sides.

Then

PLATE I. Then multiply 28.968 by 96 = half the sum of the sides; and the product will be 1780.928 the area required.

PROB. VI.

To find the Area of a Circle.

As mathematicians consider a circle as a polygon of an infinite number of equal sides, the circumference will be equal to the sum of all the sides; If this then be multiplied by half the radius, the product will be the area, the same as if it were a polygon; but before this can be done, the circumference must be found, by investigating the proportion it bears to the diameter, which has been calculated to great exactness by several eminent mathematicians, and is universally allowed to be as 1 is to 3.141592, &c. in practice, as 1 to 3.1416; hence the area of a circle may be found by the following rule, *viz.*

Multiply the diameter by 3.1416, and the product will be the circumference; this again multiplied by $\frac{1}{4}$ of the diameter (which is the same as half the radius) will give the area, which will be the same as if the diameter be squared, and the $\frac{1}{4}$ of the product multiplied by 3.1416, as will appear by the following:

EXAMPLE.

What is the Area of a Circle whose Diameter is 128.

$128 \times 3.1416 = 402.1248$; then $\frac{1}{4}$ diameter $32 \times 402.1248 = 12867.9936$; the area required: $= 128 \times 32 \times 3.1416$; for when several numbers are to be multiplied into one another, it will be indifferent in what order the operations are performed; but if instead of 32 we take $\frac{128}{4}$, which is equal to it, and multiply the diameter by this fraction, it will be $\frac{128 \times 128}{4}$ the square of the diameter 16384 divided by $4 = 4096$, also $128 \times 32 = 4096$, and $4096 \times 3.1416 = 12867.9936$, the area as before; now because 4 will always be a divisor, and 3.1416 a multiplier; we may take $\frac{1}{4}$ of 3.1416, which is 7854, for a multiplier, and then there will be no occasion for a divisor; and the area of any circle will be found by multiplying the square of the

the diameter by .7854, and the product will be the area 128 × 128 PLATE
= 16384 the square of the diameter, and 16384 × .7854 = I.
12867.9936 the area as before: Hence the areas of circles will be in the same proportion as the squares of their diameters: And because the square of 1 is 1, the area of a circle whose diame- will be .7854; or, if the circumference, which is 3.1416, be multiplied by a ¼ of the diameter (1) or .25, it will give .7854 the area as before.

PROB. VII.

To find the Area of a Sector.

Rule. Find the area of the whole circle; then say, as 360, the degrees contained in the whole circumference; are to the area of the whole circle; so is the number of degrees contained in any Fig. 20.
arch, as A D B to the area of the sector A C B D.

PROB. VIII.

To find the Area of the Segment of a Circle, as A *b* B D.

Rule. Find the area of the triangle made by the radii A C, B C, and the chord A B, which subtract from the area of the sector A C B D; the remainder will be the area of the segment A *b* B D.

But it often happens that we have the segment of so large a circle, that a small part of the circumference may be taken for a right line; and when the plane will not contain the radius, it will be difficult to know whether the curve be an arch of a circle, or of an ellipsis, or it may be neither; for as the periphery of the ellipsis falls within that of the circle, so there may several curves fall between them, or within the ellipsis, which mathematicians have given no rules to investigate. In such cases, we presume the following method may do for practice, and be pretty near the truth in most cases.

Draw several lines perpendicular to the chord, or right line, as T S, at equal distances from one another; suppose 1 foot. These will divide the whole surface, whether it be the segment of a cir- Fig. 45.
cle, or of any other curve, into as many trapezia, less one, as there are perpendiculars, and two right angled triangles besides; now the areas of all these added together, will give the area of

PLATE I. the whole segment; and as every one of the trapezia have two parallel sides, and every perpendicular is a side to two trapezia, or to a trapezium and a triangle; the sum of all the perpendiculars multiplied by the distance between each, will be the area of the whole; and if the distance be 1 foot, the sum of all the perpendiculars measured in feet, will be the area in feet of the whole space contained within the right line and curve.

Fig. 45.

But as one side of every trapezium is a curve; if this side should differ perceptibly from a right line, draw a chord to it, which will cut off a small segment; in which, if there be drawn other two chords to meet at the middle of the arch, we shall have a triangle, the area of which must be added to that of the trapezium; these segments will be greatest at each end, if it be an arch of a circle, which may be reduced to two or more triangles; but there are some curves that approach so near a right line at each end, that a small part may be taken for such, without any sensible error.

PROB. IX.

To find the Area of an Ellipsis.

As every ellipsis is equal to a circle, whose diameter is a mean proportional betwixt the transverse and conjugate diameters, and this is found by multiplying the transverse diameter by the conjugate, and extracting the square root of the product; the square of this mean diameter multiplied by .7854, will be the area of the ellipsis; hence the following rule:

Multiply the transverse diameter by the conjugate, and that product by .7854; the last product will be the area required.

EXAMPLE.

Let the transverse diameter be 48, and the conjugate 32.

Then $48 \times 32 = 1536$, and $1536 \times .7854$
$= 1206.3744$, area required.

It remains to be proved, that the mean proportional to the transverse, and conjugate diameters, will be the diameter of a circle equal to the ellipsis; for which take the following.

DE-

PLATE I.

DEMONSTRATION.

Let the ellipsis T C S E, be inscribed in a circle whose diameter is T S; then the area of the semicircle will be equal to the sum of the areas of all the trapezia contained in it; now the ellipsis contains the same number, but smaller trapezia, and the sum of their areas is the area of the semi-ellipsis; therefore the area of a circle, is to the area of the ellipsis; as a trapezium in the semicircle, is to the same trapezium in the semi-ellipsis; but a trapezium in the semicircle is to a trapezium in the ellipsis, as the transverse diameter is to the conjugate; by the construction of the ellipsis; hence as the area of the circle : area ellipsis : : T S : C E, and multiplying the two last terms by T S, their products will still be proportional to the two first terms; that is, as the area of the circle : area of the ellipsis : : T S × T S : C E × T S, but T S × T S, is the square of the diameter of the circle, and C E × T S is the square of the mean proportional. Fig. 45.

SECT. IV.

Of Mensuration of Solids.

A Solid is that which has three dimensions, *viz.* length, breadth, and thickness; and as the areas of all surfaces are estimated in squares, so are the contents of any solid estimated in cubes.

A cube is a solid limited by 6 equal square surfaces, like a die. If the side of the square be 1 inch, foot, yard, &c. the side of the cube will be the same; and the solid is said to be a cubic inch, foot, or yard. Fig. 46.

A parallelopipedon is limited by two parallel and equal squares, called its bases, and four rectangles for its sides, which may be called the heighth or length, and if they are perpendicular to the bases it is a right one; but when the sides are oblique to the bases, then it is an oblique parallelopipedon; the like may be said of the following figures.

A prism is limited by two parallel and equal polygons for its bases, and as many rectangles as the polygon has sides; if the bases be triangles, it is called a triangular prism, but if squares, it is a parallelopipedon, &c. Fig. 47.

PLATE I. Fig. 48. A pyramid is a priſm, taper'd to a point at the top; it has but one baſe, which is a polygon, and as many iſoſceles triangles as the polygon has ſides, meeting at the top, which is called the vertex; it is exactly one third part of a priſm of the ſame baſe and altitude.

Fig. 49. A cylinder has two equal and parallel circles for its baſes, and the ſpace betwixt them is limited by one curve ſurface. If one ſide of a rectangle be fixed as an axis, the oppoſite ſide moved round, will deſcribe the curve ſuperficies, and the other two ſides will deſcribe the baſes; of this form is a rolling ſtone.

Fig. 50. A cone is, in reſpect of a cylinder, what a pyramid is in reſpect of a priſm, and is exactly one third of a cylinder of the ſame baſe and altitude; it has but one baſe, which is a circle, and tapers to a point at the top, called its vertex, like a ſugar loaf. If the perpendicular of a right angled triangle be fixed immoveably as an axis, the hypothenuſe, turned round, will deſcribe the curve ſurface, and the baſe of the triangle will deſcribe the circular baſe of a cone.

A ſphere or globe, is a ſolid contained under one curve ſurface, every part of which is equally diſtant from one point, called its center, and may be formed by the revolution of a ſemicircle round the diameter. It is exactly two thirds of a cylinder, whoſe altitude and diameter of its baſe are equal to that of the globe.

In order to find the contents of theſe regular ſolids, let us examine how they may be compoſed; and if we may not be allowed to ſay, that a great many plain ſurfaces of one inch ſquare, and infinitely thin, laid upon one another, will conſtitute a cubic inch, becauſe the ſum of ever ſo many cyphers will not make one unit, yet it is very plain, that if ſeveral dies, or cubes of one inch, be laid upon one another, they will compoſe a parallelopipedon, containing as many cubic inches as it is inches in heighth; but if the heighth be only one inch, the ſolid will contain juſt as many cubic inches as the baſe contains ſuperficial. If the ſide of the baſe be 12 inches, the area will be 144, and it is plain it will take 144 cubic inches to cover this baſe. It will require 12 ſuch ſquares of one inch high to compleat the cube, ſo that a cubic foot will contain 1728 cubic inches, that is 144×12; hence the following rules may be deduced.

PROB.

PLATE I.

PROB. I.

To find the solid Content of a Cube, Parallelopipedon, Prism, or Cylinder.

Rule. Find the area of the base by the problems in the preceding section, which multiplied by the perpendicular distance betwixt the bases, gives the solid content in the same kind of measure as the dimensions are taken. Or multiply the length, breadth and thickness (all taken by one kind of measure) into one another; the product is the content in cubes of the same measure; hence 1 : area base : : length : content. or 1 : length : : depth × breadth : content.

PROB. II.

To find the solid Content of a Pyramid or Cone.

Rule. Multiply the area of the base by ⅓ of the height; the product is the content 3 : area base : : heighth : content.

PROB. III.

To find the solid Contents of a Globe.

Rule 1. Find the area of a circle whose diameter is equal to that of the globe. Fig. 56.

Rule 2. Multiply the area by double the diameter, and divide the product by 3; the quotient is the content of the globe. 3 : area circle × 2 : : diameter of the globe : content.

PROB. IV.

To find the solid Content of the Frustum of a Pyramid or Cone.

Note. If either of these be cut, by a plane, parallel to the base, the top cut off, will be a pyramid or cone, and the remaining part its frustum, which will have two bases; the small one is that of the cone or pyramid cut off; the great one, is the base of the whole cone or pyramid before it is cut: Hence, if we can find the content of each, and subtract the lesser from the greater, the remainder is the content of the frustum; so all that is wanting is to find the heighth of each, and to do this:

1st. Sub-

PLATE I. 1*ſt*. Subtract ½ the diameter of the leaſt from ½ the diameter of the greateſt baſe: then

2*dly*. Multiply the perpendicular diſtance betwixt the baſes by ½ the diameter of the greateſt baſe.

3*dly*. Divide this product by the difference of half the baſes, and the quotient is the heighth of the whole cone before it is cut, and ſubtracting the diſtance betwixt the baſes from this, we have the heighth of the cone cut off.

DEMONSTRATION.

Fig. 50. Let $nmAC$ be the fruſtum; draw the perpendicular sm; the line Cm is the difference of half the baſes, and the triangles mCs, and tCx are ſimilar; therefore $Cs : ms :: Ct : tx$, and $\frac{ms \times Ct}{Cs} = tx$, but tx is the heighth of the cone; therefore ms the heighth of the fruſtum.

PROB. V.

To find the ſolid Content of irregular Solids, which are limited by ſeveral curve and plain Surfaces.

To do this, let the ſolid be ſuppoſed to be cut by ſeveral planes, parallel to the baſe, and at one foot, or inch diſtance from one another. Every ſection will form two equal plain ſurfaces. Half the ſum of all the areas, including the areas of both baſes, will nearly be the content in feet or inches; but if the ſolid has no plain ſurfaces parallel to one another, let two ſmall parts be cut off, which may be meaſured as parts of a globe, cone or pyramid.

In all the foregoing problems it will be convenient to take the length, breadth and thickneſs, in the ſame kind of meaſure in which the content is required; but very often it happens that the content is required in feet, and the length given in feet; but the breadth and thickneſs in inches, or partly in feet, and partly in inches; as in, timber, bales, caſes, &c. The value of timber is eſtimated by the load of 50 feet, and the freight of bales, &c. is by the tun of 40 feet. Such of our readers as are not well acquainted in decimals or croſs multiplication, may reduce the feet into inches, ſo firſt find the content in inches, this divided by 1728, gives the content in feet; and to render this method as uſeful and expeditious as poſſible, we have ſubjoined three tables: the

the first is for dividing a number by 1728; the other two for finding the value of a remainder, and may be of use in dividing by 144, or by 12; and tho' most of our readers may be presumed to have the last table by heart, yet as all may not, we choose to insert it.

Table 1.		Table 2.		Table 3.	
1	1728	1	144	1	12
2	3456	2	288	2	24
3	5184	3	432	3	36
4	6912	4	576	4	48
5	8640	5	720	5	60
6	10368	6	864	6	72
7	12096	7	1008	7	84
8	13824	8	1152	8	96
9	15552	9	1296	9	108
		10	1440	10	120
		11	1584	11	132
		12	1728	12	144

EXAMPLE.

Required the content of a case:

Length 7 feet 5 inches, or 89 inches; breadth 3 feet 5 inches, or 41 inches; 2 feet 5 inches, or 29 inches depth.

Now $89 \times 41 \times 29 = 105821$, and when this is divided by 1728, the quotient will be 61 feet, and 413 remaining; to find the value of this, look for it, or the number next less in the second table, which is 288; against which is 2, that is $\frac{2}{12}$ of a cubic foot, which are called inches; again there will be a remainder of 125; the next number less in the third table is 120, against which is 10, that is $\frac{10}{12}$ of an inch, and a remainder of 5, which is $\frac{5}{12}$ of $\frac{1}{12}$ of an inch; so the content is 61 feet, 2 inches, 10 primary parts, and 5 secondary parts; all concisely expreſs'd thus, 61 . 2 . 10 . 5.

It must be observed, that by one inch is understood 144 cubic inches, being the 12th part of a cubic foot; by one of the first parts 12 cubic inches, and by one in the last part is understood 1 cubic inch.

But when the length is given in feet without any odd inches, and the other two dimensions in inches, the operation may be performed without reducing the feet to inches; only dividing by 144.

EX-

EXAMPLE.

What is the content of a piece of timber 24 feet long, 18 inches broad, and 14 inches deep; $24 \times 18 \times 14 = 6048$, and $6048 \div 144 = 42$, the content in feet; after the same manner any other piece of square timber may be measured; but in practice it is not always required to find the exact contents of timber, for sometimes the computed is less, and sometimes more than the real content.

It would be very difficult to find the exact contents of a tree, but as it generally grows pretty near round and tapering, it will be somewhat like a frustum of a cone; notwithstanding which, it is measured as if it were a parallelopipedon, and to find the square base in some places, the circumference of the tree is taken by girting it with a line pretty near the middle, and $\frac{1}{4}$ of this is accounted the side of the square; now it is plain that the area of such a square will be above $\frac{1}{5}$ less than the area of the circle, and the tree measures so much less than the true contents.

In other places the tree is hewed somewhat in the form of an irregular prism of four flat sides and four round; the base will be an octagon, contained under four equal chords, and four arches of circles, but in measuring the tree the chords are supposed to be produced till they meet, and form a square; the area of this, multiplied by the length, is accounted the content, tho' it is plain, the tree thus hewed, does not contain near so much, because there is wood wanting at the corners, these are called wanes, and the flat sides are called squares; besides the tree may be hewed in such a manner as to make it contain more than the real contents of the tree, even if it were allowed to be a cylinder, so that there may be very great impositions on the purchasers; to prevent which, the government contract, that the tree shall be hewed in such a manner, that what is to be called the side of the square shall bear a certain proportion to the diameter of the tree, which may be easily discovered by the callipers; for if they be applied to the wanes, we have the diameter of the tree, and if to the flats, the side of the square, or the thickness; now because the larger the wanes are, so much more will the tree measure; it must be hewed so that two wanes shall not exceed one square. What is meant by a wane should likewise be expressed, for it is generally allowed to be the round part

part of the tree where the wood is wanting to compleat the ſquare, or the chord of it, which may be taken with a pair of compaſſes, as in *Fig.* 53. B E is the wane, and is exactly half the ſquare T B; but in ſome contracts the portions of the chords, which are produced without the circle to compleat the ſquare, are called wanes, as in *Fig* 52. D T = half T E. Plate I.

It is very difficult to hew a tree exactly to this ſtandard, and very often the wanes are as big as the ſquares, as in *Fig.* 54. where the ſquares divide the circumference into 8 equal parts; by which means the content of the tree, meaſured as a parallelopipedon, would be to the real content meaſured as a cylinder, nearly as 34142 to 31416; for which reaſon, before it is meaſured, it muſt be reduced to its proper thickneſs at the meaſuring place, which is nearly the middle of the tree: For tho' all trees taper, and conſequently are greater at the butt than the top end, yet they are allowed to be cylinders, the diameters of which are taken at the middle. But there will be no occaſion to hew the tree, as the proportion is known, which the thickneſs of the tree, when properly hewed, ſhall bear to the whole diameter. All that is neceſſary, is only to conſtruct a line of equal parts, which ſhall have the ſame proportion to a line of inches, that the diameter of the tree has to this thickneſs. If the tree happens to be thicker one way than the other, a mean proportional muſt be found for the diameter.

The conſtruction of a line of equal parts, that ſhall have the ſame proportion to a line of inches, that the diameter ſhall have to the thickneſs when the tree is hewed ſo that the flat ſhall be double the wane, will admit of two caſes.

Caſe 1. When by the wanes are underſtood the portions of the chords, produced without the circle to compleat the ſquare.

1*ſt*. Erect a perpendicular at K, and from K to C ſet off any number of inches, and from K to O double the line K C; then draw the line C O, with which as radius, from the center O, deſcribe a circle. Fig. 52.

2*d*. Divide the line C O into the ſame number of equal parts as the line O K contains inches; then will 21 $\frac{1}{2}$ of thoſe diviſions be equal to 24 inches very nearly; that is a tree whoſe diameter is 24 inches, will be 21 $\frac{1}{2}$ inches thick when hewed.

Now if the chords be produced till they meet, they will form

PLATE I. Fig. 52. a ſquare: And it is very plain that C O is half the diameter of the tree, and K O half the thickneſs; and becauſe the wane C D, or its equal C K, is half the flat C B, the tree is properly hewed according to the contract. Hence it is evident, that when the tree is ſo hewed, the diameter will contain as many equal parts of the line C O, as the thickneſs taken upon the flat will contain inches.

Caſe 2. When by the wanes are underſtood the chords of the arches, or the round parts of the tree, where there is no wood taken off, as B E, D N, I G, F T; and it be required to hew the tree; ſo that the flats T B, F I, G N, D E be double thoſe wanes.

1ſt. Make an angle of 45 degrees, at the point B, or which is the ſame thing, an angle of 90 degrees at M; and taking the points B and E, equally remote from M; draw the line B E, and make B A and A T each equal to B E; ſo ſhall B T be double of B E.

2d. Thro' the points T, B, E, deſcribe a circle; and thro' the center C draw C A perpendicular to T B; ſo will the flat T B be double the wane B E, the line C B half the diameter of the tree, and the line C A half the thickneſs: And if the line C B be divided into the ſame number of equal parts, as the line C A contains inches; it is plain the diameter, when meaſured on this line, will contain as many equal parts of this, as the thickneſs contains inches. The following example will ſuffice to illuſtrate what has been ſaid on this head.

EXAMPLE.

Fig. 52. Let there be a piece of timber 20 feet long, and 24 inches diameter, to be hewed ſo as to make the flats, according to *Caſe* 1. and let us ſuppoſe that when the timber is ſerved in for meaſurement, it is found, by applying the callipers to the flats, to be 22 $\frac{1}{2}$ inches thick. Now to know if it be properly hewed, meaſure the diameter by a line graduated, as C O, which will be found to be nearly 21 $\frac{1}{2}$; which ſhews there ſhould be one inch more hewed off; and therefore 21 $\frac{1}{2}$ muſt be taken for the ſide of the ſquare baſe, which will make the content in feet 64; for the thickneſs is not quite 21 $\frac{1}{2}$, it being only 21.466.

Now 144 : ſquare of the thickneſs in inches : : length in feet : content in feet: That is, 144 : 460.789156 : : 20 : 64.

The content may be found by meaſuring the diameter by a line

line of inches, for which the following proportion must be taken; as 180 is to the square of the diameter in inches so is the length in feet to the contents in feet; for 144 is to 180 as the square of the thickness to the square of the diameter, which may be thus proved: $144 \div 4 = 36$, and $36 \times 5 = 180$, now $\frac{1}{4}$ of the square of the thickness taken upon the flat, multiplied by 5 will be the square of the diameter; for the squares of K O, and K C both together, are equal to the square of O C, by *Prop.* 20. *Chap.* 3. but the square of K C is $\frac{1}{4}$ of the square K O; therefore five times the square of K C will be equal to the square of O C. In this example the square of the thickness is 460.789156, which is the second term in the proportion, when 144 is the first: But if $\frac{1}{4}$ of the second term be multiplied by 5, and that product taken for the second term, it is certain, to preserve the same proportion; that $\frac{1}{4}$ of the first term must likewise be multiplied by 5, and the product made the first term. Now this is the very case here, when the square of the diameter is taken for the second term; $460.789156 \div 4 = 115,197289$; this $\times 5 = 575.986445$; and $24 \times 24 = 576$; then 180 : 576 : : 20 : 64 the content as before; tho' it is plain this exceeds the real content, because of the wood that is wanting at the corners. It is even more than the whole tree would measure, allowing it to be a cylinder of 24 inches diameter; for the square of the diameter $576 \times .7854 = 452.3904$ the area of the base in inches. Again, 144 : 452.3904 : : 20 : 62.8, the content in feet.

PLATE I. Fig. 52.

If it be contracted that the tree is to be hewed, as in *Case* 2. when the diameter 24 inches is applied to the line constructed for that purpose; it will measure 20.71. Then $144 : 20.71 \times 20.71$: : 20 : 59.5 the content in feet. This may likewise be done by taking the square of the diameter in inches for the second term, if 193.3 be taken for the first: $193.3 : 24 \times 24$: : 20 : 59.5, as before: And that 144 : is to 193.3 : : as the square of the thickness taken on the flat : is to the square of diameter, may be thus proved: Let the radius C B be 10000; then will the sine of the angle A B C be 8268; and $8268 \times 8268 : 144$: : 10000×10000 : 193.3. By this it appears that there will be 7 *per Cent.* difference in hewing the tree by this method.

Fig. 53.

The works of the several artificers relating to building, whether superficial or solid, may be measured by the preceding rules: But as all the operations require multiplication and division; this, in ſome caſes, is deemed too tedious for practice, on which account they make uſe of the ſliding rule. But before the lines neceſſary for that purpoſe can be conſtructed, there muſt be ſome method found to multiply and divide natural numbers, by adding or ſubtracting artificial ones: This is moſt effectually done by the Logarithms; which ſhall be the ſubject of the next chapter.

CHAP. V.

Of LOGARITHMS.

IT is not our buſineſs here to conſtruct tables of theſe admirable numbers, they being already calculated to great exactneſs. The learned are obliged for this uſeful diſcovery to the indefatigable labour of the noble inventor, Lord *Neper*. We ſhall only explain ſo much of the nature of them, as is neceſſary for underſtanding the uſe and conſtruction of the line of numbers.

LOGARITHMS are artificial numbers adapted to natural numbers, and ſo contrived, that by adding the logarithms of any two numbers, their ſum will be the logarithm of the product of theſe two numbers, or by ſubtracting the leſs from the greater, the remainder will be the logarithm of the quotient of the one divided by the other. From this deſcription, the following inferences will eaſily be deduced, *viz.*

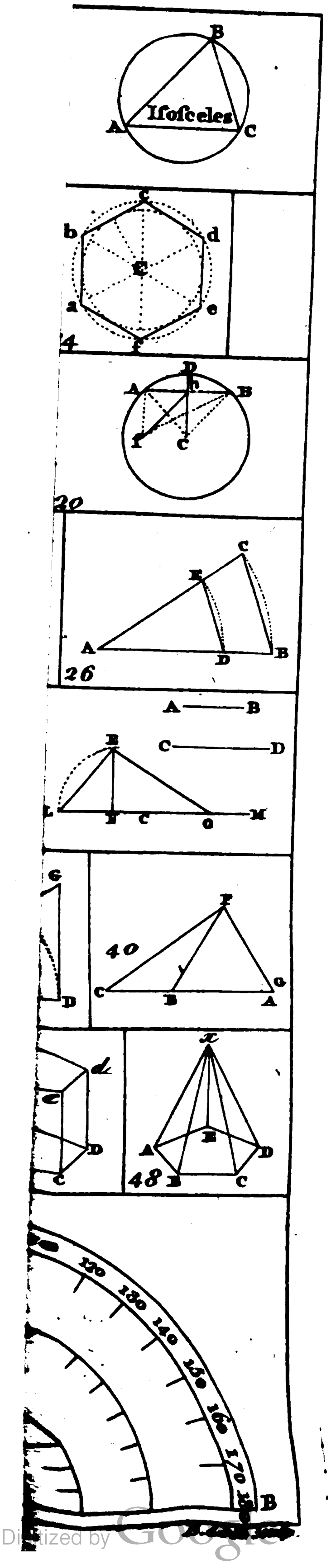
1. Every natural number muſt have a proper logarithm, and therefore a table ſhould be made to find it by inſpection.

2. If the logarithm of any number be increaſed, the correſpondent natural number will be increaſed likewiſe.

3. If the logarithm of any number be added to itſelf, (or which is the ſame thing, if it be doubled) the ſum will be the logarithm of the ſquare of the natural number.

4. If the logarithms of any two numbers are known, the logarithm of the product of thoſe two numbers may with certainty be found: For, if the two known logarithms are added together, their ſum will be the logarithm of the product.

By a careful attention to theſe inferences, we may eaſily make loga-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160
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B

logarithms to the following rank of natural numbers in a continued geometrical proportion, *viz.* 1 : 2 : 4 : 8 : 16 : 32 : 64 : 128, *&c.* which may be continued to any number of terms. Here 1 is the firſt term, and 2 the ratio; ſo every term is the product of the preceding term multiplied by 2, as will appear by bare inſpection.

Unity (or 1) is the firſt natural number, and its logarithm muſt be a cypher (*by Inf.* 2.) for unity neither multiplies nor divides any number; ſo its logarithm muſt neither increaſe nor diminiſh any other logarithm.

The logarithm of 2 may be aſſumed at pleaſure, but this will determine the logarithm of all the reſt: Suppoſe it 10; the next natural number is 4. Now 4 is equal to 2 × 2, therefore its logarithm will be 10 + 10 = 20, which muſt be the logarithm of 4. The next natural number is 8, or 4 × 2: Add therefore the logarithms of theſe two numbers, *viz.* 20 and 10, and their ſum 30 will be the logarithm of 8 (*by Inf.* 3 *and* 4.).

It is eaſy to obſerve, that as the rank of natural numbers is formed by a continual multiplication of each preceding term by the ratio; ſo their logarithms are formed by a continual addition of the logarithm of the ratio: And as this logarithm may be aſſumed at pleaſure, ſo there may be different ſorts of logarithms, as in the following, *viz.*

1 :	2 :	4 :	8 :	16 :	32 :	64 :	128 :	256 :	512 :	1024	Numbers.
0 :	10 :	20 :	30 :	40 :	50 :	60 :	70 :	80 :	90 :	100	Logarithms.
0 :	15 :	30 :	45 :	60 :	75 :	90 :	105 :	120 :	135 :	150	Logarithms.

It is evident, that either of theſe ranks of logarithms will anſwer the propoſed end: For if it were required to multiply 32 by 8, the logarithm of 32 is 50, the logarithm of 8 is 30, and 30 + 50 = 80, which is the logarithm of 256 = 32 × 8.

This may ſuffice to ſhew, that if there were a table of logarithms to all the natural numbers we ſhould have occaſion for, there would be no need of multiplication or diviſion. The difficulty will be, to make ſuch a table for all the intermediate numbers, which I preſume the inventor might effect in the following manner.

Inſtead of aſſuming the logarithm of 2, he might chuſe 1.0000000 for the logarithm of the natural number 10, the double of which would be 2.0000000; for the logarithm of 100, and

and the treble 3.0000000; for the logarithm of 1000, and so on, as in the following table.

Natural Numbers.	Logarithms.
1	0.0000000
10	1.0000000
100	2.0000000
1000	3.0000000
10000	4.0000000
100000	5.0000000

It is evident from the foregoing table, that the logarithms of all the natural numbers between 10 and 100 would begin with 1; between 100 and 1000 with 2; and between 1000 and 10000 with 3, *&c.* these initial figures are called characteristicks, and denote how many places the first figure of the natural number stands from unity: It is also evident, that the logarithm of any natural number under 10, would be less than 1, with 7 cyphers annexed, and therefore would begin with 2, 3, *&c.* with 6 figures more annexed. But to make it contain the same number of figures as the logarithms of the numbers above 10, he prefixed a cypher to it, which is the characteristick of all the natural numbers under 10.

Having thus assumed 1.000000 for the logarithm of 10, the half of it 0.5000000 would certainly be the logarithm of the square root of 10, which the inventor with great care and pains must have extracted to 7 decimal places. If this root were multiplied by 10, the logarithm of the product would be 1.5000000, the half of which 0.7500000, would certainly be the logarithm of the square root of that product. In this manner, I presume, he proceeded to find the proportionals between 1 and 10, till the root came to more than 9, and then found mean proportionals betwixt that and the next root less than 9, till at last, after a great number of trials, he came to the root, or absolute number 8.9999999, which is so very near 9, that it may be taken for the same, the logarithm of which he found, by the same number of additions and halvings, to be 0.9542420.

In the same manner he might proceed to find the logarithms of 5 and 7, and having found these, the logarithms of 2, 3, 4, 6, 8, might easily be found, for half the logarithm of 9 would be the logarithm of 3, and if the logarithm of 5 is subtracted from the loga-

logarithm of 10, the remainder will be the logarithm of 2, the double of which is the logarithm of 4, and that doubled again will be the logarithm of 8; and if the logarithm of 2 is added to the logarithm of 3, the sum will be the logarithm of 6.

By these means, I presume, after a great deal of indefatigable pains, and an uncommon application, he at last finished his table, which was justly esteemed one of the most useful discoveries in the art of numbers, and has accordingly been universally received by all mathematicians, and the lord *Neper* is allowed the whole honour of the invention without any rival.

Other methods have been proposed by authors who have wrote on this subject, whereby the operations may be shortened in the construction of the table; but, as our design in this place is only to make the reader acquainted with the coherence of the logarithms and natural numbers, being the same with that of numbers in arithmetical and geometrical progression; I think the preceding method the most likely to answer that purpose, as being the most intelligible, and the fundamental principle, upon which those methods that have been found to shorten the work must be grounded. Tables being already calculated by the inventor, as well as by several succeeding mathematicians, to a great exactness, there is now no necessity for that trouble, we shall therefore, in the following propositions, shew the manner of finding logarithms in the tables, and some of their various uses in arithmetical operations.

PROP. I.

To find the logarithm of any given number.

Rule, Look for the number in the first column, (under N°) and if it consists of less than 3 figures, its logarithm will be found in the first page, with its proper characteristick. If it consists of 3 figures, it will be found in the following pages in the first column, and right against it in the column under 0, you will find its logarithm, with a proper characteristick.

If the given number consists of 4 places, find the first three as before, and look for the last figure at top, and in the column under that, right against the three first figures, you will find the proper logarithm: Only you are to observe, that in this case the characteristick will be 3: For the absolute number must always contain

contain one place of integers more than the characteristick does of units. If one, or more, of the last figures are decimals, the logarithm will be the same: The difference will be only in the characteristick, which, as we have observed before, always denotes how many places the first figure of integers stands from unity; the following examples will be sufficient under this head.

Num.	Logar.	Num.	Logar.
8	0.903090	7569.	3.879038
88	1.944483	756.9	2.879038
699	2.844477	75.69	1.879038
5463	3.737431	7.569	0.879038

PROB. II.

To find the absolute number corresponding to any given Logarithm.

Rule, Without regarding the characteristick, look for the given logarithm in the table; and right against it, in the first column, under N°, you will find the three first figures, and at top, the fourth figure of the number required. But, if the number thus found should consist of fewer places than is expressed by the characteristick, the deficiency must be made up by annexing cyphers: And if it consists of more places, one or more of the last figures must be decimals, as in the following examples.

EXAMPLE I.

Let the given logarithm be 3.914872. Against .914872; in the table you will find 822, in the first column under N°; so that, as the characteristick is 3, and the logarithm is found in the column under 0 at top, the number sought will be 8220. But if the characteristick had been 2, the number would have been 822; and if it had been 1, the last figure would have been a decimal, and only the two first figures integers, *viz.* 82.2.

If the above logarithm .914872 had not been found under 0, in the table, the fourth figure would not have been a cypher, but one of those at top of the table under which it had been found.

EXAMPLE II.

Let the given logarithm without the characteristick be .018345. This will be found in the column which has the figure 6 at top; the

the absolute numbers must therefore be taken to 4 places of figures at least, the characteristick always denoting how many of those figures must be reckoned as integers, as in the following, *viz.*

Logarithms	Numbers
5.918345	828600
4.918345	82860
3.918345	8286
2.918345	828.6
1.918345	82.86
0.918345	8.286

If the given logarithm cannot be had exactly in the tables, we must take the nearest to it; suppose it 3.861080; the natural number corresponding thereto will be more than 7262, but less than 7263. But because the given logarithm is nearer to that of 7262, that may be taken for the required number: Those who incline to more exactness may find a figure of decimals by the following method.

From the given logarithm	3.861080	difference
Subtract the next less —	3.861056	is 24
From the next greater log.	3.861116	difference
Subtract the next less —	3.861056	is 60

Then say, as 60 (the difference betwixt the two nearest logarithms to the given one) is to 10 so is 24 (the difference betwixt the given and next less) to 4, the decimal required: So 7262.4 is the natural number corresponding to 3.861080.

That the natural number is by this method found to great exactness, may be proved by adding the logarithms of any two numbers together whose product is equal to it.

Thus, 605.2 x 12 = 302.6 x 24 = 7262.4

Num.	Log.	Num.	Log.
605.2	2.781899	302.6	2.480869
12	1.079181	24	1.380211
7262.4	3.161080		3.861080

The reason of this is plain, for if to the logarithm of 7262, be added 60, the natural number will be increased a whole unit; but if only 1 tenth, 2 tenths, &c. of 60 be added to it, the natural number will be increased only 1 tenth, 2 tenths, &c. of an unit.

 Hence,

Hence, by the reverſe of this method we may find the logarithm of a number of five figures; for, after finding the logarithm of the firſt four figures, ſubtract that from the next greater logarithm in the tables; then ſay, as 10 is to the difference betwixt the logarithms ſo is the fifth figure in the natural number to the number to be added to the logarithm of the firſt four figures.

Let the number whoſe logarithm is required be 72624.

	Log.	
7263	3.861116	
7262	3.861056	10 : 60 : : 4 : 24, and
		24 + 3.861056 = 3.861080
	difference 60	

But becauſe the natural number has five figures, the characteriſtick muſt be 4.

PROP. III.

Multiplication and Diviſion by Logarithms.

Rule, Add or ſubtract the logarithms of the natural numbers, their ſums will be the logarithms of the products, and their remainders the logarithms of the quotients; and as the rule of Three requires both theſe operations, we ſhall refer thereto for examples.

PROP. IV.

The Rule of Three, by Logarithms.

Rule, Add the logarithms of the ſecond and third terms together, and from the ſum ſubtract the logarithm of the firſt term; the remainder will be the logarithm of the fourth term required.

EXAMPLE. I.

If 64 give 21, what will 72 give?

			Log.
ſecond term	21		1.322219
third term	72		1.857332
product	1512	ſum	3.179551
firſt term	64		1.806180
fourth term	23.62		1.373371

As

As the last logarithm cannot be had exactly in the tables, we must (as already observed) take the nearest to it, which is, 373280. against which, in the column under *number*, is 236, and the figure at the top is 2; so that 23.62 will be the nearest, for the characteristick being 1, the two last figures will be decimals.

We shall in the next examples show the use of the logarithms, when any of the terms are mixed numbers, or decimal fractions; and here we think it needless to perplex our readers with negative signs, as the whole business may be done by using the same process as if they were all integers; for then the characteristicks will all be positive, and denote how many places of figures are contained in the product, or quotient; and we may find how many are decimals, by the very same rule that is made use of when the operations are performed by multiplication and division of the natural numbers.

EXAMPLE II.

If 16.5 give 3.75, what will 49.5 give?

We shall work this as if the terms were all integers, and likewise as mixt numbers; the difference will be only in the characteristick.

3.75	Log.	2.574031		0.574031
49.5		2.694605		1.694605
185.625		5.268636	or	2.268636
16.5		2.217484		1.217484
11.25		3.051152		1.051152

In the first operation, when the logarithms of the second and third terms are added together, the characteristick is 5: This shews there will be six figures in the product: But then, because there is one place of decimals in the multiplicand, and two in the multiplier, there must be three places of decimals in the product, and only the first three figures are integers. And this is agreeable to the characteristick, in the second operation; which being 2, shews there will be three places of integers; but this does not determine how many places of decimals will be requisite to compleat the product. Again, when the logarithm of the first, is subtracted from the sum of the other two, in the first operation the characteristick is 3, which shows there will be four figures in the quotient. But because there are three decimal places in the dividend, and but one in the divisor, there must be two decimals in the quotient. So the first two figures will be integers, and the last two decimals: It is the same by the second operation; where the characteristick is 1. The first operation seems to have the advantage of the second, because it discovers how many decimals will be in the product or quotient.

EXAMPLE III.

If 165 give ,375, what will ,495 give?

The figures in this being the same with the former, the operation will also be the same; the difference will be only in the value of the figures in the product, and quotient. The second and third terms being decimals, their product will likewise be decimals; and the characteristick being 5, it will consist of six places: But when this comes to be divided by the first term, which is 165, all integers, the dividend will contain six places of decimals more than the divisor, and therefore the quotient must likewise have six decimal places; whereas, by the preceding operation, the characteristick of the logarithm of the quotient is three, which shows it will contain only four significant figures; to which there must be two cyphers prefixed, to make up the deficiency, and then it will be the same as if the operation was performed by natural numbers; $.375 \times .495 = .185625$, and $.185625 \div 165 = 001125$: But if the divisor is a fraction, as ,165, and the dividend the same as before, then it will contain only three decimal places more than the divisor; so the quotient must have three decimal places, $.185625 \div ,165 = 1,125$.

PROP. V.

Extraction of Roots, by Logarithms.

Rule, Divide the logarithm of the power by the index of the power, the quotient will be the logarithm of the root; but if the root be given, and the power required, multiply the logarithm of the root by the index of the power; the product will be the logarithm of the power.

N B. The index of the square is 2, of the cube 3, &c. See *Chap.* 1. *Sect.* 1.

EX-

EXAMPLE I.

What is the ſquare root of 576?

	Log.	Index	Log.
Power 576	2.760422 ÷	2 =	1.380211, the natural number

correſponding to which is 24, the root required.

EXAMPLE II.

What is the cube root of 13824?

This number, conſiſting of five figures, cannot be found in the tables, therefore we muſt make uſe of the method in *Example* 2. *Prop.* 2. of this Chap. *viz.* find the logarithms of 1382, and of 1383, their difference will be 314; then 10 : 314 : : 4 : 125.6

	Log.			Log.	
1383	140822	Dif. 314	1382	140508	Log. of 13824 is 4.140633, $\frac{1}{3}$ of which is 1.380211, and
1382	140508				
314 × 4 = 1256 ÷ 10 = 125.6				125	

the natural number correſponding to this laſt logarithm is 24, which is the cube root of 13824.

EXAMPLE III.

Admit two cylinders of equal length; the diameter of the one 32 inches, and its content 4096 cubic inches, the diameter of the other 16 inches, required the content in cubic inches?

Here, as the lengths are equal, the contents will have the ſame proportion to one another, as the areas of their baſes, which being circles, it will be as the ſquares of their diameters; that is,

As the ſquare of 32 (whoſe Log. 1.505150 × 2 is	=	3.010300)
Is to the contents 4096 (whoſe Logarithm is	=	3.612360)
So is the ſquare of 16 (whoſe Log. 1.204120 × 2 is	=	2.408240)
	ſum	6.020600
To the required contents 1025 (whoſe Logarithm is	=	3.010300)

EX-

EXAMPLE IV.

Admit 93 feet to be the length of the keel of a ſhip of 508 tons, and a ſhip of 400 tons to be built, exactly ſimilar to the other; required, the length of her keel?

In order to ſolve this is muſt be obſerved, that the contents of ſimilar ſolids have the ſame proportion to one another, that the cubes of their ſimilar ſides have, therefore, the following will be a general proportion in all caſes where the dimenſions are ſimilar, *viz.*

As the tonnage of any ſhip, or the ſolid contents of any body, is to the cube of the keel, or any other part; ſo is the tonnage of any other ſimilar ſhip, or the contents of any other ſimilar body, to the cube of her keel, or any other ſimilar part. Hence, 508 : cube of 93 :: 400 : cube of the required keel, the cube root of the fourth term muſt be extracted, for the length of the keel: Firſt, to cube 93, by the logarithms.

Log.		
93 1.968483 x 3 =	5.905449	Log. of the cube of 93
400 its Log. is	2.602060	
ſum of 2d and 3d terms	8.507500	
508 firſt term, its Log.	2.705864	
Log. of the cube, divide by 3)	5.801636	
85.88 the required keel (or 86 nearly)	1.933878	

Here the uſefulneſs of logarithms is very evident, for the cube of 93 would conſiſt of 6 places, as appears by the characteriſtick; and this again being multiplied by 400, the product would conſiſt of 9 places; and when this product is divided by 508, the quotient will have 6 places; and the cube root of this muſt be extracted to four places at leaſt; for 85.88 is the length of the keel required, being the neareſt natural number to the Log. 1.933878.

What has been already ſaid, we preſume, is ſufficient to recommend the practice of theſe admirable numbers to our readers, though they may be extended to the ſolution of moſt queſtions which require an arithmetical calculation. But they do not ſtop here; for they diſcover a method of performing the foregoing operaperations, even without the help of numbers: This is effected by the line of numbers invented by Mr *Gunter*, which we ſhall treat of in the next chapter, and ſhall only here remark, that numbers may be added or ſubtracted, by a ſcale of equal parts and a pair of compaſſes, as in the following examples, where we ſhall make uſe of the ſame ſcale of equal parts before deſcribed, as in *Plate* 2. *Fig.* 1.

EXAMPLE I.

Let the two given numbers be 36 and 48.

Rule, Extend from the point A (where the line A B begins) to either of the given numbers, ſuppoſe 36; ſet the ſame extent forwards from the other given number 48, and it will reach to 84, the ſum required in the ſame line A B.

EXAMPLE II.

Let it be required to find the ſum of 3010 and 4771. As theſe numbers cannot be had on the line A B, find them in the diagonals, and transfer them to the line A B, in the points *x*, *z*; the extent from A to *x* will reach from *z* to *y*. Now, to find the value of *y*, take the diſtance of the point *y* from figure 7, in the line A B, and ſet it off from figure 7, in the line C D; then a ruler laid from this point to *y*, in the line A B, will interſect the diagonal next before *y*, in the required point, which will be found to be 7781.

As ſubtraction is only the reverſe of addition, it will be needleſs to give any examples, this being not intended for practice.

The reaſon of the operation is ſo plain, as to require no demonſtration: For if two rulers, one of ten inches, and another of fourteen, be laid ſo as to make one ſtrait edge when joined to one another, they will make 24 inches; and if there be ſix inches cut off from a ruler of 24 inches, there will remain only 18 inches.

CHAP.

CHAP. VI. SECT. I.

Construction of the Line of Numbers.

Plate I. Fig. 1. THIS line may be of any length, but as there must be a particular scale adapted to it, we shall fix upon the line A B, which being divided into 10,000 equal parts, will answer our purpose.

The intent of the line of numbers is only to add or subtract logarithms, so that all that is necessary to this end is to place the logarithms properly upon the line.

The logarithm of 10, by the table at the end of the book, is 1.00000. But because our scale contains only 10,000, we shall fix upon that number for the logarithm of 10, and all those under 10 will be as in the margin. Find them all amongst the diagonals, and transfer them to the line A B in the points *x*, *z*, *t*, *p*, *y*, *s*, *r*, *n*.

1-0000
2-3010
3-4771
4-6020
5-9990
6-7781
7-8451
8-9030
9-9542
10-1000

Draw the line G H, parallel and equal to the line A B, and transfer the points *x*, *z*, *t*, &c. to this line from the line A B.

We have now the logarithms of all the numbers from 1 to 10 upon the line G H; and if to the end of it be joined another line, of the same length, and graduated and numbered properly, we shall have all the numbers from 1 to 100. But as our scale will not admit of these; draw the line E F parallel and equal to G H; and instead of doubling the line G H, take E N, (half the line E F) and make it the length of a line of numbers, by transferring the logarithms, as was done on the line G H, but there must be a scale of equal parts adapted to the line E N, which must contain 10000 equal parts, if we make use of the same table of logarithms as before. And by this means the line A B would contain 20000 equal parts, which would require double the number of parallels. Instead then of making a new scale, we may make the same diagonals answer our end: For it is only taking

taking half the logarithms. We ſhall therefore accommodate the logarithms to our ſcale, as in the margin; transfer all theſe from the diagonals to the line E N in the points 2, 3, 4, 5, 6, 7, 8, 9, 1; then graduate and number the line N F, the other half of the line E F, exactly as the line E N. Theſe laſt will be the logarithms of 20, 30, *&c.* for the logarithms of 20, 30, *&c.* are the logarithms of 2, 3, *&c.* added to the logarithm of 10; but E N is the logarithm of 10; E 2, E 3, the logarithms of 2, 3, *&c.* therefore, if N 2, N 3, *&c.* be made equal to E 2, E 3, *&c.* E N 2, E N 3, *&c.* muſt be the logarithms of 20, 30, *&c.* They may be transferred to the line N F from the diagonals, if to each of the logarithms in the margin we add 5000. So the logarithm of 20, will be 6505, the half what it is in the tables. Now to find the units, or intermediate points betwixt 10 and 20, 20 and 30, *&c.* find in the tables, the logarithms of 11, 12, 13, *&c.* to 20, and the logarithms of 21, 22, *&c.* theſe muſt be divided into two equal parts, to accommodate them to our ſcale; and being found in the diagonals, they may from thence be transferred to the line N F.

PLATE II. Fig. 1.

1-0000
2-1505
3-2385
4-3010
5-3495
6-3890
7-4225
8-4815
9-4771
10-5000

We have now the whole line E F divided into 18 unequal parts; 1 at E, and the figures 2, 3, *&c.* denote ſo many units; 1 at N, and the figures 2, 3, *&c.* to F, denote ſo many tens; 1 at F 100: The intermediate diviſions betwixt the figures in the line N F are units; ſo the 6th diviſion betwixt the figures 2 and 3 is 26; the firſt betwixt figure 1 and 2 is 11, and ſo of all the reſt.

If the ſpaces betwixt the figures in the line E N be graduated, as thoſe in the line N F, they will be tenths of units: And becauſe the difference betwixt the logarithms of 1 and 10, the logarithms of 10 and 100, and of 100 and 1000 are all equal; 1 at the point E may be accounted 10, and 1 at N 100, at F 1000. The figures in the line E N will now be tens, and thoſe in the line N F hundreds. The intermediate diviſions betwixt the figures in the line E N will now be units, and thoſe in the line N F will be tens. So 3 in the line E N will be 30, and in the line N F 300. The diviſions in the line E N betwixt 5 and 6, will be 51, 52, *&c.* in the line N F 510, 520, *&c.*

In order to find the points for 101, 102, 121, 122, *&c.* we

 muſt

PLATE II. Fig. 1. must find the logarithms of those numbers, and transfer them from the diagonals as before directed; which would require the spaces betwixt the figures to be divided into 100 equal parts; but the length of our scale will not admit of this. The divisions betwixt the figures 1 and 2 are sub-divided into five, by transferring the logarithms of 102, 104, 122, 124, *&c.* and the divisions betwixt 2 and 3 are only sub-divided into two, by transferring the logarithms of 205, 215, 235, *&c.* The spaces betwixt the other figures are only divided into ten; so the units can only be had by taking $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, *&c.* as near as the judgment can direct.

The line being thus constructed, it will be easy to find any number upon it: And that this may be done with all possible expidition, where the spaces are divided into ten parts betwixt the figures, every fifth is distinguished by longer strokes than the tens. Again, where the spaces can admit of being divided into more than ten parts, the sub-divisions are distinguished by shorter strokes than the tens. Now the value of these strokes are determined by the value of the figures, which being arbitrary, they must be determined before we can find any number upon the line: If the number be less than 100, 1 at E may be unity; then 1 at N will be ten, 1 at F 100. The strokes representing the tens in the line E N, will be tenths of units, and those in the line N F will be units. The short strokes betwixt the tens are estimated according to their number; for if there were 9 intermediates, each would be 100th part of an unit in the line E N, and tenths of units in the line N F. If there be only 4 intermediate strokes, each will be 200th parts, or two tenth parts of unity: And if there be but one stroke, it will be 500th, or 5 tenths.

If 2 were required, look for that figure in the line E N; if 20, it will be at 2 in the line N F; if $3\frac{1}{2}$, or 3:5, count five strokes beyond the figure 3 in the line E N; this, as was before observed, will always be longer than any of the others. If 400 were required, 1 at E must be accounted ten, 1 at N 100: So figure 4 in the line N F will be 400; 470 will be 7 strokes beyond figure 4; 475 will be in the middle betwixt the 7th and 8th stroke beyond figure 4; if 473, we must take little more than $\frac{1}{4}$ of the space betwixt the 7th and 8th stroke, but this cannot be had to a great niceity.

The line G H is called a single line of numbers, and the line E F a double one: This last containing double the numbers that the

the former does; and it is only two lines of numbers joined to one another, both graduated and numbered alike. PLATE II.

It is very plain that the figures 2, 3, *&c.* in the line G H are double the distance from one another, that the same figures are in the line E N or N F; so that we may, by inspection, find either the square or root of any number on these lines. A few examples will illustrate this. Fig. 1, 2.

Let it be required to find the square of 6. To do this, is to multiply it by itself, or to double its logarithm. Now this is at the point 6 on the line E N, extend therefore from E to figure 6, the same extent will reach from 6 to 36 = 6 × 6: But 36 is double the distance from E that 6 is; and 6 in the line G H is likewise double the distance from G that 6 in the line E N is from E: So we shall have no occasion for compasses; we need only look for the root on the line G H, and the square will be on the line E F right against the root: If the root be less than 10, 1 at G and at E may be units, but if it exceed 10, 1 at G must be 10, and 1 at E 100: If the square be given, and the root required; look for the square on the line E F, and the root will be against it on the line G H. Let the square root of 81 be required. 81 will be found in the line N F, and will be double the distance from E, that the root is from E. We must therefore find, by compasses, half the distance from E to 81, and whatsoever figure or point is at this middle point in the line E F; the figure of the same name, or point of the same value in the line G H, will be double that distance from G, and therefore must be against 81; and in this case 9 is the square root of 81.

As the square of any number is double the distance of the root from E; so is the cube of any number triple the distance of its root from the point E. And in order to find the cubes or roots of numbers by inspection; draw the line I K equal, and parallel to the lines E F, and G H. Divide it into three equal parts in the points L, M. Make each of these a line of numbers, either by adapting a scale of equal parts to it, or taking one third of the logarithms, and making use of the same diagonals as before. This line is called a triple line of numbers. Now if the cube root of any number be required; look for the cube on this line, and its root will be right against it on the line G H; for the same reasons, that the square is on the line E F. If the root be given, and Fig. 2, 3.

M 2 the

the cube required; look for the root on the line G H, and its cube will be on the line I K, right againſt the root.

Note. Becauſe the lines G H, E F and I K, are at too great a diſtance from one another to find the cubes, ſquares and roots, without a pair of compaſſes; there is a double line drawn cloſe tothe ſingle line, as in *Fig*. 2. and a ſingle line drawn cloſe to the triple line, as in *Fig*. 3. ſo that they may ſerve for a table of cubes and ſquares.

S E C T. II.

Of the Uſe of the double Line of Numbers.

IT is evident from the conſtruction of the line of numbers, that the logarithm of any number may be had by a pair of compaſſes: Thus, extend from the beginning of the line to the number whoſe logarithm is ſought; that extent meaſured on the ſcale of equal parts, will give the logarithm required.

E X A M P L E.

Let it be required to find the logarithm of 4.

Extend from the beginning of the line to 4; that extent meaſured on the ſcale of equal parts will give .6021, which is the logarithm of 4. The like may be ſaid of any other number; which is very plain, being only the reverſe of the method by which the line was conſtructed.

The intent of finding the logarithms of any numbers in this manner, is in order to add, or ſubtract them. But if this can be done by the line of numbers only, we ſhall have no occaſion for the ſcale of equal parts.

We have already ſhewn how to add any two numbers by the ſcale of equal parts; therefore we may add the logarithms of any two numbers in the ſame manner; and the ſum will be the logarithm of their product.

E X A M P L E.

Let it be required to add the logarithm of 3 to the logarithm of 4.

This cannot be done by the ſcale of equal parts without having a table of logarithms; but this defect is ſupplied by the line of numbers. The logarithm of 3, by the table, is 4771, and the logarithm of 4 is 6021. Now the points 3 and 4 upon the line of numbers, are the ſame diſtance from the beginning of that line, that the numbers 4771 and 6021, are from the beginning of the ſcale of equal parts: So that it will be the ſame thing to extend from the beginning of the line of numbers to 3, that it would be to extend from the beginning of the ſcale of equal parts to 4771, and when this extent from 1 to 3 is ſet forward from 4, it will reach to 12, which point is 10792 equal parts from the beginning of the line of numbers; and that number being the ſum of 4771 and 6021 (the logarithms of 3 and 4) it is the logarithm of 12, as appears by the table.

Multiplication by the Line of Numbers.

Rule. Extend from 1 to either of the given numbers, that extent will reach from the other given number to the product: A few examples will ſuffice to illuſtrate this.

EXAMPLE I.

Let it be required to multiply 8 by 6.

1 : 6 :: 8 : 48.

The extent from 1 to 6 will reach from 8 to 48.

EXAMPLE II.

Let it be required to multiply 98 by 8.

Here the diſtance from 1 to 8, when ſet forward from 98, will go beyond the end of the line; for, if 1 at the beginning of the line be unity, all the figures on the firſt part will be units, and thoſe on the ſecond, tens, and 98 will be within two diviſions of the end of the line. In this caſe, 1 at the beginning of the line muſt be accounted 10, ſo 98 will be found on the firſt part of the line; and becauſe the extent from 1 to 8 is the ſame as from 10 to 80; when this is ſet forward from 98, it will reach to 784, the product required.

$$\left.\begin{matrix} 1 : 8 :: \\ 10 : 80 :: \end{matrix}\right\} 98 : 784.$$

A

A slider having a line of numbers upon it, exactly the same with that on the rule, will perform the office of a pair of compasses; and being readier for practice, we shall shew how to work by it.

As in the first example, let it be required to multiply 6 by 8.

Set 1 upon the slider, against 6 upon the rule, look for 8 upon the slider, and against it, is 48 upon the rule; and when the slider is thus set, we have the product of any number multiplied by 6; for against 2 is 12, against 6 is 36, against 10 is 60, against 16 is 96; but 17 on the slider goes beyond the end of the line upon the rule. In this, and in such like cases, the value of the figures must be alter'd, as observed before; and 1 at the beginning must be 10, and 17 will be found in the first part of the line upon the slider, and against it you will find 102 upon the rule. If the number had been 170, the operation would have been exactly the same; it would be only calling the 1, 100, and adding a cypher to the product 102, which would make it 1020.

Division by the Line of Numbers.

This is only the reverse of multiplication, for the extent from the divisor to the beginning of the line, set back from the dividend, will reach to the quotient. Or by the slider; set the divisor on the slider against 1 on the rule, and against the dividend on the slider, you will find the quotient on the rule.

EXAMPLE.

Let 48 be divided by 8.

8 : 1 :: 48 : 6.

Set 8 on the slider, against 1 on the rule; and against 48 on the slider is 6 on the rule; and, without moving the slider, we have the quotient of any number divided by 8, by inspection; as the reader will easily perceive upon examination.

Hence, to reduce a vulgar fraction into a decimal, add a cypher to each part of the fraction; and if the denominator upon the slider is set against 10 on the rule; then against the numerator you will find the decimal fraction upon the rule. Thus, the vulgar fraction $\frac{6}{8}$ or $\frac{60}{80}$ will be found .75 in decimals.

The

The Rule of Three by the Line of Numbers.

This is only to find a fourth proportional to three given numbers.

Rule. Place the numbers, or suppose them to be placed, as in the rule of three direct; then extend from the first to the second; that extent set the same way from the third, will reach to the fourth number required.

By the slider, set the first term upon the slider against the second term upon the rule; and against the third term upon the slider you will find the 4th term required upon the rule.

E X A M P L E.

Let the given numbers be 12 : 20 : : 27, to which a fourth is required that shall bear the same proportion to 27, that 20 does to 12.

Set 12 upon the slider against 20 on the rule; and against 27 upon the slider, you will find 45 upon the rule, which is the fourth term required; for 12 : 20 : : 27 : 45, the product of the extremes ($12 \times 45 = 540$) being equal to the product of the means ($20 \times 27 = 540$).

In order to demonstrate the reason of this rule, it will be proper to observe; that to perform the operation by figures, 20 must be multiplied by 27, and the product divided by 12.

By the method already shewn for multiplication by the line of numbers, the extent from 1 to 20, set forward from 27, will reach to 540, the product; but then, as this product is to be divided by 12, the extent from 12 to 1 must be set back from this product 540. Now it is very plain, that in effect, we only set the distance between 12 and 20 forward from 27: For as we are obliged after we have set forward the distance between 1 and 20, to set back the distance between 1 and 12; it is plain, that betwixt this last point and 27, there will be exactly the same distance as there is between 12 and 20.

It must be observed, that in extending, if the second term is greater than the first, the fourth term will be to the right hand of the third; but if it be less, it will be to the left hand of the third: When we use the slider, it is indifferent whether the first term be taken on the slider, or on the rule, provided the third term be taken on the same line as the first is. Neither is it material which of of the means is taken for the ſecond term; for 12 : 20 : : 27 : 45; and 12 : 27 : : 20 : 45.

Having now fully explained the conſtruction and uſe of the line of numbers, we ſhall give ſome examples in caſes that moſt commonly occur to the ſhipwrights. And as the ſlider is moſt expeditious, we ſhall always make uſe of it.

EXAMPLE I.

Suppoſe it were required to know how much an artificer would gain in 30 days, at the rate of 3 ſhillings per day?

To reduce this to the rule of three, it will be 1 : 3 : : 30 : 90, and when the ſlider is ſo ſet, that is 1 againſt 3, then will 90 be againſt 30. But as the anſwer to theſe, and ſuch like queſtions, is ſometimes required in pounds, this muſt again be divided by 20; and the operation by the pen would be $3 \times 30 \div 20 = 4.5$. Now, here are two numbers to be multiplied by one another, and divided by a third, therefore it will be 20 : 3 : : 30 : 4 . 5 : And inſtead of ſetting 1, ſet 20 againſt 3, and againſt 30 you will find 4, and 5 of the ſmall diviſions, which are tenths; each of which, in this caſe, muſt be reckoned 2*s.* ſo that, 4 and 5 tenths will be 4*l.* 10*s.* 0*d.*

EXAMPLE II.

What is the 3 fifths of 45?

As the $\frac{3}{5}$ths of 5 is 3, ſay by the rule of three.

If 5 gives 3; what will 45 give? The anſwer will be 27, for 5 : 3 : : 45 : 27.

From this example, take the following rule for finding the quarters of maſts and yards, having the partners and ſlings given; and alſo the fraction, that the quarters muſt be of the partners, or ſlings.

Rule. Set the denominator of the fraction againſt the numerator, and againſt the ſlings; will be the quarter required.

EXAMPLE III.

If a yard is 25 inches at the ſlings, what will it be at the yard arm, the proportion being $\frac{2}{5}$ths of the ſlings?

Set 5 againſt 2, and againſt 25 you will find 10. 5 : 2 : : 25 10.

EX-

EXAMPLE IV.

If a ship of 69 feet by the keel, be 23 feet broad; how broad will a ship of 75 feet be, that is built in the same proportion?

Set 69 against 23, and against 75 you will find 25; and the slider being thus set, we have by inspection, the breadth of any ship, if the length of the keel is known, and the proportion, the same as above. If the keel is in feet and inches, it will be proper first to work for the feet, and then for the inches.

EXAMPLE V.

What will be the breadth of a ship whose keel is 75 feet 9 inches, the proportion being as 3 to 1?

Set 3 against 1, and against 75 you will find 25; and without moving the slider, against 9 (inches) you will find 3; so the breadth required is 25 feet 3 inches.

EXAMPLE VI.

Suppose a ship to be of the following dimensions, viz.

Length of the keel	— —	93 feet	4 inches.
Extream breadth	— —	32	0
Breadth at the transom	—	18	4
Breadth at the top timber line	—	26	6

And suppose several other ships are to be built in the same proportion, and the lengths of the keels are given, as follows; the other dimensions will, by the foregoing method, be found to be as in the columns, viz.

Lengths of the keels.		Extreme Breadths.		Breadth at the tran.		Bre. at the top. line.	
f.	in.	f.	in.	f.	in.	f.	in.
108	10	37	4	21	4½	30	3
117	8	40	2	23	0	32	7
123	0	42	0	24	0¼	34	0

What has been said in *Example* 4. will suffice for finding all these dimensions.

EXAMPLE XVII.

The length of the keel, and extreme breadth, being given; to find the tunnage.

The general method is to multiply the length of the keel by the breadth, and that product by the half breadth; then divide by 94; and the quotient will be the tunnage required: Or, which is the same thing, multiply the breadth by the $\frac{1}{2}$ breadth; then say as 94: is to this product:: so is the length of the keel: to the tunnage.

Let the length of the keel be 93 feet 4 inches, and the breadth 32 feet.

The operation by the pen will be 93 feet 4 inches × 32 = 2986.8 × 16 = 47786.8 ÷ 94 = 508 $\frac{34}{94}$.

The first step by the slider will be, set 1 against 93.4 (or 93 feet $\frac{4}{12}$ = 93.33), and against 32 will be 2986.8. Now as this product is to be multiplied by 16, and divided by 94, it will be 94 : 2986.8 :: 16 : 508 $\frac{34}{94}$; therefore, if you move the slider 'till 94 is against 2986.8, the tunnage 508 nearly, will be against 16. In finding the first product, there is no occasion for estimating the number, only let it be marked, so that 94 may be moved to it; we shall shew in another place how this may be done at once by the slider.

By a careful attention to the manner of solving these questions, it will be easy to apply the slider to any other question in the rule of three, whether direct or inverse; or any thing else that is performed by multiplication or division.

We shall only add a few examples in measuring plank and timber.

Of measuring Plank.

We observed in *Chap.* 4. *Sect.* 3. that all plank is considered as an oblong square, and measured as a plain surface, without any regard to the thickness; and that all the varieties thereof may be reduced to the rule of three by the following proportions.

1 : length in feet :: breadth in feet :	area in feet.
12 : length in feet :: breadth in inches :	
144 : length in inches :: breadth in inches :	

EX-

EXAMPLES.

Let there be 5 planks of the following dimensions.

L.	B.	L.	B.	Area.	L.	B.	Area.	L.	B.	Area.
f.	in.	f.	f.	f.	f.	in.	f.	in.	in.	f.
20	9	1 : 20 ::	,75 :	15;	or 12 : 20 ::	9 :	15;	or 144 : 240 ::	9 :	15
40	6	1 : 40 ::	,5 :	20;	or 12 : 40 ::	6 :	20;	or 144 : 480 ::	6 :	20
36	12	1 : 36 ::	1 :	36;	or 12 : 36 ::	12 :	36;	or 144 : 432 ::	12 :	36
30	15	1 : 30 ::	1.25 :	37.5;	or 12 : 30 ::	15 :	37½;	or 144 : 360 ::	15 :	37½
15	18	1 : 15 ::	1.5 :	22.5;	or 12 : 15 ::	18 :	22½;	or 144 : 180 ::	18 :	22½

Here every example is done 3 different ways; which method may be very useful for proving the examples, of which our readers may furnish themselves for practice with as many as they please, the above containing 15 different questions in the rule of three, which we presume sufficient for our purpose: Their solutions will be found as before directed: For if 1 at the beginning of the slider be accounted 1 tenth, 1 in the middle will be unity. If then this 1, in the middle of the slider, be set against 20, on the rule; then will 15 on the rule be against 75 on the slider: Or, if 12 be set against 20; then 15 will be against 9: And if 144 be set against 240, 15 will likewise be against 9. The like may be said of all the rest.

Of measuring Timber.

We have shewn before how this may be done by the pen, *viz.* by finding the superficial content, as if it was plank. This multiplied by the thickness in inches, and the product divided by 12, the quotient will be the content in feet. So that here there will be two operations: The proportions are,

1st. 12 : length in feet : : breadth in inches : area in feet.
2d. 12 : area in feet : : thickness in inches : contents in feet.
Or, 1 : breadth in inches : : thickness in inches . a fourth number.
And 144 : fourth number: length in feet : contents in feet.

That is, multiply the breadth by the thickness, if both be inches, and this product by the length in feet; divide the last product by 144, the quotient will be the content in feet.

EXAMPLE.

Required the content of a piece of timber 20 feet long, 18 inches broad, and 15 inches thick.

1st. 12 : 20 : : 15 : 25 Or, 1 : 18 : : 15 : 270
Then 12 : 25 : : 18 : $37\frac{1}{2}$ 144 : 270 : : 20 : $37\frac{1}{2}$

Here we must draw out the slider twice; first 12 against 20, then 25 will be against 15; secondly 12 against 25, and $37\frac{1}{2}$ will be against 20, or 1 against 18; 270 will be against 15, and if 144 be set against 270, $37\frac{1}{2}$ will be against 20 as before. There will be no occasion to estimate the value of the fourth proportional to the three first numbers; it will be sufficient to mark it so as that the slider may be moved till 12 or 144 be against this point: But as this will be attended with some inconveniency, it will be best to make use of the inverted line, which performs it at once without moving the slider twice.

Description and Use of the inverted Line.

The slider is fitted betwixt two double lines of numbers, of which the lower one is inverted, in such a manner, that 12 upon it, is exactly against 12 upon the upper line; so 20, 30, *&c.* upon the inverted line, are as much to the left hand of the point 12, as 20, 30, *&c.* are to the right hand of the point 12 upon the upper line. In reading the inverted line, we begin at the right; and because the distance betwixt 1 and 12 is more than that betwixt 12 and 100, the inverted line begins at 1,4, for 1 would extend beyond the end of the ruler.

Now the slider having two double lines of numbers, graduated exactly as the upper line, it will follow that whatever way the slider be moved, the point 12 upon the upper line on the rule, and the point 12 upon the lower line on the slider, will be both against the same number.

To measure Timber by this Line, when the Breadth is not the same with the Thickness, and both given in Inches, and the Length in Feet.

Rule. First find any of the three given numbers upon the inverted line; then as this number upon the inverted line : is to either of the two given numbers upon the slider : : so is the third given

given number upon the upper line : to the content in feet upon the ſlider; obſerving that the upper line upon the ſlider, compares with the upper line upon the rule, and the lower line upon the ſlider with the inverted line.

As in the foregoing example; ſuppoſe a piece of timber 20 feet long, 18 inches broad, and 15 inches thick; ſet 15 on the inverted line, againſt 18 on the ſlider, and againſt 20 on the upper line, you'll find 37½ upon the ſlider; the content the ſame as by the two operations.

The reaſon of this will appear very plain, only by conſidering in what manner it is performed by two operations, and making uſe of the ſame ſlider, and the upper line with which it compares: For firſt, to find the fourth proportional to 12 ; 15 : : 18, we draw the ſlider out till 15 upon it is againſt 12 upon the upper line, and when in this poſition, 12 upon the ſlider will be againſt 15 upon the inverted line. Now the fourth number will be on the ſlider againſt 18 upon the upper line, which will be 22½. And becauſe 18 on the upper line, is the ſame diſtance from 12 upon the ſame line, that 18 is from 12 upon the ſlider; or which is the ſame thing, that 18 upon the ſlider is from 15 upon the inverted line; when we draw the ſlider to the left hand, to bring 22½, the fourth number, to 12 upon the upper line; the point 18 upon the ſlider will likewiſe come as far to the left, and therefore will be againſt 15 upon the inverted line; which is the very thing we are directed to do by the rule : and when it is thus ſet, 12 upon the upper line will be againſt the fourth number on the ſlider; and the content, without moving the ſlider, will be found upon it againſt 20 upon the upper line.

Tho' all plank is meaſured as a ſurface, the value is eſtimated by the load, which is 50 ſolid feet: The following proportion will ſerve to find how many ſuperficial feet of plank will make a load, *viz.* As the inches thick : is to 12 : : ſo is 50 the ſolid feet in a load : to the ſuperficial feet.

E X A M P L E S.

How many ſuperficial feet of 2, 3, 4, inches plank will make a load.

2 : 12 : : 50 : 300
3 : 12 : : 50 : 200
4 : 12 : : 50 : 150

SECT.

SECT. III.

Of the single Line commonly called the girt Line.

IT was observed before in the construction of this line, that the figures upon it are double the distance from one another, that the same figures are upon the double, and triple the distance that they are upon the triple line: Also that the cube and square roots were had by inspection. We shall now shew the use of it in measuring timber: And first,

Case 1. When the timber is square, that is, when the breadth and thickness are both alike, and given in inches, and the length in feet; to find the content in feet.

Rule. Set 12 upon the girt against the length on the double line, and the content will be on the double line, against the thickness on the girt.

EXAMPLE.

Required the content of a piece of timber 8 inches thick, and 9 feet long. Set 12 upon the girt, against 9 upon the double line; and against 8 upon the girt will be 4 upon the double line; the required content in feet. The reason of this will appear very plain. If we work it by the double line, the proportion will be 144 : (8×8) 64 : : 9 : 4; so the extent from 144 to 64 will reach from 9 to 4; and if we move the slider till 144 upon it, be against 9 upon the double line, then will 4 be against 64. Now, if instead of 144 and 64 upon the double line on the slider, we take 12 and 8, the roots of these numbers upon the girt line; they being the same distance from one another, they will perform the same office; and because 144 is always the first term, and the square of the thickness the second term, the rule will be general.

This might likewise be performed by the double line without moving the slider twice: The proportions will be 12 : 8 : : 9 : 6. Now when 12 is set against 8, then will 6 be the fourth proportional to the three first numbers; and in the next three numbers, the first and second terms being the same as before, there will be no occasion to move the slider; only look for 6, the fourth numberber before found upon the same line with the 12, and against it you'll find 4, the content, upon the same line with the 8.

Case 2. When the breadth and thickness are unequal, to find the contents by the girt line.

First find a mean proportional between the breadth and thickness, in inches; then set 12 upon the girt, against the length in feet upon the double line and against the mean upon the girt; will be the contents in feet upon the double line.

E X A M P L E.

Required the contents of a piece of timber 20 feet long, 18 inches broad, and 15 inches thick. Before this can be done we must find the mean thus.

Look for either of the given numbers, suppose 18, upon the double line, and move the slider till this number is opposite to 18, the same number, upon the girt line; then look for 15, the other given number, upon the double line, and against it you'll find the mean required upon the girt line, which will be a little more than 16.4; and when 12 upon the girt is set against 20 on the double line; against the mean on the girt is 37 $\frac{1}{2}$, the content on the double line.

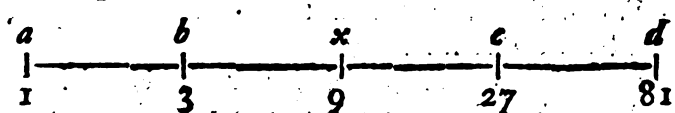
But it is plain that this method requires two operations; one to find the mean, and the other to set 12 to the length; so that it will be better to use the inverted line, as before directed.

To demonstrate the reason of this method of finding the mean, it must be observed, that to do it by the pen, the two numbers must first be multiplied into one another, and then the square root of the product will be the mean required. Let the two numbers be 3 and 27; their product is 81, the square root of which is 9, the mean required; for $3 \times 27 = 81 = 9 \times 9$.

Now to do this by the line of numbers, we must first extend from 1 to 3; that from 27 will reach to 81: And to find the square root of 81, we must find the middle point betwixt 81 and the beginning of the line; but this will be exactly in the middle betwixt 3 and 27; for let the extent from 1 to 3 upon the line of numbers, be represented by the line *a b*; and let *c* be the point 27; the extent *a b* set forward from *c*, will reach to the product 81 at *d*. Now to find *x*, the middle of the line *a d*, divide *b c* into two equal parts, which will give the point required: For

a b,

a b, and *c d*, being equal by construction, they will be equally distant from the point *x*, the middle of the line.



The figures on the girt line, as was observed before, are double the distance from one another, that the same figures are upon the double: Therefore when 27 on the double line, is set against 27 on the girt, 3 on the double line, which then will be against 9 on the girt, must be the middle point betwixt 3 and 27 upon the girt, which may be proved by a pair of compasses; though these two numbers 3 and 27, are not to be found upon the girt line, which begins on most of the sliding rules at 4, and ends at 40: Now it is very plain, that if the line was produced to the left, of a sufficient length to begin with 1, that point would be as far to the left of 4, as 10 is to the left of 40; and 2 and 3 would likewise be as far to the left of 4, as 20 and 30 are to the left of 40; and if the extent from 40 to 30 be added to that betwixt 9 and 4, these two will be found equal to the extent betwixt 27 and 9.

The reason I presume, for calling the single line the girt line, is, because when the quarter of the circumference is taken for the side of the square, the tree is girted with a line, and the breadth and thickness being supposed equal, the content will be readily found by this line; and the reason for beginning at 4 will appear by the following examples.

EXAMPLES.

Let there be 3 pieces of timber of 30 feet each, their breadth and thickness equal as below.

Long.	Thick.	Contents.
30	8	13.3
30	9	16.9
30	23	100

If the girt line begins at 1, when 12 upon it is set against 30 upon the double line, then 8 and 9 will be beyond the end of the double line; so the content cannot be had, unless we observe what point of the girt line is against 1 at the end of the double line, and then bring 1 at the beginning of the double line against

the

the ſame point; whereas by beginning at 4, we can have the content for any thickneſs from 4 to 40.

But this line may be adapted to ſeveral other uſes: We ſhall only mention the three following, *viz.*

1ſt. The length and breadth of a ſhip being given; to find the tonnage.

It was obſerved in *Exam.* 7. *Sect.* 2. that the method of doing this required two operations, *viz.* Firſt to multiply half the breadth by the breadth; then, as 94 is to this product; ſo is the length of the keel to the tonnage. Now if we double the firſt and ſecond terms, their products will be proportional to the third and fourth terms as before; the double of the ſecond term is equal to the ſquare of the breadth, and 188 is the double of 94; therefore 188 is to the ſquare of the breadth; as the length of the keel, is to the tonnage. So that if 188 upon the double line of numbers on the ſlider, be ſet againſt the length of the keel upon the double line on the rule; then will the tonnage be upon the rule, againſt the ſquare of the breadth upon the ſlider. But if, inſtead of the firſt and ſecond terms upon the double line, we take their roots upon the girt line, becauſe they are the ſame diſtance from one another; the tonnage may be found without moving the ſlider twice, as in the following example, where we ſhall take the ſame dimenſions as before.

E X A M P L E.

Length of the keel 93 feet 4 inches, breadth 32 feet; required the tonnage.

Rule. Set the tonnage point upon the girt againſt 93 feet 4 inches, upon the double line; then againſt 32, the breadth upon the girt, is 508 upon the double line; the required tonnage.

Note. The tonnage point upon the girt may be found by ſetting 10 upon the girt, againſt 10 upon the double line; then againſt 188 upon the double line, make a mark upon the girt, which will be the tonnage point. Hence, if the length and tonnage be given, and the breadth required; ſet the tonnage point againſt the length upon the double line; then the breadth will be on the girt, againſt the tonnage on the double line. But if the tonnage and breadth are given, and the length required; ſet the tonnage upon the double line, againſt the breadth on the girt; then will the length be on the double line, againſt the tonnage point upon the girt line. As if it were required to find the breadth of a ſhip of 300 tons, the length of the keel being 78 feet: When the tonnage point is ſet againſt 78, then will 27, the required breadth upon the girt, be againſt 300 upon the double line: Or ſuppoſe the breadth 26 feet, and tonnage 280; required the length of the keel. Set 280 upon the double, againſt 26 on the girt line; then againſt the tonnage point is 78 on the double line, the required length of the keel. So in this caſe, one foot in breadth will increaſe the tonnage 20 feet, which may be ſeen without moving the ſlider.

2*d*. To find the content of a tree by the girt line, the diameter and length being given, and ſuppoſing it to be hewed ſo as that the two wanes ſhall be equal to one ſquare.

We obſerved before, that if by the wanes be underſtood, what the flat wants to compleat the ſide of the ſquare, the proportion would be, as 180, is to the ſquare of the diameter in inches; ſo is the length in feet, to the contents in feet. Inſtead of the two firſt terms upon the double line, take their ſquare roots upon the girt: The root of 180 will be found nearly 13.4, but there is no occaſion to eſtimate the value, but only to find the point, which will be done by ſetting 10 on the girt againſt 10 on the double line, and the point will be upon the girt line againſt 180 upon double line.

E X A M P L E.

Suppoſe a tree 20 feet long, and 30 inches diameter: Required what the content will be when hewed, ſo as that the flat ſhall be equal to half the thickneſs?

Set the point upon the girt, found as now directed, againſt 20 on the double line; and againſt 30 on the girt, is 100 upon the double line; the required content in feet.

But if by the wanes, be underſtood the round parts of the tree where there is no wood taken off; the proportion, as was before obſerved, will be, as 193 nearly, is to the ſquare of the diameter; ſo is the length to the content. In this caſe we muſt make uſe of the ſquare root of 193; for which purpoſe we may find a point upon the girt, in the ſame manner as the point for the root of 180 was found. Now if the point for 193 be ſet againſt 20; then againſt 30 upon the girt, will be 93 on the double line; which is 7 *per Cent*. leſs than the former: A cylinder of ſuch dimenſions would meaſure only 98.17; ſo that by hewing the tree till the ſquares are half the thickneſs, it will meaſure near 2 *per Cent*. more than the full contents of the tree, if there had been no wood taken off.

3*d*. There are two points generally marked on this line; one W G, the other A G, for wine and ale gallons, their uſe is in gauging; for a wine gallon containing 231 cubic inches, and a circle whoſe area is 231, having for its diameter 17.14 inches: It is plain a cylinder of that diameter

meter will contain as many wine gallons as it is inches in height; therefore if the length and mean diameter of any cask be given, the wine gallons that it will contain, may be found by the following proportion.

As the square of 17.14 is to the square of the mean diameter; so is the length, to the content in wine gallons.

Hence if the gauge point W G upon the girt line, (which is the square root of the first term) be set against the length on the double line; then the contents in wine gallons will be found on the double line, against the diameter of a cylinder, or the mean diameter of any cask upon the girt line; but if ale gallons be required, we must make use of the point A G, which should be exactly at 18.94, the diameter in inches of a circle, whose area is 282, the cubic inches in an ale gallon.

S E C T. IV.

Of the triple Line of Numbers.

IT was shewn in the construction of this line how to find the cubes, and their roots by inspection. We shall now shew how to find the dimensions of similar solids of different contents, as for instance: Suppose a ship of 508 tons to be 93 feet 4 inches by the keel, and 32 feet broad, and it be required to find the length of the keel, and extreme breadth of a ship of 400 tons.

This was performed by the logarithms in *Chap.* 5. *Ex.* 4. where it was observed that the proportion is, as the tonnage of one ship, or the content of any solid, is to the cube of the keel, or of any other part; so is the tonnage, or the content of any other similar solid, to the cube of the required keel, or any other similar part: Hence 508 : 400 : : cube of 93.4 to the cube of the required keel; and the extent from 508 to 400 upon any line of numbers, will reach from the cube of 93.4 to the cube of 86, the length of the required keel. But there will be no need of finding the cube, (which is the fourth proportional to the three given numbers) if the root can be found: Now the cubes of any two numbers are three times the distance from one another that the roots are upon the same line; and because the two tonnages are the first and second terms, they will be the same distance from one another that the cubes of the keels are, which are the third and fourth terms, and therefore their roots will be one third of the distance from one another that the tonnages are; which in this case are 508 and 400. Let the distance then betwixt these two numbers be divided into three equal parts, one of which being ſet back from 93.4, will reach to 86, the length of the keel required.

Now, as the roots are the ſame diſtance from one another upon the ſingle line, that the cubes are upon the triple line; the keels will be the ſame diſtance from one another upon the ſingle line, that the tonnages are upon the triple line. Therefore the following method may be uſed where there is a cube line adapted to the ſingle line.

Rule. Set 94 feet 4 inches, or 93.33, the length of the given keel, upon the ſingle line, againſt 508, the tonnage upon the triple line; then againſt 400 upon the triple line, is 86 upon the ſingle line, which is the length of the keel required. And when the ſlider is thus ſet, we have, by inſpection, the lengths of the keels of all ſhips that are ſimilar to this, be the tonnage what it will: For if the tons are found on the triple line, the lengths of their correſponding keels will be againſt them on the ſingle line.

The like method may be uſed in finding the other dimenſions, as in the followings table, where the dimenſions of a ſhip of 508 tons are ſuppoſed to be as in the columns in the upper line, and are pretty near to thoſe of a ſhip of 20 guns; the other tonnages are nearly thoſe of 40, 50, *&c.* guns. We have in each column ſet down the real dimenſions in feet and inches, below thoſe found by the rule, which are in decimals, that our readers may ſee that ſome dimenſions are pretty near ſimilar in all ſhips, and others arbitrary.

Guns.	Tons.		Keels.	Extreme Breadth.	Heighth of the Breadth	Tranſ. Breadth	Tranſ. Heigh.	Length in Hold.	Length of gun Deck.
20	508	real	93: 4	32: 0	13: 8	18:4	16:4	11: 0	113: 0
	814	rule	109, 0	37, 5	16, 0	21,5	19,3	12, 9	132; 0
40		real	108:10	37: 6	16: 6	22:8	22:0	16: 0	133, 0
	1052	rule	118, 5	40,75	17, 5	23,5	21,2	14,16	144, 0
50		real	117: 8½	41: 0	18: 0	25:0	24:1	17, 8	144: 0
	1191	rule	123, 5	41,49	18, 2	24,4	22,0	14,65	150: 0
60		real	123: 0½	42: 8	19, 4	26:0	25;2	18: 6	150: 0
	1414	rule	131, 4	45, 0	19, 2	25,7	23,2	15, 4	159, 0
70		real	131: 4	45, 0	20: 4	27:6	26:3	19: 4	160: 0
	1585	rule	136, 0	46,75	19.85	26,9	24,2	15, 9	165 0
80		real	134:10½	47: 0	21: 0	30:5	27:0	20: 0	165 0
	1730	rule	140, 3	48, 4	20, 0	27,6	24,8	16, 6	170, 0
90		real	138: 4	48: 6	21: 9	31:5	27:9	20: 6	170, 0
	2000	rule	147, 0	50, 5	21, 6	29,0	26,1	17, 4	178, 0
100		real	144: 6½	51: 0	22: 9	33:0	29:0	21: 5	178: 0

It muſt be obſerved, that as every figure is three times placed upon the triple line, it will be indifferent in what part of the line the tonnages are taken. The only thing to be regarded is, that the ſlider muſt be ſo placed, that all the tonnages whoſe dimenſions are required, be againſt ſome part of the ſingle line.

Now, if the firſt 5 on the triple line be accounted 500; when 93.4 is ſet to 508; 40 on the ſingle line, which is now 400, will be againſt 40000 on the triple line; and 64, which is at the beginning of the triple line, will be againſt 46.6 on the ſingle. So that in this poſition, the numbers upon the triple line betwixt 64 and 40, will not be againſt any part of the ſingle line; for there will be no numbers leſs than 64 upon the triple line, without altering the value of the figures; but 4 at the beginning of the ſingle line, will always be the ſame diſtance from 64, the beginning of the triple line, that 40 at the end of the ſingle line, is from 64000 at the end of the triple line. As the ſlider is now ſet, 4 at the beginning of the ſingle line is accounted 40; and becauſe the diſtance betwixt 64000 and 40000, is the ſame with that betwixt 64 and 40; the value of the figures may be alter'd, and 64000 at the end of the triple line may be called 64, and 40000 will be 40. But we muſt likewiſe alter the value of the figures on the ſingle line; and 4 at the beginning of the line muſt be four units, and 40 on the triple, will be againſt 40 on the ſingle line; 20 on the triple againſt 31.7 upon the ſingle, *&c.* as in the columns: To find the figures betwixt 64 and 40, let the ſecond 5 upon the triple line be 500, and the ſlider ſet as before directed; then will 40 on the triple be againſt 4, which is at the beginning of the ſingle line, but is now accounted 40; againſt 50 on the triple, is 43 on the ſingle; againſt 60 on the triple, is 45.75 on the ſingle, *&c.*

In the ſame manner the extreme breadths to any aſſigned tonnage may be found, ſuppoſing 32 feet to be the extreme breadth of a ſhip of 508 tons: Let the third 5 on the triple line be 500, and when 32 on the ſingle is againſt 508 upon the triple, then we have all the numbers below 640 upon the triple line; and 1 upon the triple will be againſt 4 on the ſingle. But if the ſecond 5 be 500, and the ſlider properly ſet, 990 on the triple line, will be againſt 40 on the ſingle; and as in this poſition we cannot find the dimenſions correſponding to 1000, and the numbers above it; we muſt in theſe, and ſuch like caſes, obſerve what point of the triple line is againſt 40 at the end of the ſingle line; and draw out the ſlider till 4 at the beginning of the ſingle be againſt the ſame point, which in this example is 990: And as the value of the figures upon the triple line are not altered, 4 upon the ſingle line muſt be accounted 40; and then againſt

against 1000 is 40,1, against 2000 is 50,5, &c. the breadths corresponding to those tunnages. Again, let 5 be some given dimension of a ship of 508 ton ; if the slider is properly set, 260 on the triple, is against 4, the beginning of the single line; and when 40 at the end of the single line is accounted 4, and brought against 260 on the triple; then against 100 on the triple, is 2,91 on the single, against 10 on the triple, is 1,35, against 1 on the triple, is 6,25 on the single. All the other dimensions in the columns are found by the same method, *viz.* by setting the given dimension to its proper tonnage; and to prove the work, after the length of the keels are found, we may use the double lines of numbers as before directed in *Ex.* 6. *Sect.* 2. of this *Chap.* Here the keel of a ship of 400 tons is found to be 86; when this is set against 93.4, the keel of 508 tons, against 32, the breadth of 508, is 29.6, the breadth of a ship of 400 tons: And as all the dimensions in the columns for a ship of 508 are known, look for them on the same line that her keel is taken, and against them will be found the corresponding dimensions for a ship of 400 tons.

The Tonnage of a Ship being given to find the Length of the keel, and extreme Breadth.

Before this can be done, the proportion that the length of the keel bears to the extreme breadth must be determined; which suppose as 3 to 1, and then six times the half breadth will be the required length of the keel. Now, because in finding the tonnage, when the length and breadth are given, we are directed to multiply the length by the breadth, and that product by the half breadth, and then divide this last product by 94, and the quotient will be the tonnage; it is certain, if the tonnage be multiplied by 94, the product will be the same as if the length, breadth, and half breadth, were multiplied into one another; and if any of these three be given, the others are found by the given proportion they bear to one another. We shall therefore work for the half breadth, which may be found by the following rule, *viz.*

First multiply the given tonnage by 94; then divide that product by 12, and lastly extract the square root of the quotient; and that root will be the half breadth required.

E X A M P L E.

Let the given tonnage be $127\frac{62}{94}$; then $127\frac{62}{94} \times 94 = 12000$, and $12000 \div 12 = 1000$, the cube root of which is 10, the half breadth; so 20 will be the extreme breadth, and 60 the length.

The

The reaſon of dividing the product by 12, is becauſe 12 times the cube of the half braadth, is always equal to the product of the heighth of the keel, breadth, and half breadth multiplied into one another, when the length is three times the breadth, as will appear by the following proceſs.

The half breadth 10.

The breadth will be $10 \times 2 = 20$.

The length of the keel will be $10 \times 2 \times 3 = 60$.

Now, as in multiplication, when ſeveral numbers are to be multiplied into one another, it is indifferent in what order the operations are performed; ſo it is evident that $10 \times 10 \times 10 \times 2 \times 2 \times 3 = 10 \times 20 \times 60 = 12000$, but $10 \times 10 \times 10$, is the cube of the half breadth, and $2 \times 2 \times 3$ is 12; therefore 12 times the cube of the half breadth will be equal to the product of the length, breadth, and half breadth, multiplied into one another.

By the ſliding rule, ſet 12 upon the double line on the ſlider, againſt 94 upon the double line; on the rule look for the tonnage $127\frac{62}{94}$, or 127,66 upon the ſlider, againſt which is 1000. Then to find the cube root of this, ſet the ſlider ſo that 64 upon the triple, is againſt 4 on the ſingle; then againſt 1000 on the triple, is 10 on the ſingle; the half breadth.

But as the length does not bear any conſtant proportion to the breadth; after finding the dimenſions by this rule, the breadth may be altered, and the length of the keel can be had by the tonnage point. There is no invariable rule to determine the breadth, but in ſhips under 500 tons, this method will always bring it to leſs than a foot, and therefore may be very uſeful in determining proper dimenſions for a ſhip of any number of tons.

CHAP.

CHAP. VII. SECT. I.

Of the Construction and Use of several Lines on the Shipwright's Rule.

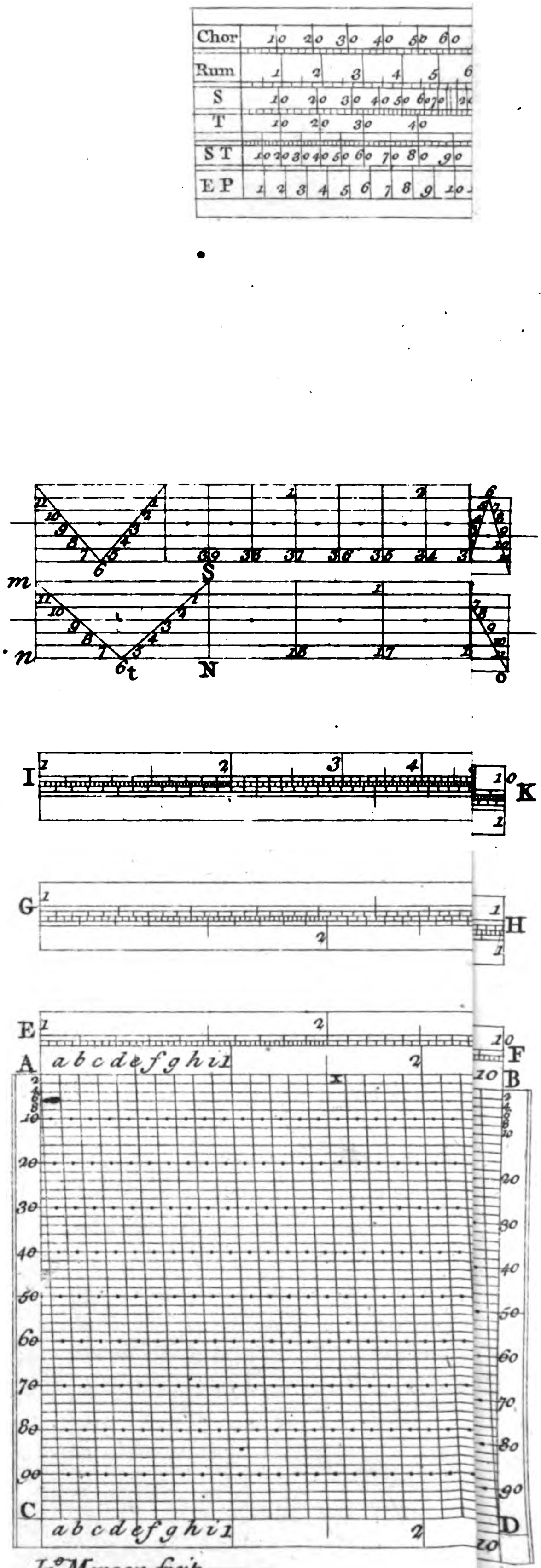
Of SECTOR LINES.

THE instrument called a sector, is only a rule with a good joint, containing several different lines, each divided into some given proportion; as sines, chords, equal parts, *&c.* Every line upon one leg has a corresponding one upon the other leg, both exactly of the same length, and divided in the same manner; and all the lines on both legs meet at the center of the joint.

It is very useful in all the practical parts of the mathematicks, especially in dividing a line into any number of equal parts, or into any given proportion. As for instance, if it was required to divide a line, as A B, into any number of equal parts, suppose 9, (See *Plate* 3. *Fig.* 1, 5.) it is only opening the rule till the distance betwixt 9 and 9, in the line of equal parts, be equal to the line to be divided; and then the extent from 1 to 1, from 2 to 2, *&c.* in the same lines will be one ninth, two ninths, *&c.* parts of the line A B; for by opening the rule, the point 9, and every other point in the lines E, P, in effect, describe arches of circles; and if chords be drawn to these arches, they will form so many isosceles triangles, whose bases will be parallels to one another: Therefore C 9 is to 99, as C 1 is to 11, but C 1 is the ninth part of the line C 9; therefore 11 is the ninth part of the line 99.

In like manner, if it were required to make the line D F a line of numbers, when the rule is opened till the distance betwixt the extremities of the lines of numbers be equal to the line D F; then if the several distances from 1 to 1, from 2 to 2, *&c.* be set off from D towards F, the line D F will be a line of numbers.

Another method of dividing a line, as B D (See *Plate* 3. *Fig.* 4.) in the same proportion as the line A B is this. At the point B, the end of the divided line, make an angle, and set off the line to be divided from B to D; then draw the line A C parallel to A B, making D 3 equal A 3,



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D 2 equal to A 2; D 1 equal to A 1: Draw the lines A D, 33, 22, 11: Then the triangles B 1 *a*, B 2 *b*, B 3 *c*, B A D, being similar, B A : B D : : B 3 : B *c* : : B 2 : B *b* : : B 1 : B *a*.

But before any of these methods can be used, the lines on the rule must be divided by some proportion given in numbers, or from the equal divisions of the arch of a circle.

The sector lines on the shipwrights rules, are for making masts and yards, of which there are four on each leg, divided into the same proportion, that the diameters at the several quarters bear to that at the slings; which being given in numbers, and expressed by the fraction, each quarter is of the slings, they may be constructed in the following manner:

Make an equilateral triangle A B C. (*Plate* 3. *Fig.* 6.) From the point A set off, on the lines A B, and A C, the equal parts expressed by the several denominators of the fractions, and draw lines across at these divisions. Then set off, on these lines, the equal parts expressed by the respective numerators of the fractions, and draw lines from A thro' these points to intersect the line B C. So if the side of the triangle be supposed to be the diameter at the slings, the several divisions of the line B C, from the point B, will be the diameters at the quarters. It will be proper to raise the fractions, so their denominators do not exceed 100.

E X A M P L E.

Let it be required to construct the line for yards, the quarters being the following fractions of the slings.

1st. $\frac{27}{28}$, or $\frac{81}{84}$. *2d.* $\frac{9}{10}$, or $\frac{90}{100}$. *3d.* $\frac{7}{10}$, or $\frac{70}{100}$. Yard arm $\frac{2}{5}$, or $\frac{40}{100}$.

Having constructed the triangle A B C, set off 84 equal parts, the denominator of the first quarter, from A to *a*; and from A to *b* draw the line *a b*, which will be parallel to B C. Upon the line *a b* set off 81 equal parts, the numerator of the first quarter, from *a* to *c*; and draw the line A *c* to intersect the line B C in the first quarter. Again, for the second quarter, its denominator is 100; therefore set off 100 equal parts from A to B, and from A to C, which in this case is already done, because the side of the equilateral triangle was made 100 equal parts: It remains only to set 90 equal parts from B to the second quarter, and draw a line from A to this point. The denominators of the third quarter and yard arm, are likewise 100; the numerator for the third quarter is 70, which set off from B to the third quarter, and draw a line from A to this point:

The numerator for the yard arm is 40, which ſet off from B to *y* A, and draw a line from A to this point; by this means, if B C be ſuppoſed the diameter of a yard at the ſlings, B *y* A will be that at the yard arm; B 3 qr. that at the third quarter; B 2 qr. that at the ſecond; and B 1 qr. that at the firſt quarter; they being by conſtruction $\frac{2}{3}$, $\frac{7}{10}$, $\frac{9}{10}$, $\frac{27}{28}$, of the line B C.

The line being thus divided, may be transferred to each leg of the rule: But if the rule will not contain the length of the line, take ſuch a length as may ſuit the rule, and with a pair of compaſſes, ſet off that length from A to G, and from A to F; and draw the line G F, which being parallel to B C, will be divided in the ſame proportion, and may be transferred to the rule. In the plate (*Fig.* 7.) the lines are only drawn to *y* A, but if continued, would meet in the center of the joint.

The line being thus conſtructed, and transferred to the rule, we ſhall ſhew the uſe of it in making a yard; (*Plate* 3. *Fig.* 8.) which ſuppoſe 71 feet long, of which A B is one half, and S S, the diameter at the ſlings 17 inches: Having divided the line A B into four equal parts at the points 1 qr. 2 qr. 3 qr. open the rule till the diſtance betwixt S and S at the extremities of the lines on the rule, be equal to 17 inches, the diameter of the yard at the ſlings; then the extent from the dots on the one leg, to thoſe correſponding on the other leg, will give the diameters at thoſe quarters, and at the yard arm, from which they may be ſet off upon the yard. But it muſt be obſerved, that only the half of each of thoſe diameters thus found, muſt be taken, and that ſet off on each ſide of the middle line upon the yard; for which reaſon, in practice, it will do better to take half the diameter at the ſlings, and ſet the rule by that; and then we have half the diameters at the quarters by the rule.

The lines A B, and A C, (*Fig.* 6.) and the lines drawn from the point A for the quarters, may be produced to any length, and drawn upon a board. If then the lines A B, and A C, be divided into inches, halfs, and quarters, there will be no occaſion to transfer them to the rule: For ſuppoſe the diameter at the ſlings 21 inches, the half is $10\frac{1}{2}$; lay a ruler, or ſtrait edged batten acroſs the board, from $10\frac{1}{2}$ in the line A B, to $10\frac{1}{2}$ in the line A C, and make a mark upon the batten at each interſection with the lines drawn from A for the quarters. By this means we have, upon the batten, half the diameter at each quarter and yard arm; and theſe being ſet off on each ſide of the middle line upon the yard, in their proper places, will give the whole diameter at thoſe places.

After the ſame manner are the lines for maſts, bowſprits, and mizen yards conſtructed, by an equilateral triangle, and from thence transferred to to the rules; the proportions, or fractional parts, by which the lines are constructed, are as follows:

	1	2	3	Hounds.	Head.	Heel.	Cape.	Yard Arm.
Quarters Masts	$\frac{60}{61}$	$\frac{14}{15}$	$\frac{5}{6}$	$\frac{9}{11}$	$\frac{4}{7}$	—	—	—
Bowsprits	$\frac{30}{31}$	$\frac{9}{10}$	$\frac{3}{4}$	—	—	$\frac{2}{3}$	$\frac{1}{2}$	—
Mizen Yard lower arm	$\frac{21}{22}$	$\frac{11}{12}$	$\frac{5}{6}$	—	—	—	—	$\frac{9}{11}$
Mizen Yard upper arm	$\frac{14}{15}$	$\frac{13}{15}$	$\frac{2}{3}$	—	—	—	—	$\frac{1}{2}$

Another method of proportioning the quarters to the slings, is taken from the divisions of the quarter of a circle, but the diameter at the yard arm must be first determined; which suppose $\frac{2}{5}$ of the slings, as before, and let G F (*Plate* 3. *Fig.* 3.) represent the diameter at the slings. Now to find the diameter at the quarters, take the following rule.

1st. With the radius G F, describe the quadrant G F E.

2d. Upon the line G E, set off $\frac{2}{5}$ of the diameter at the slings, from G to *e*, and thro' *e* draw a line parallel to G F, to intersect the arch in *a*; from *a* draw a line parallel to G E, to intersect the line G F in *y*; then will *y a* be the diameter at the yard arm.

3d. Divide the line G *y* into four equal parts, in the points 1, 2, 3, and draw the lines 1 qr. 2 qr. 3 qr. parallel to G E, which will be the diameters at those quarters.

There is also another line on each leg; these meet at the center of the joint, from whence they are numbered 1, 2, &c. to 12; the space betwixt the figures is nearly $\frac{3}{4}$ of an inch, each divided into 12 equal parts: So the spaces betwixt the figures may represent feet, and the intermediate divisions inches. But if it was required to make a scale of feet and inches, where one quarter of an inch, one eighth of an inch, or any other space shall represent one foot; it may be done by the same lines, as before directed. Thus, take with the compasses 12 times the space intended to represent one foot, and open the rule till that reaches from 12 upon one leg to 12 upon the other leg. Now, whereas the feet and inches of the scale on the rule, are taken from the center of the joint; those of the other scale, must be taken across at those points.

SECT. II.

Of the Construction of the Ten, Twelve, and Eight square Lines.

THE use of these lines, is in order to hew a piece of timber so that it shall be a prism contained under 10, 12, or 8 equal planes, which are called squares; their bases will be polygons of the same number of equal sides.

The first thing to be done, is to hew the piece four square; the four sides for a ten square will not be equal, for its base will be a rectangle; whereas the bases for a twelve and eight square will be squares; so they must first be hewed into parallelopipedons.

There are three lines necessary for making a ten square, *viz.* The first for determining the longest side of the rectangle; the second for determining the side of the polygon; and the third for determining how much wood must be taken off the corners to reduce the rectangle to a polygon. Their chief use is for making barrels of capstans.

To Construct the Ten square Lines.

Upon the line A B (*Plate* 3. *Fig.* 9.) erect a perpendicular C F, which make any number of inches, suppose one and a half. Thro' F draw the line D G parallel to A B: From C draw the lines C *m* and C *n*, making each an angle of 18 degrees with the line C F, so shall *m n* be the side of a polygon of ten equal sides; for the angle at the center is 36 degrees, the tenth part of 360. With the radius C *m* (equal to C *n*,) describe the semi-circle A *m n* B, and erect the perpendiculars A D, B G; so shall the rectangle A D G B, be half the base of the piece when hewed into four squares. Now it is plain, that the piece will be thicker one way than the other; for C F is one half of one of the diameters, and C B half the other; and F *m* half the side of one of the ten squares: To reduce this rectangle to a polygon, make *m s*, *s* A, *n t*, *t* B, equal to *m n*, and produce the lines *m s* and *n t*, to *b* and *o*, and cut off the triangles D *m b* and G *n o*; cut off also the triangles *b s* A and *f o* B, so shall A *s m n t* B, be half the barrel of a capstan of three inches diameter. If F *m* be divided into three equal parts, we shall have the divisions of the line marked B S, (*Fig.* 2.) which may be continued to any length. The use of this, is to find the side of the

the ten ſquare, which is done by ſetting off as many diviſions of this line as the barrel is inches thick on each ſide of the middle line on the two big ſides.

If the line A *b* (*Plate* 3. *Fig.* 2.) be divided into three equal parts, we ſhall have the diviſions of the line marked S S, which ſerves to ſhew how much muſt be ſet off from the middle line on the two ſmall ſides, *viz.* one of theſe diviſions, for every inch of the thickneſs of the barrel, which in this caſe will be to *b* and *o*; and when the wood is taken off from *m* to *b*, and from *n* to *o*, we may make *m s* and *n t*, each equal to *m n*, alſo equal to S A and *t* B.

If the line A B be divided into three equal parts, it will give the diviſions of the line L D, (*Fig.* 2.) which ſerves to ſhew how much the barrel muſt be the biggeſt way.

In making the barrel, the firſt thing to be done, is to hew it to the deſigned thickneſs in inches; which is what is called ſideing of it, and afterwards to ſquare it; that is to hew it the other way. In order to do this, we muſt take as many diviſions of the line L D, as the barrel is to be inches thick, ſo when it is thus hewn, it will have two big, and two ſmall ſides: There muſt be a middle line ſtruck on each ſide, and then proceed as directed, by ſetting off the proper diſtances from the middle lines; and when all the wood is taken off, there will remain nothing of the ſmall ſides but the middle line; but of the two big ſides there will remain the ſide of the ten ſquare.

In conſtructing theſe lines. it will be proper to produce the lines C F and C *m*, till C F is ſix inches or more; then will F *m* be half the ſide of the ten ſquare of a barrel of 12 inches thick, which may be divided into 12 equal parts; the ſame muſt be done with the lines S S and L D.

To Conſtruct the Lines for a 12 *Square.*

Theſe are likewiſe chiefly for making barrels of capſtans, (*Plate* 3. *Fig.* 10.) but here the piece muſt be hewed exactly 4 ſquare; for when all the wood is taken off, the whole ſide of the 12 ſquare will remain on all the four ſides.

There are only two lines neceſſary for reducing the 4 ſquare to a 12, and formed after the ſame manner as thoſe for a 10 ſquare; for ſince the angle at the center of a polygon of 12 equal ſides is 30 degrees, if at C two angles of 15 degrees each be made, and the lines C *r* and C *s* be drawn, it is plain *r s* will be the ſide of the 12 ſquare, of which F *r* is one half; which being divided into 3 equal parts, will give the diviſions of the line 4 S: (*Fig.* 2.) There muſt be as many of theſe diviſions ſet off from the

the middle line on all the 4 ſides, as the barrel is inches thick, and therefore muſt be ſet off from A to *b*, and from B to *o*, as well as from F to *r* and *s*; but the whole wood from *r* to *b* muſt not be taken off. It muſt firſt be taken from *r* to *c*, the point where the ſide *r d* produced interſects the line A D, which is half the ſide of the ſquare. A *c* divided into three equal parts, will give the diviſions of the line 2 S, (*Fig.* 2.) which are to be ſet off from the middle line, on two of the oppoſite ſides, as from A to *c*, and from B to *n*; and when the wood is taken off from theſe points to *r* and *s*, the points *d* and *t* may be found, by making *r d* and *s t* equal to *r s*; and when the wood is taken off from *d* to *b*, and from *t* to *o*, we have A, *b*, *d*, *r*, *s*, *t*, *o*, B, the half of a 12 ſquare of 3 inches thick; but the lines C F and C *r*, may be produced as directed in forming the lines for the 10 ſquare.

To Conſtruct the Eight ſquare Lines.

Theſe are on moſt of the ſhipwright's rules. Their uſe is in making maſts and yards, and ſo well known, that we need give no directions how to apply them; and ſhall only remark, that tho' maſts and yards are round, they muſt be firſt hewed four ſquare, each ſide being equal to the diameter of the maſt or yard; which, ſuppoſe A B, (*Plate* 3. *Fig.* 11.) then will the rectangle A G D B, be half a four ſquare, in which a ſemi-circle may be inſcribed of the ſame diameter with the maſt or yard. If from the center C be drawn the lines C *m* and C *n*, making each an angle of 22 ½ degrees with the line C F, we ſhall have *m n* the ſide of a polygon of eight equal ſides, of which F *n* is one half; and if this is divided into as many equal parts as the diameter has inches, it will give the diviſions of the 8 ſquare line; one of which for every inch diameter muſt be ſet off from the middle line on all 4 ſides: And becauſe the diameter is ſuppoſed 3, when that number is ſet off from the points A, B and F, we ſhall have the points *r*, *m*, *n*, *b*: But theſe points may be found by ſetting off their diſtance from D and G, the ſide of the ſquare; for which purpoſe the line *n* G or *m* D, muſt be divided into 3 equal parts; ſo either of the lines may be uſed.

Theſe lines are on foot rules; but there are two lines on the ſlider of the two foot rules; the one for finding the diameter at the quarters; the other for finding the length and diameter at the ſlings of maſts and yards. We have already ſhewn how to find the quarters by the ſliding rule, which requires an operation for every quarter; but this line ſhews them all at once, only by ſetting a point in the ſlider to the given diameter of the

ſlings;

slings. It is the upper line upon the slider, and has a space for masts, one for yards, one for bowsprits, and one for each arm of a mizen yard.

To find the Quarters by this Line.

For masts, set P on the slider against the diameter at the partners, and against the figures 1, 2, 3, you will find the diameters of the 1st, 2d, and 3d quarters; against H S will be the diameter at the hounds, and against *m h* the diameter of the lower mast head, but if it is a top mast, the diameter will be against T *m*.

After the same manner the quarters of the yards may be found, by setting S against the diameter at the slings; and against the figures 1, 2, 3, will be found the diameters of 1st, 2d, and 3d, quarters; and against YA, the diameter at the yard arm. The same may be said of the bowsprit and mizen yard: One example will be sufficient to illustrate the whole.

Required the bigness at the Quarters, and Yard Arm, of a Main Yard 30 Inches at the Slings.

Set S against 30, then against 1 will be 29, nearest, for the first quarter; against 2 will be 27, for the second; against 3 will be 21 for the third; and against Y A will be 12 for the yard arm.

Note. The slider must be applied to a double line of numbers.

Before this line can be constructed, the proportion that the quarters bear to the slings must be determined, which suppose to be as before.

		In decimals.
First quarter	$\frac{27}{28}$	.964
Second quarter	$\frac{9}{10}$	.9
Third quarter	$\frac{7}{10}$	.7
Yard arm	$\frac{2}{5}$	.4

At any convenient place upon the slider, assign a point for S, and draw a score at that point for the slings; set that point or score against 28, and against 27 draw a score upon the slider for 1; then move the slider till S is against 10, and against 9 draw a score upon the slider, for 2 against 7 draw a score for 3; then move S to 5, and against 2 draw a score for the yard arm.

The reason of this will appear very plain, if we find the quarters by a pair of compasses; for then we take the distance betwixt 28 and 27, (which will always be equal to the distance betwixt the diameter at the slings, and that at the first quarter) but this is equal to the distance betwixt

twixt S and 1 upon the slider; therefore if S be set against the diameter at the slings, 1 will be against that at the first quarter; the like may be said of the rest.

If the fractions are all reduced to decimals, the strokes for the quarters, *&c.* may be remarked, without moving the slider after S is set to 1, as will be found upon examination; and the construction of the line will appear plainer by the decimals; for the points 1, 2, 3 and Y, are exactly the same distance from S, that the numbers .964, .9, .7, .4 are from 1 on the line of numbers. There is no difference therefore between the line on the slider, and the line of numbers, only when the proportion of the quarters, *&c.* is determined by numbers; as the distances properly set off from 1, to decimal parts, will reach from S to the points 1, 2, 3 and Y A; these points are used instead of the numbers .364, .9, .7, .4.

The lower line on the slider, is for finding the lengths of masts and yards, but before this can be done, there must be a given proportion; the following is generally allowed for masts, *viz.* As 100 is to 76, so is the extreme breadth in feet, to the length of the main mast in yards.

The lengths of the other masts are proportioned to the main mast, and the points or strokes upon the slider, are laid down from B *r b*, by those established proportions, in the same manner as the diameters at the quarters of the yards are proportioned to that of the slings.

The main mast is allowed to be in proportion to the fore mast, as 100 to 89; in like manner there must be proportions for the mizen mast, bowsprit, main, fore and mizen top masts, topgallant mast and sprit sail top mast; by these proportions, the points or strokes, are set off for the different masts at proper distances from B *r b*.

To find the Lengths of the Masts by the Line.

Set *B r b* against the extreme breadth in feet; then the lengths of all the masts will be against their respective divisions, which are all distinguished by letters peculiar to each; and at the same time, the lengths of their heads and hounds may be found, there being peculiar letters for that purpose, and those for distinction's sake are across the slider.

Thus *Mhd* } For main mast head.

Mhs } For main mast hounds.

EX-

E X A M P L E.

Required the lengths of the masts of a ship whose extreme breadth is 50 feet.

Set B *r h* against 50; then the lengths of all the masts, mast heads and hounds, will be found against their respective divisions, distinguished by letters as follows, *viz.*

		Lengths, Yards.		Head, Feet.		Hounds, Feet.	
M	Main mast	38	,0	15	.8	6	.6
F	Fore mast	33	$\frac{3}{4}$	14	.1	5	.3
Z	Mizen mast	32	$\frac{1}{4}$	12	.2	4	.85
B	Bowsprit	24	$\frac{1}{4}$	——		——	
M*t*	Main top mast	22	$\frac{3}{4}$	6	.75	2	.7
F*t*	Fore top mast	20	$\frac{3}{4}$	6	.1	2	.4
Z*t*	Mizen top mast	16	,1	4	.75	1	.9
M*g*	Main top gallant mast	11	,0	23	.25	1	.28
F*g*	Fore top gallant mast	9	,9	2	.9	1	.26
S*t*	Spritsail top mast	8	,6	——		——	

The lengths of the yards are proportioned to the length of the gun deck, and estimated in the same measure that the gun deck is, and are found in the same manner as the masts are.

Set G D against the length of the gun deck, and the lengths of all the yards will be found against their proper letters, as in the following examples, *viz.*

Admit the gun deck to be 174 feet; required the lengths of all the yards.

Set G D against 174; then the several lengths will be found as follows, *viz.*

		Feet.
M	The main yard	103
F	The fore yard	90 $\frac{1}{2}$
Z	The mizen yard	82
M*t*	The main top sail yard	71
F*t*	The fore top sail yard	62 $\frac{1}{2}$
M*g*	The main top gallant yard	49
Z*t*	The mizen top sail yard	47
F*g*	The fore top gallant yard	43 $\frac{1}{4}$

To find the diameter of any mast or yard, first find the length in feet. Set F L against that length, and you will find the diameter in inches a-

 gainst

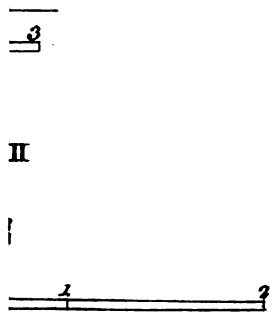
gainft the letters, reprefenting the maft or yard, whofe diameter is required as in the following, *viz.*

	Feet.		Diameter.
Main maft	114	L *m*	38
Fore maft	102	ditto	34
Bowfprit	97 ½	B	48 ¾
Main top maft	69	T *m*	20 ½
Fore top maft	62 ½	ditto	18 ¾
Mizen top maft	48	ditto	14 ·4
Main top gallant maft	33	—	9 ·9
Fore top gallant maft	30	—	9 ·0

The letters L *m* fignify all lower mafts. B the bowfprit. T *m* all top mafts, and top gallant mafts.

The line for finding the lengths and diameters of mafts and yards, is put upon the infide of the flider on the foot rules.

The End of the Firft Part.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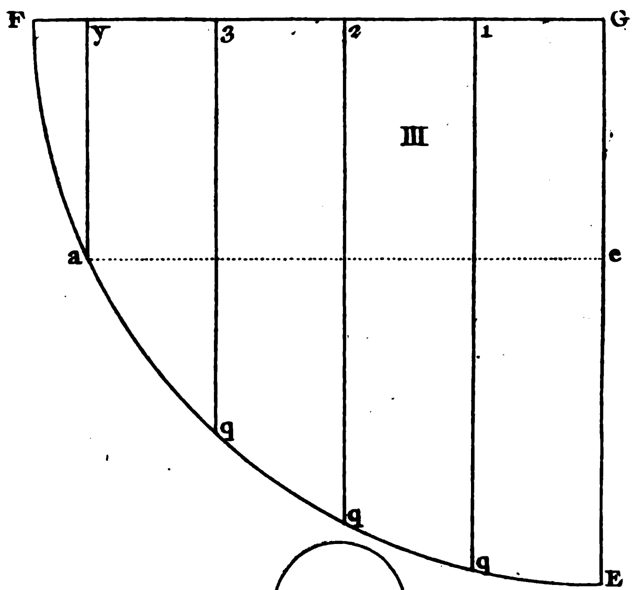
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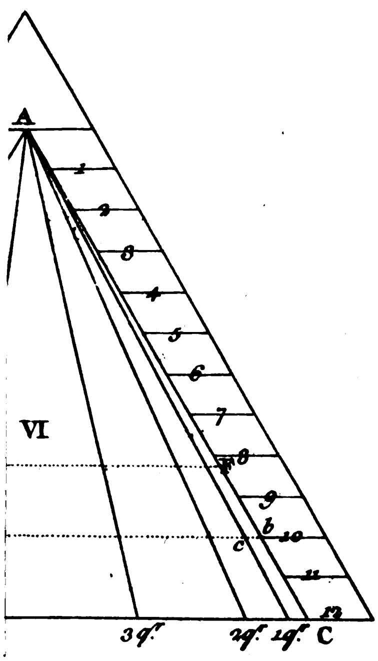
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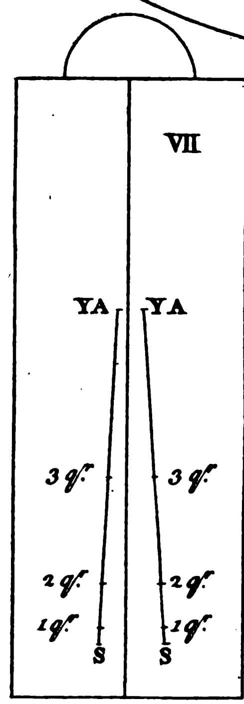
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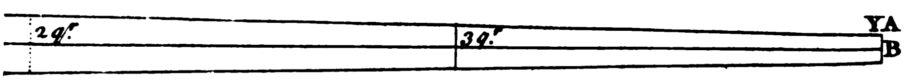
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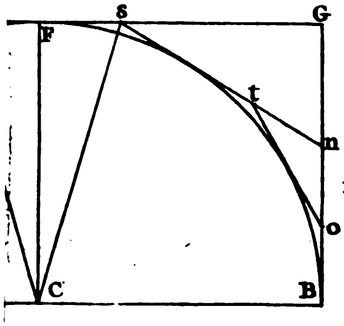
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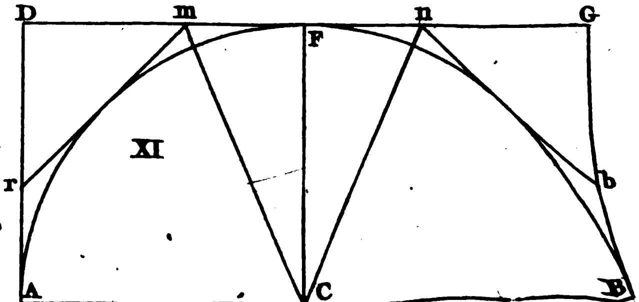
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THE THEORY OF

SHIPBUILDING and NAVIGATION.

PART II.

CHAP. I.

Of the Orthographick projection of Solids on a Plane.

THE chief defign of delineating a houfe, fhip, or any other folid upon a plane, is to fettle the juft dimenfions, and fymetry of its parts according to the fcheme of the builder. When this is done by mathematical rules, we can find the exact length, breadth and heighth, not only of the whole, but alfo of any particular apartment on a fheet of paper. However, as a plane has but two dimenfions, *viz.* length and breadth, and a folid three; they cannot all be reprefented by only one projection on the fame plane.

A plane is an even furface, to which a right line may be every way applied, and upon which there are feveral ways of projecting folids. We fhall only treat of the orthographic projection, as beft fuited to our purpofe.

Before any folid can be reprefented by this way of projection upon a plane, it muft be fuppofed to be cut by feveral planes: Thefe are called plain fections, and will form even furfaces, which having but two dimenfions, may be delineated upon a plane: And when the folid is cut fo as to form an uneven furface, it is always fuppofed to be covered with an even one before it can be reprefented upon a plane; fo that in effect, we only reprefent one plane upon another.

The thing to be reprefented is called the original, and the plane upon which it is to be reprefented, the plane of the projection.

When ſeveral lines parallel to one another, are drawn from all the parts of an original, to cut the plane of the projection; they will upon it deſcribe a figure, which is called the projection of that original. The lines producing this figure, are called the projecting lines or rays; and this manner of repreſenting any object, is called the orthographick projection of that object.

This paralleliſm of rays is the eſſential property which diſtinguiſhes the orthographick from all other kinds of projection: And tho' it is indifferent in what direction the projecting lines are drawn; it will be more convenient to make them perpendicular to the plane of the projection, and when this is parallel to the horizon, the length and breadth of any ſolid can be found by a plummet carried round it with a thread, ſo as to touch all the parts of it; but the heighth cannot be repreſented by this operation. This is what is called a plan of a building.

If another plane be erected perpendicular to the horizon, and the ſolid in the ſame poſition, ſuppoſed to be cut length ways by ſeveral planes parallel to one another, and perpendicular to the horizon; we can upon it, repreſent the true lengths and heighths of all theſe ſections; but inſtead of a plummet, we muſt make uſe of a ſquare. This is what is called the plane of elevation, or ſide view of a building.

If another plane be erected perpendicular to the two former, we can upon it, repreſent the heighth and breadth of any ſection, cutting the ſolid right acroſs, perpendicular to the horizontal and ſide planes. This, in a building, is called the profile, being an end view; in a ſhip, the head or ſtern view. By theſe three planes all the parts of a ſolid may be repreſented; and if two of the planes be known, the third may be found without having recourſe to the ſolid.

By this deſcription, it may ſeem that a houſe or ſhip, cannot be thus delineated till actually built. But it muſt be obſerved, that the extreme length, breadth and heighth, muſt be determined; by which the three planes aforeſaid may be delineated. Theſe may be called the out lines. The ſeveral parts contained within them may be delineated ſo as to anſwer the intended uſe; by which means we ſhall have a diſtinct view of the whole deſign, and may diſcover any inconveniencies that may attend ſuch a diſpoſition of the parts, which may be eaſily remedied upon paper; and the true dimenſions of every particular may then be had upon the draught: Whereas, if we go to erect the ſtructure without a draught, we run the hazard of pulling down ſeveral parts in order to make them uniform and convenient for the reſt.

The

The delineating a ſhip upon a plane is called drawing, and the repreſentation is called a draught.

Properties of the Orthographick projection.

1. All right lines upon an original plane, that are perpendicular to the plane of the projection, will be repreſented by points.

2. All lines on an original plane, that is inclined to the plane of the projection, will be repreſented by ſhorter ones, but if they be parallel on the original, they will be ſo when projected; and when the original is parallel to the plane of the projection, they will be exactly of the ſame length in both.

3. All planes, whether limited by right or curve lines, when perpendicular to the plane of the projection, will be repreſented by right lines.

4. All planes parallel to the plane of the projection, will be repreſented by equal and ſimilar planes; but if they be inclined, their repreſentations will be leſs than their originals.

5. The mutual interſection of two planes, is a right line common to both planes, and is called their common ſection; and if ſeveral planes interſect one another in the ſame line, it will be common to all the planes.

6. In the orthographick projection, the diſtance of the original from the plane of the projection, makes no alteration in its repreſentation.

7. Before any plane can be repreſented upon another, its poſition with reſpect to the plane of the projection, muſt be determined.

8. The inclination of one plane to another, is the angle formed by two lines, one in each plane, both drawn perpendicular to the common ſection; where they meet at the ſame point.

In order to illuſtrate and demonſtrate theſe properties, we ſhall in the following problems, ſhew the different repreſentations of a plane according to its poſition, in reſpect of the plane of the projection; and to aſſiſt the imagination in conceiving why the ſame plane will have different repreſentations; the figures are ſo contrived that they may be cut, and erected to any required angle with the plane of the projection.

PROB. I.

Given, the plane A *r d a* B C, (*Fig.* 5.) upon which are drawn the lines *r* 1, *d* 2, *a* 3, parallel to A C, and perpendicular to B C; required, its projection upon the plane B M N T, to which it is ſuppoſed perpendicular; ſo that *a* C G ſhall be a right angle.

Draw

Draw the line C *g* perpendicular to C G, and make it equal to C B; (*Plate* 4. *Fig.* 5.) then will C *g* be the projection of the plane A *r d a* B C. But if it were required to project it so that A C D shall be a right angle; draw the line C B perpendicular to C D, and B C will be the required projection of the plane A *r d a* B C: This will be manifest when the plane is erected, till the point A is perpendicular to the point C; for then parallel lines drawn from the points *r*, *d*, *a*, will fall upon the line C B, and be represented by the points 1, 2, 3; and the whole plane A *r d a* B C, will be represented by the right line B C, as by properties 1 and 3: For A C and C D being perpendiculars to C B, the common section of the two planes, will form a right angle at C, when the plane A *r d a* B C, is turned upon the axis B C till the point A is perpendicular to C; and if then a thread be stretched through the points A and D, we shall have a right angled triangle, of which the thread is the hypothenuse, A C the perpendicular, C D the base, and A C D the right angle; tho' it was a right line before the plane was erected.

Note. The arch B *a d r* A, and the radius A C, are to be cut through, and then the plane may be turned upon the axis B C.

PROB. II.

Given the plane A *r d a* B C, to find its representation on the plane B M N T, to which it is supposed parallel.

This is only making a plane equal and similar to the given one. Therefore produce the lines A C, *r* 1, *d* 2, *a* 3, to D, *t*, *f*, *c*; make C D equal A C, 1 *t* equal *r* 1, 2 *f* equal *d* 2, and 3 *c* equal *a* 3; so shall C B *c f t* D be the plane required. For if the plane A *r d a* B C, be turned round upon the axis B C, till it is parallel to the plane; the point A will come to the point D, *r* to *t*, &c. and the parallels *r* 1, *d* 2, *a* 3, will be projected into equal and parallel lines, as by properties 2 and 4. And the projection will be the same, if the original is lifted up to any distance above the plane of the projection, so it be parallel to it, tho' the projecting lines be oblique to the plane: As if it were required to project the plane A *r d a* B C upon the plane R P O S, so that the point H shall represent the point B. Draw the line B H, and parallel to it, lines from the points A, *r*, *d*, *a*, 3, 2, 1, C. Make these lines each equal to the line B H; so shall H 6 5 4 *k* K, be the plane required, equal and similar to the plane A *r d a* B C; we have omitted drawing the lines from 3, 2, 1 and C to avoid confusion.

If the original is inclined to the plane of the projection, its representation will be less than the original, but its true dimensions may be found

found by the following problem; provided its inclination and representation be known, and the direction of the projecting lines.

PROB. III.

Let the angle G C E be the inclination, and B *b e s* E C, the representation of a plane, and the projecting lines perpendicular to the plane B *b e s* E; required A *r d a* B C, the original plane.

It is evident by the properties before described, that when the original is inclined according to the given angle, the projecting lines let fall from A, *r*, *d*, *s*, will fall in the points E, *s*, *e*, *b*, to which they are supposed perpendicular; and of consequence E C, *s* 1, *e* 2, *b* 3, will the bases, and *r* 1, *d* 2, *a* 3, hypothenuses of right angled triangles; of which the angles and bases being given, the hypothenuses may be found by constructing the triangles; or if the base and hypothenuse is given, the angle of inclination may be had in the triangle.

At the point E erect the perpendicular E G; at C make the given angle, and draw the line C G to intersect the perpendicular in G; then will the hypothenuse C G be equal to A C. This will easily be conceived by erecting the triangle G C E, till it is perpendicular to the plane B *b e s* E; and when the plane A *r d a* B C is inclined to it, according to the given angle, the lines A C and C G will coincide: In like manner the lines *r* 1, *d* 2, *a* 3, may be found, by erecting perpendiculars at the points *s*, *e*, *b*, and drawing lines parallel to C G, from the points 1, 2, 3, to intersect these perpendiculars, which will form so many right angled triangles: Their hypothenuses will be equal to the lines *r* 1, *d* 2, *a* 3.

Note. The lines C G and G E, in the triangle are to be cut through.

PROB. IV.

Given, the plane A *r d a* B C, and its inclination (the angle G C E) to the plane B M N T, to project it upon that plane.

This is only the reverse of the former, for here the hypothenuses and angles, are given to find the bases. Make therefore the given angle at C, and C G equal A C, from G let fall the perpendicular G E; so shall the point E be the representation of the point A. Having thus constructed the triangle G C E, right angled at E; the base C E will be the projection of the line A C. In like manner, the bases 1 *s*, 2 *e*, 3 *b*, may be found; by making G 1, G 2, G 3, in the triangle, equal *r* 1, *d* 2, *a* 3, in the plane A *r d a* B C: Then draw the lines 1 *s*, 2 *e*, 3 *b*, parallel to C E: These set off from the points 1, 2, 3, in the line B C, upon perpendiculars drawn to those points on the plane B M N T, will give the lines

lines 1 *s*, 2 *r*, 3 *b*; ſo that B *b e* E C, will be the projection of the plane A *r d a* B C.

Our readers ſhould be well acquainted with theſe problems: For we ſhall have frequent occaſion to find the inclination of one plane to another. The next thing neceſſary to be underſtood is, when ſeveral planes are given, and their inclinations to one another, to find the dimenſions of another plane which will interſect them in any aſſign'd poſition.

This will admit of ſeveral varieties; in order to explain which, we ſhall ſhew how to lay down a ſolid upon a plane. For this purpoſe we ſhall chuſe one limited by ſix planes, like a cheſt, as being the ſimpleſt, and eaſieſt to be repreſented of all ſolids; for if the ends be two equal right angled parallelograms, ſuppoſed to be parallel to one another, and perpendicular to the bottom; the ſides will be equal, and parallel planes; the top and bottom will alſo be equal. But in order to make all the planes different, we ſhall ſuppoſe it broader at one end than the other, and the top ſloping.

Let then C *s u* A, be the narrow, and D *t b* B, (*Plate* 4. *Fig.* 1, 4.) the broad end, right angled at *u* and A, alſo at *b* and B; their heighth at the back ſide B D and A C equal. The ends being ſuppoſed perpendicular to the bottom and back ſide, will occaſion the ſides likewiſe to be perpendicular to the bottom, tho' not parallel to one another; the angle T D *t*, or its equal *s* C S, will give the ſlope of the top; or its inclination to the back ſide. Now all the planes are different, and before their dimenſions can be determined, the length of the cheſt muſt be given; which ſuppoſe the line A B. This will determine the dimenſions of all the planes. And firſt to find the bottom. At the points A and B, draw the perpendiculars A E and B F; make B F equal B *b*, and A E equal A *u*, and draw the line E F; ſo the plane A B F E, will be the bottom. Secondly, to find the back ſide, produce the perpendiculars E A and F B, to C and D; making A C and B D, equal the given heighth of the ends, and draw the line C D; ſo ſhall the plane A C D B, be the back ſide. The bottom is the horizontal plane, and the back ſide, the plane of the elevation; and when this is erected perpendicular to that, the angles D B F and G A E, will be right. And if the plane D *t b* B be turned about upon D B as an axis; and the plane C *s u* A, turned round upon the axis C A, till they are perpendiclar to the horizontal and elevation planes: Then perpendicular lines drawn from the points D and *t*, will fall in the points B and F, and from C and *s* in the points A and E; ſo that the plane A E F B, will be the projection of the top on the horizontal plane; which will be leſs than the original, becauſe it is not parallel to the plane upon which

which it is projected. But as its inclination is given, the true dimensions may be had by *Prob.* 3. The triangles being constructed, produce the perpendiculars B D and A C, to K and I, making D K equal to the hypothenuse D *t*, and C I equal to the hypothenuse C *s*; and draw the line I K; so shall the plane C I K D be the top.

In like manner, the foreside may be projected upon the plane of elevation, which will be the plane A S T B: But this will be too short. Therefore from the points F and E, draw the perpendiculars E G and F H; making F H equal to *t b*, and E G equal to *s u*, and draw the line G H; so shall the plane E F H G, be the foreside. The planes being thus found, they may be cut by the dotted lines, and erected to their proper position; and when the line I K, and the dotted perpendiculars are cut, the top may then be turned round upon the line C D, till it lies flat upon the plane A C D B, where it may be pressed down; by which means the line C D will remain immoveable, and acquire such a crease or fold, that the plane I K D C, may be turned round upon it as upon a hinge. In like manner, the dotted lines C *s*, *s u*, *u* A, (*Fig.* 1.) and the dotted lines D *t*, *t b*, B *b*, (*Fig.* 4.) may be cut; and the ends turned round upon the lines C A and D B, till they are laid flat upon the plane A C D B. And when the top is likewise laid flat upon these, the plane A C D B, may (together with the ends and top) be turned round upon the line A B, till it lies flat upon the plane A G H B; after which all the planes may be turned round, and laid in their first situation; and so they will all lie in one plane. We may now easily erect the chest; and first erect the ends till they are perpendicular to the back side; and the top may be turned round till it lays upon the lines C *s* and D *t*. And if the tongue *x* (*Fig.* 1.) be put in the slit *x* in the top, near I; and the tongue *a* (*Fig.* 4.) in the slit *a* in the top near K; then the backside, together with the ends and top, may be turned round upon the line A B, till it comes to be perpendicular to the bottom; and the point *u* (*Fig.* 1.) will be in the point E: And to keep it fast, the tongue *y* may be put in the slit *y*. In like manner, the point *b* (*Fig.* 4.) will be in the point F, and the tongue *e* may be put into the slit *e*; so we have now the ends, top and backside fastened to the bottom: And if the dotted lines E G, G H, H F be cut, the plane E G H F may be erected perpendicular to the bottom. The point G will be in the point *s*, and the point H in the point *t*; and the tongue L, which may represent the hasp of a lock, may be put into the slit L in the foreside.

The chest being thus erected, it will be easy to make any partitions within it. But if the position of these partitions be known, their true dimensions, form and inclination may be found before the planes are erected; provided the dimensions and inclination of these planes to one another be given.

The horizontal and elevation planes are always perpendicular to one another. Now a plane may intersect these two in three different positions.

1st, When perpendicular to both. Its intersection in both planes will then be in a right line perpendicular to the line AB, the common section of the two planes, as *Fig*. 1. which intersects the elevation plane in the line C A, and the horizontal in the line A E. This is what the shipwrights call a square plane.

2dly, When inclined to the plane of elevation, but perpendicular to the horizontal, as *Fig*. 2. which intersects the plane of elevation in the line GH perpendicular to A B, and the horizontal in the line H *b*: So its inclination to the plane of elevation, will be the angle *n* H *b*. This the shipwrights call a canted plane. But if the plane A E F B, be the plane of elevation, and the plane A C D B, the horizontal; the plane intersecting them in the lines G H and H *b*, would then be called a raking plane.

3dly, A plane may be inclined to both, as *Fig*. 3. intersecting the plane of elevation in the line M N, and the horizontal in the line N *q*. This plane is said to cant and rake. Their positions being thus determined, their dimensions are likewise determined.

PROB. V.

To find the Dimensions of a square Plane, intersecting the common Section of the Horizontal and Elevation Planes in the Point A.

Through A draw the perpendicular I E, intersecting the common section of the top and backside in the point C, and the common section of the bottom and foreside in the point E. Make a right angle at A, because the required plane is supposed perpendicular to the horizontal; and make A *u* equal to A E, the breadth of the bottom at that place. Again, because the foreside is perpendicular to the bottom, make a right angle at *u*, and *u s* equal to E G, which is the breadth of the foreside at that place: Lastly, draw the line *s* C; so shall C *s* be equal to C I, the breadth of the top at that place; and C *s u* A the required plane, the angle S C *s*, being the inclination of the top.

The profile, is the proper plane to project all square planes upon, as being parallel to it. Now the plane D t *b* B, is the profile: And if it were required to find the dimensions of a plane, intersecting the horizontal in the line A E, and the elevation in the line A C; it is only setting

off

off A E from B to *p*, and drawing *p o* parallel to B D; then will D *o p* B, be the plane required, equal to C *s u* A; and *r o* equal to S *s*. For B D and C A, are equal by ſuppoſition.

PROB. VI.

To find the Dimenſions of a Cant Plane, interſecting the Horizontal Plane in the Line H *b*, *and the Elevation in the Perpendicular* G H.

From the point *b* let fall the perpendicular *b w*, and produce it to interſect the line S T in W, and draw the line G W; then will G W *w* H, be the projection of it upon the plane of elevation. Which will be leſs than the original, becauſe it is inclined to it; but its true dimenſions may be found by *Prob.* 3. Thus, make H *n* equal to H *b*, and raiſe a perpendicular at *n*: Make *n m* equal to *w* W, and draw the line G *m*; ſo ſhall G *m n* H, be the required plane. The angle *n* H *b*, its inclination to the backſide; and the angle H *b* E, the inclination to the foreſide. The inclination of the top will be the ſame as that of the bottom, which in this is a right angle.

PROB. VII.

To find the Dimenſions of a Plane that Rakes and Cants; interſecting the Horizontal in the Line N *q*, *and the Elevation in the Line* M N.

1ſt. From *q* let fall the perpendicular *q z*; thro' *z* draw the line P *f*, perpendicular to M N, produced to *f*. From the center N, with the radius N *q*, interſect the perpendicular *f p* in P; and draw the dotted line N P. Again, thro' M draw the line M R parallel, and equal to N P, and draw the line P R; ſo M N P R, would be the required plane, if the bottom were as broad at the point *v*, as it is at the point *z*, and the top parallel to the bottom. But as this is not the caſe here, the required plane will be narrower at the top than at the bottom.

2d. From M let fall the perpendicular M *v*, and draw the line *v* V, parallel to N *q*; make M *m* equal to *v* V, and draw the line P *m*. So M N P *m*, would be the required plane, if the top was parallel to the bottom.

3d. Make N P in the line N *q*, equal to *v* V, aud draw the perpendicular P *r*, and make M *n* equal to N *r*, So a line drawn from *r* to *n*, would be parallel to M N, and would interſect the line S T ſomewhere; and a line drawn from that point of interſection to M, would be the projection of a plane interſecting the plane of elevation in the line M N; and the horizontal plane in the line N P. But there will be no occasion to draw this line, because N *q* is the length of the line of interfection of the required plane; and *z* the projection of the point *q*. Therefore a line must be drawn from *z* to *n*, which will interfect the line S T in *x*; so shall M N *z x*, be the projection of the required plane, upon the plane of elevation, which will be less than the original. To find which, thro' *x* draw a perpendicular to the line M N, to interfect the line P *m* in the point *d*; so shall M *d* P N, be the plane required. And to find its inclination to the plane of elevation, draw the line Z *z* parallel to *f* M, and produce it to *t*: Make *f t* equal to *f p*; so shall *z f t*, be a right angled triangle; and the angle *z f t*, the inclination to the backside. For if the triangle be erected perpendicular to the plane, as mentioned in *Prob.* 3. the lines *f t* and *f p*, will coincide. To find the inclination to the foreside, make the angle *z* N *o*, equal to the angle *z f t*; so shall N *o* V, be the required angle. For if the lines A B and E F were parallel; the inclination to the backside and foreside, would be equal. Lastly, to find its inclination to the bottom; thro' the point *v* draw a perpendicular *a b* to N *q*, produced to *a*. From N, with the radius N M, interfect the perpendicular in the point *b*. With the radius *a b* interfect the line *v* V, produced in the point *c*; so shall *a v c*, be a right angled triangle, and the angle *c a v*, the inclination to the bottom. For if the plane M N P R, was projected upon the horizontal plane, the point M would be elevated till a perpendicular from it would fall upon the point *v*; so *a v* would be the base, and *a b* the hypothenuse of the right angled triangle *a v c*.

DEMONSTRATION.

At N erect the perpendicular N X, make N *s* equal to N *q*; then will N *s* Y X, be the true dimensions of a plane interfecting the elevation in the line X N, and the horizontal in the line N *q*, by the preceding problem; and the angle X N *q*, a right angle. For when the plane A B C D, is erected perpendicular to the plane A B F E; if the plane X N *s* Y, be turned round upon the axis X N, the point *s* will describe the semicircle *s q c b*, as if a door be opened, and turned upon the hinges till it lies against a partition or wall, the bottom of it will describe a semicircle up the floor. But if the upper hinge, suppose at X, be moved to M, it is plain the point *q* in the bottom of the door will not touch the floor.

And if the door be turned round upon the axis M N, till it lies flat upon the partition, or plane X N *s* Y; the line N *q* will lie upon the perpendicular N *e*; the angle M N *q* being always a right one; for the bottom

tom of the door is ſquare. But the required plane muſt touch the floor when it is in the direction of the line N *q*; therefore it cannot be ſquare: And becauſe the extremity of the bottom, when the plane is in its proper poſition, will be ſo far elevated above the plane M N *s* R, that a perpendicular from it will fall in the point *z*; it is obvious, that in turning the door upon the axis M N, a perpendicular let fall from the extreme point of the bottom, will always meet the plane M *f* P R, ſomewhere in the perpendicular *f z* produced; and therefore when the door is laid flat upon the partition, or plane M N P R, that extreme point muſt lie upon the point P, becauſe the dotted line N P, is made equal to N *q*; and when P R is drawn parallel to M N, and M R parallel, and equal to N P, we ſhall have the plane M N P R; ſo the angle M N P, will be that which the bottom makes with the ſide of the door. By which means the triangle N *k* P, will be added to the ſquare bottom; and when the door is turned upon the axis M N, till it is in the direction of the line N *q*; the dotted line N P will lie cloſe upon the floor on the line N *q*: When the door is in this poſition, if the ſtock of a ſquare be laid upon the plane M N *s y*, which may repreſent the partition of a room, and the tongue erected perpendicular to the plane, we may deſcribe the line *z n*, by keeping the ſtock of the ſquare perpendicular to the line M N; and moving it along the plane in that direction, the tongue always touching the ſide of the door. All which will appear very plain when the planes are cut by the dotted lines, and erected to their proper poſitions.

Thus I have endeavoured to explain the principles of the orthographick projection, by laying down the ſimpleſt ſolid that can well be thought of, or conceived: And if I have not done it in ſuch a manner as to make it intelligible to all capacities, it may be owing to their want of a ſufficient knowledge of the principles of geometry and trigonometry.

If they cannot attain this by what has been ſaid on thoſe ſubjects in the firſt part, I would adviſe ſuch to have recourſe to a maſter for inſtruction. If they are at a loſs how to lay down a ſolid limited by ſix planes, which requires no curves, it will be in vain for them to proceed any farther. For it will be impoſſible for them to comprehend the reaſon of the methods uſed in laying down irregular ſolids limited by various ſurfaces, ſome of which are plain, and others very irregular curves.

In laying down any irregular ſolid, we may ſuppoſe it to be incloſed within ſix planes, the oppoſites of which may be equal and parallel, and all the planes right angled; ſo we may then proceed upon the ſame principles as before, for we ſhall have the dimenſions of the three neceſſary planes,

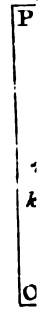
planes, *viz.* horizontal, elevation and profile. And if the true form and dimenſions of two planes parallel to the profile, and likewiſe their diſtance from one another be given; we may find the true form and dimenſions of any intermediate planes between them, in the ſame manner as we have done in that already laid down. But it muſt be obſerved, that the form of the ſurface, which limits that part of the ſolid intercepted betwixt the two given planes, muſt likewiſe be given; which ſhall be ſhewn plainly when we come to apply what has been now ſaid to the actual laying down of ſhips on a plane.

CHAP. II. SECT. I.

Explication of the Terms and Names of the Lines uſed in drawing Ships.

WHAT is chiefly intended in drawing of ſhips, is to find the form of all the timbers. Now if a ſhip's ſide were ſtrait fore and aft, this could be done with as much certainty, as in the ſolid laid down in the preceding chapter. In that caſe we need only determine the form of the foremoſt and aftermoſt timbers, and then all the ſections, either parallel or inclined to the horizontal, cutting the ſhip lengthwiſe, would be limited by ſtrait lines; any two points of which being given, a ruler or ſtrait edged batten would find the whole line. And tho' all ſections parallel to the profile may be limited by curves, and theſe very irregular, yet if a ſufficient number of points be found in each, a thin batten of pliable wood may be bent ſo as to touch all the points; and ſo deſcribe what is called a fair curve. Now we may have as many points as we pleaſe, only by forming as many ſections, either parallel or inclined to the horizontal, as ſhall be thought neceſſary to have points: For upon ſuppoſition that the ſide is ſtrait fore and aft, theſe ſections will all be limited by ſtrait lines, their lengths will be the given diſtance betwixt the two parallel planes, and their breadths will be determined by the direction in which they cut the profile. This is ſo plain that it needs no ex-

ample,



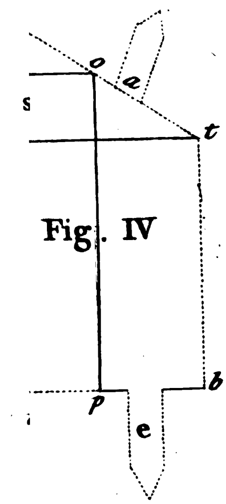


Fig. IV

ample, and a caſe that cannot happen in laying down a ſhip; which being broader near the middle than at either end, cannot have a ſtrait ſide. And therefore there will be a neceſſity of determining the form of the profile, by a ſection at the broadeſt place of the ſhip; and alſo the form of a ſection near each end parallel to the profile.

There can be no invariable rule given to determine the form of theſe three ſections, becauſe they muſt be conformable to the ſervice the ſhip is deſigned for: However, determined they muſt be, before we can begin to find the form of the intermediates; if by no other means, by repeated trials, till they pleaſe the fancy of the artiſt, aſſiſted by his judgment in diſcerning what form will beſt anſwer the propoſed deſign. The next thing to be determined, is the form and dimenſions of ſeveral ſections, either parallel or inclined to the horizontal. They will all be limited by curves, ſome of which will be very irregular. However there will be three points in each ſection given, by the direction in which they cut the given planes.

Here we think it neceſſary to explain what is to be underſtood by a fair curve; a term frequently uſed in drawing: In order to which it muſt be obſerved, that the circumference of a circle ſeems to be the only curve that can with certainty be drawn; for this requires no art. But in deſcribing an ellipſis we muſt find ſeveral points, thro' which the curve muſt paſs according to the eſtabliſhed properties of that curve; and then theſe points may be joined by a ſteady hand, which may be aſſiſted by a mould or pattern, of which the artiſt ſhould be provided with a variety of different ſorts made of pear-tree, box, or ſome other wood proper for that purpoſe. Now if one that is not very well acquainted with drawing, ſhould attempt to join theſe points without ſome ſuch aſſiſtance, he would make rather a polygon, conſiſting of ſeveral very obtuſe angles, than a curve; whereas a mould that will juſt touch three, four, or more of the points, will cut off all theſe angles and irregularities, by which means the curve will be clear of all breaches, or ſudden turnings and deviations; and this is what is called a fair curve. From this deſcription of it, we may plainly ſee that, if there be only three points of the curve given, there may be ſeveral curves drawn thro' them, and all very fair.

In forming an ellipſis, if the tranſverſe and conjugate diameters be given, we may with certainty, find any number of points, thro' which the curve muſt paſs. But the curves that are formed by the ſeveral ſections of a ſhip, are very irregular; and as they have no properties peculiar to themſelves, except that of being fair, there can be no invariable rule for deſcribing them. We ſhall therefore attempt to do it by ſector

lines

lines taken from an approved body. This will undoubtedly form lines ſimilar to the original; and the ſector is ſo contrived, that it will form very fair lines quite different from the original. It is preſumed this will be very uſeful to thoſe that are not well acquainted with drawing, for whoſe uſe it is chiefly intended. However, before we deſcribe it, we ſhall ſhew the methods generally uſed to regulate the form of all the curves that are neceſſary in drawing of ſhips; and in the firſt place ſhall explain their names in the following definitions. We ſhall make uſe of three planes, as deſcribed in the preceding chapter, but give them different names.

D E F I N I T I O N S.

1. The ſheer plane is the ſame with that of elevation, and is a ſection of a ſhip ſuppoſed to be cut, by a plane paſſing thro' the middle line of the keel, ſtem and ſtern-poſt.

2. The floor plane is the ſame with the horizontal, and is that on which the whole frame is erected: The upper ſide of the keel is in this plane.

3. The body is the ſame with the profile. It is a ſection, ſuppoſed to cut the ſhip thro' the broadeſt place, and is perpendicular to the ſheer and floor planes.

4. Water lines are ſuppoſed to be drawn on the ſurface of a ſhip, by the upper part of the water into which ſhe ſwims, and are formed by the ſection of a plane cutting the whole body lengthways, perpendicular to the ſheer plane, where they will always be repreſented by ſtrait lines; and if theſe are parallel to the keel, they will be repreſented by ſtrait lines on the body plane, called level lines. But theſe planes will be limited by curve lines on the floor plane, which in ſome caſes will be inverted at the after end, and alſo at the fore end; but this laſt is avoided as much as poſſible. Theſe curves limit the breadth of the ſhip at certain heighths, expreſſed by lines drawn on the ſheer plane for that purpoſe. But as the ſheer plane cuts the ſhip exactly in two equal and ſimilar parts, only one half of theſe ſections are laid down; ſo that one ſide will always be repreſented by a ſtrait line.

5. The heighths of the breadth lines are deſcribed on the ſheer plane, to determine the heighths at which the half breadth of the planes of the timbers are to be ſet off; in which reſpect, the water lines on the ſheer plane may be called the heighths of the breadth lines. But becauſe the extreme breadth of the plane of each timber riſes gradually from the midſhips fore and aft; the lines repreſenting their heighths will be curves.

Such

Such are the main, and top timber heighths of breadth lines, which are the principal ones that go by that name; but there may be as many more as ſhall be judged neceſſary to determine the form of all the timbers.

6. Half breadth lines are deſcribed on the floor plane, and are curves limiting the $\frac{1}{2}$ breadths of the planes of the timbers, at the heighths expreſſed by the correſponding heighth of breadth on the ſheer plane, in which reſpect the water lines may be called $\frac{1}{2}$ breadth lines; but that name is generally given only to ſuch whoſe heighths are expreſſed by curves on the ſheer plane. They are formed by ſuppoſing the ſhip to be cut length ways, in a perpendicular direction to the ſheer plane, thro' a curve heighth of breadth line. This will form an uneven ſurface; ſo the true length of it is not repreſented on the floor plane.

7. Ribband lines are either ſquare or canted.

The ſquare which is often called the horizontal ribband line, is in all reſpects the ſame with the above deſcribed $\frac{1}{2}$ breadth lines: The uſe of the ribband lines, is to faſten the timbers before the plank is brought on; for which purpoſe, they muſt be of a ſufficient ſubſtance, and formed in ſuch a manner, that they may fit the timbers without forcing or penning them. It would be very difficult to make a ſquare one, becauſe it rounds two ways; for when the ſhip is cut upon a level by this ribband, the ſurface produced, will be an uneven one. Upon this account, a plane muſt be ſo inclined to the ſheer plane, that it ſhall interſect the timbers at the ſame points with the ſquare ribband.

The cant or diagonal ribband, ſo called, becauſe it cuts the body plane in a diagonal, is formed by a plane inclined to the ſheer plane; and cutting the ſhip length ways in that direction, in ſuch a manner, that it will interſect the timbers in the ſame points that the ſquare ribband does. It will interſect the ſheer plane in a ſtrait line parallel to the keel, at the ſame heighth at the ſtem and poſt, with that of the ſquare ribband. Its repreſentation on the floor plane, will be the ſame as that of the ſquare ribband. But becauſe the plane of it is not parallel to the floor plane; this will not be the true breadth of it. This may be found by *Prob.* 3. of the preceding chapter, by which means we ſhall obtain the exact length and breadth of it; and being a plane, it will only round one way, and ſo a ribband may eaſily be made by it.

8. Sweeps, are arches of circles, deſcribed in the body plane to form the timbers, and are generally four.

1ſt. The floor ſweep, which is limited by a line drawn in the body plane, perpendicular to the middle line, a little above the keel. The diſtance of this line above the keel at the midſhip timber, is called the dead riſing; the upper part of this arch forms the head of the floor timber.

2*d*. The under breadth ſweep; the center of which is in the line that repreſents the heighth of the extreme breadth of the timber. If there is a part of the timber ſtrait, the center of the ſweep will be in the lower line. From this center extend to the point that limits the ½ breadth of the timber in the ſame line, and with that radius deſcribe a circle downwards, till it comes near to the floor ſweep.

3*d*. The reconciling ſweep, which joins the two former in ſuch a manner as to interſect neither; by which means we ſhall have a fair curve from the heighth of the breadth to the riſing line: And if a ſtrait line is drawn from the ſide of the keel at the upper edge, to touch the back of the floor ſweep, we ſhall have the form of the midſhip timber below the breadth.

4*th*. The upper breadth ſweep; the center of which is in the line that repreſents the extreme upper heighth of the breadth of the timber; from which a circle muſt be deſcribed to paſs thro' the point that limits the ½ breadth of the timber in the ſame line, and produced upwards diſcretionally to form the top timber. To theſe four ſome add a fifth, to form the hollow of the top timber; but this is generally done by a mould ſo placed as juſt to touch the above breadth ſweep, and paſs thro' the point that limits the ½ breadth of the top timber: So that now the form of the midſhip timber is determined from the keel to the top of the ſide. The radius of the underneath ſweep decreaſes, the farther the timber is from the midſhips; but the other ſweeps have generally the ſame radius for all the timbers. There is no certain rule to determine the radii of theſe ſweeps. Some do it by proportioning them to the extreme breadth of the ſhip, according to ſome given ratio. But there are various ways of forming this midſhip timber; ſometimes by two arches, and inſtead of having a ſtrait line from the edge of the keel to touch the back of the floor ſweep in ſome ſhips it is made a hollow.

9. Half breadth of the floor is the diſtance of the center of the floor ſweep from the middle line in the body plane, at the midſhip timber, which will always be leſs than the diſtance betwixt the point where the ſtrait line drawn from the ſide of the keel to touch the back of the floor ſweep is from the middle line. This laſt may be called the true ½ breadth of the floor, and in ſharp ſhips will be above the riſing line.

10. Riſing of the floor, is a curve drawn on the ſheer plane, limited at the midſhips by the dead riſing; and in flat ſhips it runs nearly parallel to the keel for ſome timbers before and abaft the midſhip, for which reaſon theſe timbers are called flats; but in ſharp ſhips it riſes gradually from the midſhip till it ends on the ſtem and poſt. To this line is ſome-

times

times adapted an $\frac{1}{7}$ breadth of the floor line. The uſe of theſe two lines is to find the centers of the floor ſweeps.

11. Cutting down line, is drawn on the ſheer plane. It is limited in the midſhips by the thickneſs of the floor timber, and abaft by the breadth of the kelſon; for it muſt be carried ſo high abaft as to leave room for the kelſon, for which purpoſe the thickneſs of the timbers muſt be known. It muſt be carried up ſo high upon the ſtem as to leave ſufficient ſubſtance for the breeches of the riſing timbers. The lower edge of the kelſon is in this line; ſo it limits the thickneſs of all the floor timbers, and likewiſe the heighth of the dead wood afore and abaft.

12. Timber and room, or room and ſpace, is the diſtance betwixt the moulding edges of two timbers, which muſt always contain the breadth of two timbers, and ſometimes two or three inches between them. It muſt be obſerved, that one mould ſerves for two timbers; the foreſide of the one being ſuppoſed to unite with the aftſide of the other, and ſo make only one line, which is actually the caſe in all the frames, which in ſome ſhips are every third, in others every fourth timber. The frames are firſt put up, and faſtened to the ribbands, and afterwards the others are put up, which are called filling timbers. The midſhip timber is called dead-flat, and diſtinguiſhed by this character ⊕; the timbers abaft the midſhip are diſtinguiſhed by the figures 1, 2, 3, *&c.* and thoſe before the midſhip by the letters of the alphabet A, B, C, *&c.*

S E C T. II.

Of WHOLE MOULDING.

THE length of the keel, extreme breadth, depth in the hold, heighth between decks, and in the waſte; and ſometimes the heighth and breadth of the wing tranſom are agreed on by contract in the merchant's ſervice: From which dimenſions the builder is to form a draught ſuitable to the trade the ſhip is deſigned for.

The firſt thing that is generally done, is to lay down the keel, ſtem and poſt, upon the ſheer planes: Then to determine the proper ſtation of the midſhip timber, where a perpendicular is erected: It is generally about $\frac{4}{7}$ of the keel before the poſt. On this line the given depth of the hold is ſet off from the upper ſide of the keel; to obtain which point, the thickneſs of the timber and plank muſt be added to that agreed on by contract. This being fixed, will enable us to determine the upper heighth of the extreme breadth at that place, which ſometimes is the very point itſelf. The lower heighth of the breadth muſt likewiſe be determined at this place. Then we may form the two main heighths of the breadth lines which nearly unite abaft and afore. Abaft, theſe curves end at the wing tranſom, or above it; and afore, they are carried up ſometimes as high as the hawſe holes. The heighth of the breadth line of the top timber muſt likewiſe be formed. This is generally done by a bow, which makes nearly an arch of a circle. It is limited in midſhips by contract, afore and abaft only by the fancy and judgment of the artiſt, according to what ſheer he deſigns: We muſt alſo form a line for the riſing of the floor; for which purpoſe we muſt determine the dead riſing, which is that of the midſhip timber. This limits it at that place, and in the whole moulding it is pretty near parallel to the lower heighth of the breadth line. Theſe lines muſt abſolutely be drawn on the ſheer plane; and correſponding to the main and top timber heighth of breadth lines; there muſt be two half breadth lines formed on the floor plane.

The main half breadth at the midſhip timber is agreed on by contract, only obſerving that the thickneſs of the timber and plank muſt be deducted out of it, becauſe it is the extreme breadth from outſide to outſide of the plank that is contracted for. Thoſe in the draughts are called moulded half breadths: Then the breadth at the wing tranſom, if a ſquare ſtern, is limited: It is generally about two thirds of the extreme breadth, but this is juſt as the artiſt ſhall think proper. He alſo fixes the breadth of the top timber, and then deſcribes the two half breadth lines. In the due formation of theſe curves on the ſheer and floor plane, the whole art of drawing chiefly conſiſts; which muſt be acquired by practice, ſo that it will be ſcarce poſſible for one that is not very well acquainted with drawing, to form them, without having recourſe to ſome other draughts. After theſe are formed, the ſtations of the timbers are fixed, if the room and ſpace, and the breadth of the midſhip timber is agreed on by contract, this will determine the ſtation of all the timbers; obſerving that the timbers abaft the midſhips muſt be ſet off from the foreſide of the midſhip timber; and the timbers before the midſhip from the aft ſide of it. At every third or fourth timber there muſt be perpendiculars drawn on the ſheer and floor planes, to the line that repreſents the lower edge of the keel, which is the common ſection of theſe two planes; tho' ſometimes the half breadth lines are deſcribed on the ſheer plane, when there is not ſpace to produce the perpendiculars till they be of ſufficient length to contain the heighth of the breadth and half breadth.

After

After the timbers are ſtationed, and the perpendiculars for the frames drawn on the ſheer and floor planes; we proceed to the body plane, and draw a line equal in length to the whole breadth moulded. This line may be called the baſe of the body plane. A perpendicular is erected at each end of it, and one in the middle, which may be produced at pleaſure. The next thing to be done, is to form the midſhip frame: The limits of it are had from the ſheer and floor planes; the lower, upper and top timber heighths of the breadth are taken from the ſheer plane at the perpendicular, repreſenting the midſhip frame, and ſet off on the middle line of the body plane from the baſe. Thro' theſe points, lines are drawn parallel to the baſe, and the reſpective half breadths correſponding to each, are ſet off on theſe lines, from the middle line in the body plane. The lower and upper main half breadths are limited by the perpendiculars already drawn at each end of the baſe. The half breadth of the top timber, is had from the floor plane on the perpendiculars repreſenting the midſhip frame. The heighth of the dead riſing is likewiſe taken from the ſheer plane, and ſet up from the baſe upon the middle line in the body plane, thro' which point a line parallel to the baſe muſt be drawn; and upon this line the half breadth of the floor, is ſet off from the middle line, at which point a perpendicular is erected. The center of the floor ſweep is in this line, from which a circle muſt be deſcribed that ſhall juſt touch the riſing line. A proper radius for the under breadth ſweep is next to be found: The center of it is in the lower breadth line, from which it is deſcribed to paſs thro' the point which limits the half breadth. After which the radius, and center of a reconcileing ſweep to join the floor, and under breadth ſweeps is found, and the circle deſcribed; and to compleat the frame below the breadth, the half breadth of the keel is ſet off from the middle line on the baſe; from which point, a ſtrait line is drawn to touch the back of the floor ſweep.

By this way of forming the frame, it is plain the centers and radii of the ſweeps are arbitrary, but they muſt be determined before any of the other timbers can be formed; if by no other means, by repeated trials, till they are made to pleaſe the fancy and judgment of the artiſt. But there are various other ways of forming this frame; ſo that, tho' ſeveral ſhips may be of the ſame breadth, depth in the hold, and dead riſing; they may all differ in the form of their timbers. After this midſhip timber is formed, a pattern or mould is made to fit exactly to the curve, and the dead riſing line. By this, and a hollow mould, all the timbers are formed ſo far as the riſing line, and lower heighth of the breadth line are parallel to one another in the ſheer plane: This is what is called whole mould-

moulding, which we ſhall illuſtrate by laying down a long boat. And becauſe in ſeveral mould lofts there is not ſufficient length for the ſheer plane, it is often laid down as if it were cut by the midſhip frame, and one part laid upon the other in ſuch a manner, that the midſhip timber of the after part ſhall coincide with a perpendicular let fall from the fore part of the ſtem.

To lay down a Long-Boat 29 *Feet* 1 *Inch long, and Breadth Moulded* 9 *Feet.* (See *Plate* 5.)

1*ſt*. Draw the ſtrait line P ⊕, and erect the perpendicular P T. From the point P ſet off 29-1, the given length of the keel. But becauſe the plate will not admit of the whole length, let the ſtation of the midſhip timber be aſſigned; at which point erect the perpendicular ⊕ M. Let M be the upper, and N the lower heighth of breadth, at that place; T the heighth of breadth at the tranſom, and draw the curve T M to repreſent the ſheer, or extreme heighth of the ſide. This in a ſhip would be called either the upper heighth of breadth line, or the upper edge of the wale. Draw alſo a curve thro' the point N, parallel to T M, to repreſent the breadth of the upper ſtrake in a boat, or lower edge of the wale, if in a ſhip. The dotted line T N may alſo be drawn to repreſent the lower heighth of breadth.

2*d*. Set off the rake of the poſt from P to *p*, and draw the line *p t* to repreſent the aft ſide of the poſt; ſo ſhall T *t* repreſent the round up of the tranſom. Set off the breadth of the poſt from *p* to *r*, and from T to *s*, and draw the line *r s* to repreſent the fore ſide of the poſt, which may either be a curve or a ſtrait line at pleaſure. Set up the heighth of the tuck from *p* to *k*. Let *k x* be the thickneſs of the tranſom, and draw the line *z x* to repreſent the fore ſide of the tranſom.

3*d*. Set up the dead riſing from ⊕ to *d*, and form the riſing line *r i s*. We may then draw the line K L parallel to P ⊕, to repreſent the lower edge of the keel, and another to repreſent the thickneſs of the plank or the rabit. The rabit on the poſt may likewiſe be repreſented, and the ſtations of the timbers aſſigned; diſtinguiſhed in the plate by their proper names, *viz.* ⊕, ①, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Thus have we compleated the ſheer plane, or ſide draught for the after body; and in like manner is that for the fore body to be done. Firſt produce the line ⊕ M to *y* the heighth of the fore part of the ſtem, and form the ſtem either by ſweeps or ſome other contrivance. The breadth of the ſtem muſt be known, and the aft ſide likewiſe formed. The ſtem being formed, we may ſet off from the fore part of it as much as the line

P

P ⊕ wanted of the whole length of the boat, which ſuppoſe ⊕ ⊕. Erect the perpendicular ⊕ F, and make it equal to ⊕ M, the heighth of the ſheer, and form the curve F S, which will repreſent the ſheer or heighth of the ſide in the fore body. We may likewiſe draw a line to repreſent the lower part of the upper ſtrake, and one for the lower heighth of the breadth. The riſing line muſt alſo be formed, and the timbers ſtationed and diſtinguiſhed by their proper names Ⓐ, ⊕, A, B, C, D, E, F, G, H. Now the whole ſheer plane is compleated; for if the line S T was drawn aſunder, till the point F came to the point M, we ſhould have the whole length of the boat.

The next thing to be done is to form the half breadth line; for which purpoſe the perpendiculars T P, 9, 8, *&c.* muſt be produced. Then, from the point where the perpendicular ⊕ M interſects the line K L, ſet off 9 feet, the half breadth: Set off alſo the half breadth at the tranſom, from the line K L, and form the half breadth line B *b*. In like manner ſet off the half breadth, from the point where the perpendicular ⊕ F interſects the line K L, and form the half breadth line R X, according to the deſigned round of the harpin.

We may now proceed to form the timbers in the body plane: Where, let A B be the breadth moulded at ⊕. Erect the perpendicular C D in the middle of the line A B; and parallel to C D draw the lines *n m*, the half thickneſs of the poſt, and *x y* the half thickneſs of the ſtem. Then take off the ſeveral portions of the perpendiculars ⊕, 1, 2, *&c.* intercepted betwixt the upper edge of the keel, and the riſing line in the ſheer plane; and ſet them up from C upon the line C D. Thro' theſe points draw lines parallel to A C; take off alſo the ſeveral lower heighths of breadth at ⊕, 1, 2, *&c.* from the ſheer plane; ſet them alſo up from C upon the middle line in the body plane, and draw lines parallel to A C thro' theſe points: Then take off the ſeveral half breadths correſponding to each, from the floor plane; and ſet them off on their proper half breadth lines, from the middle line in the body plane.

We muſt in the next place form the midſhip timber, either by two, or three ſweeps, or ſome other contrivance which muſt be left entirely to the fancy and judgment of the artiſt: If he uſes three ſweeps, a proper center of each muſt be found. That of the under breadth muſt always be in the breadth line, as in this caſe at *a*. The center of the floor ſweep, ſuppoſe at C, muſt be ſo, that when it is deſcribed, the back of the ſweep may juſt touch the riſing line. Theſe two centers being found, and the arches deſcribed, we may with certainty find the center of the reconciling ſweep, provided the radius be known, thus: Set off the given radius upon any ſtrait line, as from A upon the baſe line to R; then take the radius of the lower breadth ſweep, which ſet off from A to *a*; take alſo the radius of the floor ſweep, and ſet off from A to *c*: Then with the radius *a* R, from the center of the under breadth ſweep, deſcribe an arch; and with the radius *c* R, from the center of the floor ſweep, deſcribe another arch to interſect the former in *r*, which will be the center of the reconciling ſweep: For if with the radius A R, from the center *r*, we deſcribe an arch, it will juſt touch the under breadth, and floor ſweeps in the points *t* and *s*. A line drawn from *r* to *t* will paſs through the center of the underbreadth ſweep, and a line drawn from *r* to *s* will paſs through the center of the floor ſweep; ſo that it will be impoſſible for the reconciling to interſect either of the other two arches. The curve part of the timber being formed, a ſtrait line muſt be drawn from the ſide of the keel to touch the back of the floor ſweep; for which purpoſe the half breadth of the keel muſt be ſet off on each ſide of the point C, upon the baſe line. The form of the midſhip frame being determined, will in ſome meaſure determine the form of all the reſt. For if a mould be made on any ſide of the middle line to fit the curve part of it; and the riſing line, as that marked B E N D, and laid in ſuch a manner that the lower part of it, which is ſtrait, may be ſet upon the ſeveral riſſing lines, and the upper part juſt touch the point of the half breadth in the breadth line, correſponding to that riſing upon which the mould is placed; a curve may then be drawn by the mould to the riſing line. In this manner we may proceed ſo far as the riſing line is parallel to the lower heighth of the breadth line. Then a hollow mould muſt be made, the upper end of which is left ſtrait, as that marked H *l w*. This is applied in ſuch a manner, that ſome part of the hollow may touch the ſide of the keel, and the ſtrait part touch the back of the curve before deſcribed by the bend mould; and, beginning abaft, the ſtrait part will always come lower on every timber till we come to the midſhip timber, where it comes to the ſide of the keel. Having thus formed the timbers, ſo far as the whole moulding will ſerve; the timbers abaſt them are next formed. Their half breadths are determined by the ſheer and floor planes, which is the only fixed point thro' which the curve of theſe timbers muſt paſs. Some form theſe after timbers before the whole is moulded, and then make the hollow mould, which will be ſtraiter than the hollow of either of theſe timbers. It is indifferent which are firſt formed, or what methods are uſed; for after the timbers are all formed, tho' every timber may appear very fair, when conſidered by itſelf, it is uncertain, what the form of the ſide will be. In order to find which, we muſt form ſeveral rib-

band

band and water lines; and if these do not make fair curves, they must be rectified, and the timbers formed from these ribband and water lines. In using the hollow mould, when it is applied to the curve of each timber, if the strait part is produced to the middle line, we shall have as many points of intersection as there are timbers: And if their heighths above the base be transferred to the corresponding timbers in the sheer plane, a curve passing thro' these points is what is called a rising strait. This may be formed by fixing a point for the aftermost timber that is whole moulded, and transferring that heighth to the sheer plane. The curve must pass thro' this point, and fall in with the rising line, somewhere abaft ⊕: And if the several heighths of this line be transferred from the sheer, to the middle line in the body plane; these points will regulate what is called the hawling down of the hollow mould. The timbers being thus formed, and proved by ribband and water lines; we may then form the transom. This may be done either by ribband or water lines. Here it is by water lines, of which there are three, formed in the following manner. (*Plate* 5.)

1st. Draw three lines in the body plane parallel to the base: These are called level lines, and may be equally spaced betwixt the tuck, and heighth of the sheer, taken upon a perpendicular to the keel. They may be likewise drawn on the sheer plane at the same heighths, so they will be parallel to the keel. They are distinguished by 1*st.* 2*d.* and 3*d.* W^r.

2dly. To form them on the floor plane. Take the distance betwixt the middle line in the body plane, and the several intersections of the level lines with the timbers: Transfer these to the corresponding timbers in the floor plane, which will give the points thro' which the curves will pass: So the portion of the first level line in the body plane, intercepted betwixt the middle line and timber 9, will be equal to the distance taken upon the perpendicular in the floor plane, drawn from the point 9, to intersect the curve of the first water line. The like may be said of all the rest.

The water lines being thus formed; the next thing to be determined, is the round aft of the transom, if any; if none, produce the line T *p* to *b*, then will K *b* be the half breadth of the transom at the heighth of the sheer; So the heighth P T in the sheer plane, must be transferred to the middle line in the body plane. Thro' this point a line K *b*, must be drawn parallel to the base A C, upon which the half breadth K *b* being set off, we shall have one point thro' which the curve must pass. In like manner there must be perpendiculars let fall from the several intersections of the water lines, and aft side of the post in the sheer plane, and produced to interſect their correſponding water lines in the floor plane; which will give their half breadths. Theſe again being transferred, to their correſponding level lines in the body plane, we ſhall have the points through which the curve of the tranſom muſt paſs; obſerving at the tuck to ſet off the half breadth of the poſt; or the depth of the rabbet may be deducted out of it. The tranſom being thus formed, it is plain it will be too ſhort, by reaſon of the raking of the poſt: We muſt therefore take the heighth of the tranſom upon the rake, which will be the line *p* T in the ſheer plane; and ſet up this on the middle line in the body plane. In like manner we muſt ſet up on the middle line in the body plane, the ſeveral diſtances of the water lines, from the point *p* in the ſheer plane; and thro' theſe points draw the ſeveral dotted lines, upon which muſt be ſet off the half breadths as before: Then a curve paſſing thro' theſe points, will give the true form and dimenſions of the tranſom, as is expreſſed by the dotted curve.

If the tranſom is to round aft, as the curve K *n* G on the floor plane, it may be formed after the ſame manner without regarding the round; and after it is properly trimmed the round may be worked out: But as this will require very thick plank; in ſuch caſes it will be proper to make uſe of a faſhion piece, and wing tranſom: This faſhion piece will be formed in the ſame manner as that for a ſhip which has a ſquare tuck, ſo the ſame operation will ſerve for both.

Now the faſhion pieces being always ſided ſtrait, their planes will interſect the ſheer and floor planes in a ſtrait line. In this caſe, it will be in the line G *g* on the floor plane, which touches the tranſom in the point *n*; G *n* being ſuppoſed the thickneſs of the faſhion piece. Having thus determined its direction on the floor plane, this will likewiſe determine its direction on the ſheer plane. If the tranſom had no round, only what is called a flight or riſing like a floor timber, the plane of the faſhion piece would interſect the ſheer plane in the rabbet of the ſtern poſt; and the floor plane in a ſtrait line drawn from G to K. But here it is ſuppoſed to be in the line G *g*, which will throw the head of the faſhion piece aft to W on the ſheer plane; the point where a perpendicular erected from *g* interſects the ſheer line or breadth line produced. Now *k* being the heighth of the tuck, the line *k* W will be that in which the faſhion piece interſects the ſheer plane.

Having thus found the interſection of the plane of the faſhion piece, both on the ſheer and floor planes, it is evident it will rake aft and cant forward; ſo the true dimenſions and form of it may be found by *Prob.* 6. *Chap.* I. in the following manner.

1*ſt*. Pro-

1st. Produce all the water lines in the sheer plane, to the line *k* W in the points *a*, *s*, *h*, ; and let fall the perpendiculars *a e*, *s o*, *h u*.

2dly. From the points *e*, *o*, *u*, draw lines parallel to G *g*, to intersect each corresponding water line on the floor plane in the points 3, 2, 1.

3dly. Transfer the several points G, 3, 2, 1, on the floor plane, to the points G, 3, 2, 1, on the sheer plane, in such a manner, that lines drawn from G to G, from 3 to 3, &c. may be perpendicular to *g* L; and let the point G be in the breadth line, the point 3 in the third water line, &c. on the sheer plane; so the plane W G 3 2 1 *k* will be the projection of the plane of the fashion piece on the sheer plane. But it will be less than the plane of the fashion piece, because it is not parallel to the sheer plane. Therefore,

4thly. Thro' the points G, 3, 2, 1, in the sheer plane, draw the dotted perpendiculars G F, 3 A, 2 S, 1 H, to the line W *k*.

5thly. Make the lines W F, *a* A, *s* S, *h* H, in the sheer plane, equal to the lines *g* G, *e* 3, *o* 2, *u* 1, in the floor plane; so that they may intersect the dotted perpendiculars in the points F, A, S and H: So shall W F A S H *k* be the true form and dimensions of the plane of the aft side of the fashion piece. When it is in its proper position, the line W F will be in the same plane with the sheer line; the line *a* A in the same plane with the water line *a* 3; the line *s* S in the same plane with the water line *s* 2; and the line *h* H in the same plane with the water line *h* 1.

We have now formed all the timbers in the after body. Those for the fore body are formed in the same manner, by transferring the several heighths of the rising and breadth lines from the sheer to the body plane; the half breadths corresponding to each heighth, must also be transferred from the floor to the body plane. The same hollow mould will serve both for the fore and after body; and the level lines by which the water lines, to prove the after body, were formed, may be produced into the fore body, and by them the water lines to prove the fore body may be described.

Another method of proving the body, is by ribband lines, which are formed by sections of planes inclined to the sheer plane, and intersecting the body plane diagonally, as before observed; of which there may be as many as we shall judge needful.

Here we think it sufficient to lay down one, represented in the body plane by the lines marked dia[l]. These are drawn in such a manner, as to be perpendicular to as many timbers as conveniently may be. After they are drawn in the body plane, the several portions of the diagonal, intercepted betwixt the middle line and each timber, must be transferred to the floor plane: Thus, fix one foot of the compaſſes in the point where the diagonal interſects the middle line in the body plane; extend the other foot to the point where the diagonal interſects the timber, ſuppoſe timber 9; ſet off the ſame extent upon the perpendicular repreſenting the plane of timber 9, from the point where it interſects the line K L, on the floor plane; do the ſame by all the other timbers both in the fore and after body; and we ſhall have the points thro' which the curve muſt paſs. And if this ſhould not prove a fair curve, it muſt be altered, obſerving to conform to the points as nearly as the nature of the curve will admit: So it may be carried within one point, and without another, according as we find the timbers will allow: For after all the ribband lines are formed, the timbers muſt, if needful, be altered by the ribband lines, this is only the reverſe of forming the ribband lines; for taking the portions of the ſeveral perpendiculars intercepted betwixt the line K L, and the curve of the ribband line in the floor plane, and ſetting them off upon the diagonal, from the point where it interſects the middle line; we ſhall have the points in the diagonal, thro' which the curves of the timbers muſt paſs: So the diſtance betwixt the line K L, and the ribband at timber 3 on the floor plane, when transferred to the body plane, will extend on the diagonal, from the middle line, to the point where the curve of timber 3 interſects that diagonal. The like may be ſaid of all the other timbers; and if ſeveral ribband lines be formed, they may be ſo contrived, that their diagonals in the body plane ſhall be at ſuch diſtances, that a point for every timber being given in each diagonal, will be ſufficient to determine the form of all the timbers.

In ſtationing the timbers upon the keel, for a boat, there muſt be room for two futtocks in the ſpace before, or abaft ⊕; for which reaſon, the diſtance betwixt thoſe two timbers will be as much more, than that betwixt the other, as the timber is broad. Here it is betwixt ⊕ and Ⓐ; which contains the diſtance betwixt ⊕ and ①, and the breadth of the timber beſides.

This method of whole moulding will not anſwer for the long timbers afore and abaft: They are generally canted in the ſame manner as thoſe for a ſhip, of which we ſhall treat in their proper place; and here ſhew in what manner the timbers are moulded after they are laid down in the mould loft, by a riſing ſquare, bend, and hollow mould.

It was ſhewn before how to form the timbers by the bend and hollow moulds, on the draught. The ſame method muſt be uſed in the loft, but the moulds muſt be made to their proper ſcantlings in real feet and inches. Now when they are ſet, as before directed, for moulding each timber,

timber; let the middle line in the body plane be drawn acroſs the bend mould, and draw a line acroſs the hollow mould at the point where it touches the upper edge of the keel; and let them be marked with the proper name of the timber, as in the figure (*Pate* 5.): So the graduations of the bend mould will be exactly the ſame as the narrowing of the breadth; for the diſtance betwixt ⊕ and 7, on the bend mould, is equal to the difference betwixt the half breadth of timber 7, and that of ⊕. The heighth of the head of each timber is likewiſe marked on the bend mould, and alſo the floor and breadth ſirmarks. The floor ſirmark is in that point where a ſtrait edged batten touches the back of the bend mould, the batten being ſo placed as to touch the lower edge of the keel at the ſame time. The ſeveral riſings of the floor, and heighth of the cutting down line, are marked on the riſing ſquare, and the half breadth of the keel ſet off from the ſide of it, as in the figure.

Note. The cutting down is omitted in the plate to avoid confuſion.

The moulds being thus prepared, let it be required by them to mould timber 7.

The timber being firſt properly ſided to its breadth, lay the bend mould upon it, ſo as may beſt anſwer the round according to the grain of the wood: Then lay the riſing ſquare to the bottom of the bend mould, ſo that the line drawn acroſs the bend mould at timber 7, may coincide with the line repreſenting the middle of the keel upon the riſing ſquare; and draw a line upon the timber by the ſide of the ſquare, or let the line be ſcored, or cut by a tool made for that purpoſe, called a raſeing knife. The term raſeing is uſed when any line is drawn by ſuch an inſtrument inſtead of a pencil. This line ſo raſed will be the ſide of the keel. Then the ſquare muſt be moved till the ſide of it comes to 7 on the bend mould, and another line muſt be raſed in by the ſide of it, to repreſent the middle of the keel. The other ſide of the keel muſt likewiſe be raſed after the ſame manner, and the point 7 on the riſing ſquare be marked on each ſide of the keel, and a line raſed acroſs at theſe points, to repreſent the upper edge of the keel. From this line the heighth of the cutting down line at 7 muſt be ſet up, and then the riſing ſquare may be taken away, and the timber may be raſed by the bend mould, both inſide and outſide, from the head to the floor ſirmark: Or it may be carried lower if needful. After the ſirmarks, and head of the timber are marked, the bend mould may likewiſe be taken away; and then the hollow mould applied to the back of the ſweep in ſuch a manner, that the point 7 upon it may interſect the upper ſide of the keel, before ſet off by the riſing ſquare: And when in this poſition, the timber may be raſed by it, which will compleat the outſide

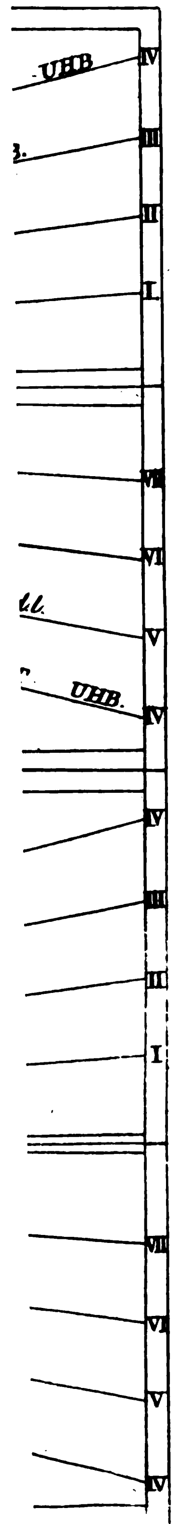
ſide of the timbers. The inſide of the timbers may likewiſe be formed by the hollow mould. The ſcantling at the keel is given by the cutting down before ſet off. The mould muſt be ſo placed, as to touch the ſweep of the inſide of the timber formed before by the bend mould, and paſs thro' the cutting down point.

The uſe of the ſirmarks, is to find the true places of the futtocks; for as they are cut off 3 or 4 inches ſhort of the keel; they muſt be ſo placed, that the futtock and floor ſirmarks may compare, or coincide: Notwithſtanding which, if the timbers are not very carefully trimmed, the head of the futtock may be either within or without its proper half breadth; to prevent which a half breadth ſtaff is made uſe of.

The half breadth ſtaff may be one inch ſquare, and of any convenient length. Upon one ſide of it are ſet off, from one end, the ſeveral half breadths of all the timbers in the after body; and thoſe of the fore body upon the oppoſite ſide. On the other two ſides are ſet off the ſeveral heighths of the ſheer; the after body on one ſide, and the fore body on its oppoſite. Two ſides of the ſtaff are marked half breadths, and the other two ſides, heighth of the ſheer, as in the figure. (*Plate* 5.)

The ſtaff being thus prepared, and the floor timbers faſtened on the keel, and levelled acroſs; the futtocks muſt next be faſtened to the floor; but they muſt be ſet firſt to their proper half breadth and heighth: The half breadth ſtaff, ſerves to ſet them to the half breadth; for which purpoſe a ſmall line, called a ram line, is ſtretched from the middle line of the ſtem, to that of the tranſom or poſt; to which a plummet is hung by a line, ſo tied round the ram line, that it may ſlip eaſily along upon it, and may be moved to the plane of any timber, and as the plummet will occaſion this line to be always perpendicular to the keel, which in a boat is generally parallel to the plane of the horizon: We may, by it, likewiſe ſet the timbers perpendicular to the keel, and then ſet them to their proper half breadth by the ſtaff; and when the two ſirmarks coincide, the futtock will be at its proper heighth, and may be nailed to the floor timbers, and likewiſe to the breadth ribband; which may be ſet to the heighth of the ſheer by a level laid acroſs, taking the heighth of the ſheer by the ſtaff from the upper ſide of the keel; by which means we ſhall diſcover if the ribband is exactly the heighth of the ſheer; and if not, the true heighth may be ſet off by a pair of compaſſes from the level, and marked on the timbers. The next thing to be explained, is the conſtruction and uſe of the bevelling board; but we ſhall firſt ſhew how to form the timbers by ſweeps, becauſe the ſame method for bevelling ſerves for both.

SECT.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SECT. III.

Of forming the Body by Sweeps. (Plate 7.)

IN ſhips of war the general dimenſions are eſtabliſhed by the authority of thoſe appointed by the government for that purpoſe, which I have collected into a table, together with the principal dimenſions of ſhips for the merchant ſervice, to which we refer our readers.

The ſheer and floor planes are laid down in this, exactly in the ſame manner as in that of whole moulding. We may have a ſufficient number of points from the tables, to determine the heighths of the breadth, and half breadth lines. A riſing of the floor line muſt likewiſe be formed on the ſheer draught. We may then go to the body plane, and form the midſhip bend or frame timber; the limits of which, we have from the ſheer and floor planes, and it muſt be formed in the ſame manner as before directed in whole moulding, either by two, three, or more ſweeps, as the artiſt ſhall think moſt ſuitable to the ſervice the ſhip is deſigned for. The lower, upper, and top timber heighths of breadth, and riſings of the floor, are ſet up on the middle line in the body plane, as in whole moulding, and lines drawn thro' theſe points parallel to the baſe upon which the half breadths are ſet off. A mould may then be made for the midſhip frame as before, and laid upon the ſeveral riſings in the ſame manner as in whole moulding, with this difference; that here an under breadth ſweep is deſcribed to paſs thro' the point which limits the half breadth of the timber; the center of which will be in the breadth line of that timber. The proper centers for all the frames being found, and the arches deſcribed, the bend mould muſt be ſo placed on the riſing line of the floor, that the back of it may touch the back of the under breadth ſweep. But the general practice is to deſcribe all the floor ſweeps with compaſſes as well as the under breadth ſweeps, and to reconcile theſe two by a mould which is an arch of a circle; its radius being the ſame with that of the reconciling ſweep, by which the midſhip frame was formed. It is uſual for all the floor ſweeps to be of one radius; and in order to find their centers, a line is formed on the floor plane for the half breadth of the floor: This, as was before obſerved, is only an imaginary one; for it cannot be deſcribed on the ſurface of the ſhip: Inſtead of it ſome make uſe of a diagonal in the body plane, to limit the half breadth of the floor upon every riſing line, and erect perpendiculars at the ſeveral interſections in the

the fame manner as for the midfhip frame, as in the draught; where it is very plain the floor fweep conftitutes no part of the after timbers abaft the fquare body.

After the fweeps are all defcribed, we muft have recourfe to moulds, or fome fuch contrivance, to form the hollow of the timbers, much in the fame manner, as in whole moulding; and when we have thus formed all the timbers, they muft be proved by ribband and water lines, as before directed; and altered, if needful, to make thefe lines fair. Hence it is obvious, that the form of the ribband lines muft be determined, before we can with certainty have the true form of the timbers. But there will be a neceffity of determining, at leaft, the form of three timbers, *viz.* the midfhip, foremoft and aftermoft, before we can form a ribband line. Thefe will give three points, thro' which the curve of each ribband muft pafs. The points in the intermediate timbers may be found by forming timbers as before directed; and by repeated trials, altering them till they make fair ribbands; for it is by them that the whole ftructure is regulated, when every frame is erected into its proper place.

S E C T. IV.

Defcription and Ufe of the Sector in forming the Body.

THE fector has feven lines on each leg, meeting at the center of the joint, numbered I, II, III, IV, *&c.* fo that every line upon one leg has a correfponding one upon the other leg, both divided and numbered alike. The after body is upon one fide, and the fore body on the other.

(Plate 6.) The Lines for the after Body are as follows.

I. Has five divifions, *viz.* ⊕, 4, 1^{ft} dl, 8, ft^{n}; and marked at the end H B T^{r}, denoting the heighth of the top timber breadth line at four timbers; ft^{n} is the ftern timber, and 1^{ft} dl, the firft diagonal in the body plane.

II. Has eight divifions, *viz.* $\frac{L^r\ C,}{u,\ A,}$ $\frac{U^r\ C^r,}{A,\ u,}$ S^{tn}, 8, 4, ⊕, and marked at the end $\frac{1}{2}$ B T^{r}, denoting the half breadth of the top timber at three timbers. L^{r} C^{r} fignifies the lower counter, and U^{r} C^{r}, the upper counter; u denotes

denotes the heighth, and A the rake of the counters, both taken from the wing tranſom; S^t is the rake of the ſtern timber, which is likewiſe taken from the wing tranſom at the heighth of the ſheer rail.

III. Has eight diviſions, *viz.* d^n, u^r S. R 1 s, ⊕, 3, 5, 7, 8: It is marked at the end L H B, for the heighth of the lower breadth line for five timbers: d^n is for the diſtance betwixt the frames, and R 1 s, for the diſtance betwixt the lower breadth line, and the dead riſing in the body plane: u^r S. is the radius of the upper breadth ſweep.

IV. Is in two parts. The innermoſt has four diviſions, *viz.* 7, 5, 3, ⊕, expreſſing the points where theſe timbers interſect the ſecond diagonal in the body plane: It is marked at the end 2 R for the ſecond ribband.

The outermoſt part has ſix diviſions, *viz.* ⊕, 3, 5, 7, 8, W T, and marked at the end U H B for the heighth of the upper breadth line at five timbers, and at the wing tranſom denoted by W T.

V. Is likewiſe in two parts. The innermoſt has four diviſions, *viz.* 7, 5, 3, ⊕, expreſſing the points where theſe timbers interſect the firſt diagonal: It is marked 1st R for the firſt ribband.

The outermoſt part has eight diviſions, *viz.* T, 8, 7, 5, 3, ⊕, *k l*, and then marked M $\frac{1}{2}$ Bth, for the main half breadth of five timbers; T for that at the wing tranſom, and *k l* for the half breadth of the keel in midſhips; without which there is another diviſion marked 3^d d l for the third diagonal.

VI. Has four diviſions, *viz.* 7, 5, 3, ⊕; for the points where thoſe timbers interſect the third diagonal, it is marked 3d R, denoting the third ribband; without which, there is another diviſion marked 2d d l, for the ſecond diagonal.

VII. Has five diviſions, *viz.* 7, $\frac{1}{2}$ B F l, 5, 3, ⊕. $\frac{1}{2}$ B F l, denotes the half breadth of the floor: The other four are for the points where thoſe timbers interſect the fourth diagonal: It is marked 4th R, denoting the fourth ribband; without which there is another diviſion for the rake of the poſt marked R^k P.

The heighth of the gun-deck is betwixt N° IV and V. G D ⊕ for that in midſhips, and G D *a* for that at the poſt. There is likewiſe another diviſion betwixt N° I and II for the fourth diagonal: It is marked $\frac{1}{2}$ 4th d l, which muſt be doubled, becauſe the length of the ſector will not contain the whole.

INDEX *to the* AFTER BODY.

		Nº			Nº
Heighth of breadth	lower	III	Ribbands	1	V
	upper	IV		2	IV
	top timber	I		3	VI
rising		III		4	VII
Half breadth	main	V	Diagonals	1	I
	top timber	II		2	VI
	floor	VII		3	V
Counter and ſtern timber, heighth and rake		II		4	betwixt I and II
Rake of the poſt		VII	Diſtance betwixt the frames		III
Upper breadth ſweep		III			

The Lines for the FORE BODY *are.*

I. Has three diviſions, *viz.* H^t ſtem, $\frac{G^d,\ H^d}{\text{rake } f^r\ G}$ denoting the heighth of the ſtem; and its rake from timber G at the gun-deck and head.

II. Has four diviſions, *viz.* *d*, *h*, and theſe marked H^t, T^r, for the heighth of the top timber line at theſe timbers; and again *d*, *h*, for the $\frac{1}{2}$ breadth of thoſe timbers; and the line marked $\frac{1}{2}$ B T^r.

III. Has four diviſions, *viz.* d^{ce} F for diſtance betwixt the frames, and *c*, *e*, *g*; it is marked L H B, denoting the heighth of the lower breadth line at theſe timbers.

IV. Is in two parts. The innermoſt has four diviſion, *viz.* *g*, *e*, *c*, ⊕; for the points of interſection of theſe timbers, with the ſecond diagonal, is marked 2^d R, for the ſecond ribband.

The outermoſt part has three diviſion, *viz.* *c*, *e*, *g*: It is marked U H B, denoting the heighth of the upper breadth line at theſe timbers.

V. Is in two parts. The innermoſt has four diviſions, *viz.* *g*, *e*, *c*, ⊕; for the points of interſection of theſe timbers, with the firſt diagonal: It is marked $1^{ſt}$ R for the firſt ribband.

The outermoſt part has four diviſions, *viz.* *g*, *e*, *c*, ⊕: It is marked M $\frac{1}{2}$ B^{th} for the main half breadth at theſe timbers.

VI. Has four diviſions, *viz.* *g*, *e*, *c*, ⊕, for the points of interſection of theſe timbers with the third diagonal: It is marked 3^d R for the third ribband.

VII. Has four diviſions, *viz.* *g*, *e*, *c*, ⊕, for the points of interſection of theſe timbers, with the fourth diagonal: It is marked 4^{th} R, for the fourth ribband. The ſweep of the ſtem is betwixt Nº III and IV; and the heighth of the gun deck betwixt IV and V.

IN-

INDEX *to the* FORE BODY.

		N°			N°
Heighth of breadth	lower	III	Ribbands	1	V
	upper	IV		2	IV
	top timber	II		3	VI
Half breadth	main	V		4	VII
	top timber	II	Distance of frames		III
Stem heighth and rake		I	Sweep of the stem betwixt		III and IV

Having thus described the lines, we shall now shew their use in laying down a ship. (*Plate* 7.)

The general dimensions being determined, and a scale adapted to the draught, take the half breadth with a pair of compasses, and placing one foot in the proper point for the half breadth of ⊕, which will be found in N° V. open the sector till the other foot reaches to the same point in the corresponding line on the other leg.

The sector being thus set, it will be indifferent whether we begin with the body or sheer plane: Let it then be the sheer.

1*st*. Draw the line X Z to represent the upper edge of the keel, and length of the gun deck; but it may be produced to the aft side of the wing transom, and fore part of the stem.

2*d*. Erect a perpendicular to the line X Z, upon which set up the heighth of the wing transom to W; taken from N° IV. on the sector.

3*d*. Take the rake of the post from N° VII. on the sector, and set it forward from the perpendicular of the wing transom to the point 7, where a perpendicular must be erected, which will be the station of that timber.

4*th*. Take the distance of the frames from N° III. on the sector, and set it off from 7 to 8; and erect a perpendicular at that point for timber 8. Draw also a line from 8 to the wing transom, to represent the fore part of the post.

5*th*. Take the heighth and rake of both counters, also the rake of the stern timber from N° II. The heighth of the stern timber is on N° I, and by these form the counters, and upright of the stern.

6*th*. Station the timbers, by taking the distance betwixt the perpendiculars at 7 and 8; which at eight times will reach to ⊕; and erect perpendiculars at 5, 3 and ⊕. Then for stationing the timbers in the fore body, we must turn the sector, and take the distance of the frames from N° III. which, set eight times from ⊕, will reach to H. Erect perpendiculars at C, E, G and H; and from G set off the distance of the gun-

 deck

deck before G. It is in N° I. on the sector; which will reach to Z; at which point erect a perpendicular, and set off the heighth of the gun-deck, taken from the sector; and from the gun-deck set up the heighth of the head of the stem, also its distance before G; both taken from N° I. We may then form the stem. The center of the sweep is in the perpendicular of timber F, and the radius of the sweep is upon the sector betwixt N° III and IV. which set up from the point F, will give the center: So the sweep will just touch the upper edge of the keel in the point F. And as the sweep will not reach to the gun-deck, we must make use of a mould to break in fair with the back of the sweep.

7th. Set up the heighths of the lower, upper, and top timber breadth lines upon the perpendiculars erected for the stations of the several timbers. The points corresponding to each, are on their proper lines on the sector.

Having thus finished the sheer plane, we may then go to the floor plane; and producing all the perpendiculars for the timbers, we may upon them set off the main, and top timber half breadths. The points corresponding to each, are on their proper lines upon the sector; which must be set off from the line W K, representing the lower side of the keel, and may be produced both ways, as far as shall be needful. We must in the next place form all the ribband lines, which are the dotted ones in the draught; beginning with the fourth ribband. But it will be more expeditious, first to draw all the diagonals in the body plane.

Let A B be the whole breadth, on the middle of which erect the perpendicular K O; so shall A K, or K B, be the half breadth. Upon the line K O, set up the several heighths of the breadth lines, taken from the sheer plane, and draw lines parallel to the base, as directed in the preceding sections; and likewise set off the half breadths, corresponding to each, taken from the floor plane. We may also set off the heighth and half breadth of the wing transom; all which may be done without the sector, but we must have recourse to it for the dead rising. This is in N° III. in the after body, and must be set off upon the line K O, from the lower heighth of breadth to *i*. Thro' *i* draw the line *r i s*, parallel to the base, and set off the ½ breadth of the floor from *i* to *r*, and from *i* to *s*: It is upon N° VII. on the sector. Then taking *r i*, set it up from *i* upon the line K O, to which point draw the dotted diagonal marked 1st Rd. This regulates all the other diagonals: For if one line be drawn from the point of its intersection, with the middle line, to the half breadth of the wing transom; and another from the point *r*, its intersection with the rising line, to the point ⊕ at the lower heighth of breadth;

breadth; each of these may be divided into four equal parts by the dotted diagonals 2^d R^d 3^d R^d 4^d R^d.

Note. The lines from the ends of the first diagonal to the lower heighth of breadth, and to the wing transom, were drawn only with a black lead pencil, and wiped out after the diagonals were drawn. The diagonals being thus drawn, we may form the midship frame, for which purpose we must find a point in each diagonal, thro' which the curve of the timber must pass. These points we have from the sector; which must be set off from the intersections of the diagonals with the line K O. That in the first diagonal is in N° I. The point in the second diagonal, is in N° VI. The point in the third diagonal, is in N° V. And the point in the fourth diagonal, is betwixt N° I. and II. This last must be doubled, because the sector will not contain the whole length. The midship frame being formed, we must in the next place form the after and foremost timber; which the sector does, by giving the distance on every diagonal betwixt these timbers, and the midship frame now formed: So that we shall have a point in each diagonal, thro which the curve of the timber must pass. To find the point in the first diagonal for the after timber; extend from ⊕ 1st R in N° V. to the corresponding point on the other leg. Set off this distance from ⊕ on the first diagonal: Do the same upon the second, third and fourth diagonals. The point on the second diagonal, is in N° IV. That on the third, in N° VI. And that on the fourth in N° VII. The curve must pass thro' these points, and likewise thro' the point for the half breadth, which was before set off from the sheer and floor planes; by which means we have determined the form of the after timber; and the foremost timber is to be formed by the same method. These two timbers being formed, we may find points in the diagonals for all the intermediate timbers. Thus, to find the point for timber 3 in the first diagonal; extend from the point 3 in the inner part of the line N° V. to its corresponding point on the other leg. Set off this in the first diagonal from the after timber, already formed; which will give the point thro' which timber 3 must pass; and to find the point in the second diagonal, we must extend from 3 in the inner part of the line N° IV. and set off this distance in the second diagonal from the after timber. The same method must be used to find the points in the third and fourth diagonals. In like manner we may find a point in each diagonal for the timbers 5 and 7, which will be sufficient for the after body: And the same process must be used to find points in each diagonal for the timbers in the fore body.

Having now found the points, before we form the timbers, it may be proper, by them, to form the ribbands: For now we may take the distance of of each point in the diagonal, from its intersection with the middle line K O, and transfer it to the floor plane upon the perpendiculars that represent the planes of the timbers, as directed in *Sect.* 2. In order to limit the ends of the ribband lines in the floor plane; we must set off half the thickness of the post, on one side of the middle line K O, and half the thickness of the stem on the other side of it, in the body plane; first deducting the depth of the rabbet out of it. We must likewise determine the inner part of the rabbet on the stem, and upon the post in the sheer plane. In the stem, it is generally in the middle betwixt the lines that represent the outside of the rabbet. It may be also *s o* on the post, from the wing to the lower transom; and from thence the line may be continued fair to intersect the line that represents the after side of the rabbet, at the upper edge of the keel; for there the rabbet is cut square into the post.

Now, it is obvious that when the plane of any diagonal ribband is in its proper place and position, the line W K will be in the sheer plane, parallel to the upper side of the keel; and its heighth will be the same with that of the point where the diagonal intersects the middle line in the body plane. But by reason of its inclination, and of the half thickness of the stem and post; the heighth of the plane of the ribband upon the post and stem, will be in the point where the diagonal intersects the line that represents the rabbet in the body plane. This then must be transferred from the body to the sheer plane, and set up from the upper edge of the keel upon a perpendicular that will intersect the line that represents the inside of the rabbet at that heighth. This perpendicular may be produced into the floor plane; and if that part of the diagonal intercepted betwixt the middle line, and the line that represents the inside of the rabbet, in the body plane, be set off upon the perpendicular, it will give the proper point for the end of the ribband line, as may be seen in the plate; where all the ribbands are dotted lines, and they are marked 1st D R, 2 D R, &c.

Note. The scale in the plate is so small, that we have taken the outside of the rabbet to limit the end of the ribband.

The ribbands being thus formed, we may from them form all the timbers below the breadth.

The next thing to be done, is to form the top timbers. We have the heighth and half breadth of each from the sheer and floor planes; and the timbers below the breadth, are carried up by a sweep, which forms the lower part of the top timber. The center of this sweep is in the upper heighth of the breadth line of the timber, and may be taken from the sector: It is on N° III. after body. The midship top timber has generally a hollow, which is left intirely to the artist; for some, especially small

ſmall ſhips, have none. The general practice is to make a mould for this hollow, either by a ſweep, or ſome other contrivance, and produce it conſiderably above the heighth of the top timber in a ſtrait line, or very near one. The midſhip timber is formed by this mould, and ſo placed, that it breaks in fair with the back of the upper breadth ſweep. All the other timbers are likewiſe formed by the ſame mould; obſerving to place it ſo that the ſtrait part of it may be parallel to the ſtrait part of the midſhip timber; and moved up or down in that direction till it juſt touches the back of the upper breadth ſweep. Some begin at the after timber after the mould is made for the midſhip one, becauſe they think it eaſier keeping the ſtrait part of the mould parallel to this, than to the midſhip timber; and by this means the top ſide is kept from winding.

Others again, make a mark upon the mould where the breadth line of the midſhip timber croſſes it; and with the ſame mould they form the after timber. This will occaſion the mark that was made on the mould, when in midſhips, to fall below the breadth line of the after timber; and ſo another mark is made at the heighth of the breadth of the after timber. The next thing to be done, is to lay the ſtrait part of the mould obliquely acroſs the breadth lines of the top timbers, in ſuch a manner that it may interſect the breadth line of the midſhip timber at one of theſe marks, and the breadth line of the after timber at the other mark. Then the ſeveral interſections of the breadth lines of the timbers, are marked upon the mould. The mould being thus marked, muſt be ſo placed in forming each timber, that the proper mark may be applied to its proper breadth; and the mould be turned about ſo as juſt to touch the upper breadth ſweep. Any of theſe methods may make a fair ſide; but it may be eaſily proved by forming another half breadth line.

CHAP.

CHAP. III. SECT. I.

Of the CANT TIMBERS.

HItherto we have conſidered the timbers, as having their planes perpendicular both to the ſheer and floor planes. Theſe are called ſquare timbers; and when they are all formed, we may from them form as many ribband and water lines as ſhall be neceſſary to form the cant timbers. Their planes are inclined to the ſheer, but perpendicular to the floor planes. The reaſon of canting theſe timbers, is that they may nearly be equally ſpaced at the breadth ribband: For if the poſt has a conſiderable rake, and the timbers all ſquare, there will be a great ſpace at the breadth ribband, betwixt timber 8 and the wing tranſom: Beſides the timber may be ſo canted, that it may be ſquare to ſome of the ribbands; whereas, if they were perpendicular to the ſheer plane, they would interſect the ribband lines ſo as to form very oblique angles; which would occaſion a very great bevelling. Another advantage that attends canting the timbers, is that they will not require ſuch compaſs timber.

It is uſual to begin the cant timbers from the aftermoſt floor timber; and ſpace them near equally on the breadth line, to the wing tranſom: And in order to ſpace them upon the keel, the cant of the faſhion piece muſt be determined. Now if we ſuppoſe the plane of the faſhion piece to interſect the ſheer and floor planes in the point F, it muſt interſect the floor plane in the line F P, becauſe the point P is ſuppoſed to be the end of the wing tranſom. So the angle *s* F P will be its inclination to the ſheer plane. It will interſect the ſheer plane in a perpendicular erected from the point F; and if the ſpace betwixt the point F, and the foremoſt cant timber upon the keel, be divided into the ſame number of equal parts, that the ſpace betwixt the ſame timber, and the wing tranſom upon the breadth line, is divided into; this will determine the cant of all the timbers only by drawing lines from all points in the line W K, to the correſponding points in the breadth line, in the ſame manner as the line F P, determines the cant of the faſhion piece.

It would be needleſs to draw all theſe lines in the plate; the only intent of drawing them being to ſhew how to form the timbers by them: And as one method ſerves for all the cant timbers, which are ſuppoſed perpendicular

dicular to the floor plane; it will be sufficient to shew the formation of the fashion piece.

Before any of the cant timbers can be formed, there must be a sufficient number of water lines, or diagonal and horizontal ribband lines formed from the square timbers; and when these are absolutely determined, we may, with certainty, form all the cant timbers, either by water, or ribband lines.

If we make use of the diagonal ribbands, which are distinguished by the dotted curves in the floor plane, we must form an horizontal ribband corresponding to each. We have only laid down one of these horizontals in the plate, *viz.* that corresponding to the third diagonal: It is marked 3[d] H R. To form this ribband, fix one foot of the compasses in the point where the third diagonal intersects the midship frame in the body plane; and extend the other foot to touch the middle line K O; so that if a line were drawn from one foot of the compasses to the other, it would be perpendicular to the line K O: This distance set off from the line W K, upon the perpendicular that represents ⊕ in the floor plane, will give the point thro' which the curve must pass at that place. The same method must be used for finding the points on all the other timbers.

Now, tho' the diagonal and horizontal ribbands seem to be quite different curves in the plate, they will make but one line upon the timbers; for the one intersects them in a direction perpendicular to the sheer plane, and the other is so inclined as to intersect the timbers in the very same points. The horizontal one is too short upon the plate, but the true length of it might easily be had by transferring to the sheer plane, the several heighths at which the diagonal intersects the timbers in the body plane. By these we might form a heighth of breadth line to correspond to this horizontal ribband, which is only a half breadth line; and the length of this heighth of breadth line may be taken by a penning batten, and all the timbers marked upon it. Now when the batten is applied to a strait line, and all the timbers transferred to this line from the batten, we may erect perpendiculars at each, and set off the same half breadths as before; by which means we may have the true length of the horizontal ribband: But as this will be of no manner of service, we shall omit forming it. We only mention it, because several imagine these two curves to be as different on the surface of the ship as they are upon the draught. The horizontal ribband corresponding to the first diagonal one, is formed to timber 7, and marked 1[st] H R; but the horizontals for the second and fourth diagonals were formed by a black-lead pencil, and only the point in which they intersect the line F P, is in the plate; which is sufficient for our purpose.

There are likewiſe five water lines formed, four of which are repreſented by level lines in the body plane; and by lines parallel to the keel in the ſheer plane: Three of them repreſent the planes of the tranſoms in the ſheer plane, *viz.* D^k, 1^a, 2^d: But the plane of the third tranſom is perpendicular to the poſt. The lower water line is drawn parallel to the keel from the ſtem to the poſt, and produced into the body plane, as in the plate, where it is marked M N: The plane of the third tranſom interſects the timbers at different heighths, which are transferred from the ſheer to the body plane, where it forms a curve.

The water lines being now drawn in the ſheer and body plane; our next buſineſs is to form them in the floor plane, where they will be curves. The points thro' which the curve of the lower water line is to paſs, are had by transferring the ſeveral portions of the level line, intercepted betwixt the line K O, and the curve of each timber, from the body plane to the correſponding perpendiculars in the floor plane, where it is marked W *a t'* L. It muſt be obſerved that the line K O, in the body plane, repreſents the ſeveral perpendiculars that are drawn in the ſheer plane to repreſent the planes of the timbers: For the ſpaces in the body plane, contained betwixt the line K O, and the curves of each timber, are ſo many different planes; and when in their proper places, they will be parallel to one another, if perpendicular to the ſheer and floor planes. Thus the plane contained betwixt the line K O, and the curve of timber ⊕, is (when in its proper place) ſuppoſed to be erected perpendicular to the ſheer plane, in the line which repreſents the plane of ⊕; and the like may be ſaid of all the reſt. The planes of the cant timbers will not be parallel to one another, becauſe they are differently inclined to the ſheer plane; but as they are perpendicular to the floor plane, they will interſect the ſheer plane in a line perpendicular to the keel: So the plane of the faſhion piece interſects the ſheer plane in the dotted perpendicular erected from the point F, which is the ſame with the line K O, in the body plane. We thought it neceſſary to take notice of this, becauſe ſome who are learning to draw, miſtake the line K O; for they imagine it only repreſents the poſt or ſtem.

Another error which they frequently fall into, is about forming the water lines when their planes are not parallel to the keel. They imagine that the half breadths muſt be ſet off from the line W K, which repreſents the lower edge of the keel; whereas it is indifferent what ſtrait line they are ſet off from, ſo the timbers be exactly ſpaced, and perpendiculars drawn to repreſent their planes. Now when the water lines are ſuppoſed parallel to the keel, all the timbers are properly ſpaced, and the perpendicupendiculars ready drawn to the line W K; which is the reason it is used in such cases: Tho' when the plane is in its true place, the line W K, will be in the line M N. But the case will be quite different when the water lines are not parallel to the keel; for then their planes will intersect the sheer plane in a strait line, forming oblique angles with the planes of the timbers; this is the case in the plane of the third transom. The distances betwixt the timbers will be more in this line than in the line W K; so the half breadths cannot be set off from the line W K, upon the perpendiculars that represent the planes of the timbers unless they be properly spaced at the same distance they are upon the line that represents the plane of the third transom in the sheer plane; upon which account we have made use of that line to set off the half breadths from, and drawn the dotted perpendiculars at the points where it intersects the planes of the timbers 8, 7, and at the point where it intersects the lower heighth of breadth line. The heighths of the points of intersection are transferred from the sheer to the body plane; and the half breadths at these heighths, transferred from the body plane to the dotted perpendiculars before drawn: The half breadth to be set off upon the perpendicular where it intersects the lower heighth of breadth line, is had from the floor plane, and the dotted perpendicular *a a*, will shew the place where the half breadth must be taken: This perpendicular, if produced, will intersect the plane of the third transom in the lower heighth of breadth.

Having now formed four diagonal ribbands with their corresponding horizontals, and also two water lines; we may by these, form the fashion piece, either upon the body plane or sheer plane: But as the plane of the fashion piece is parallel to neither of these, it will require two operations.

Now the line F P, will intersect all the ribband and water lines; but because the diagonal ribbands are not in their proper position, the line F P will not intersect them in the point where the plane of the fashion piece intersects them. The first thing then to be done, is to find the true place of the fashion piece on each diagonal ribband: And first, to find its place upon the fourth diagonal ribband, from the point *t*, where the fourth horizontal ribband intersects the line F P, let fall a perpendicular to the point *s*, and produce it to intersect the diagonal ribband in *r*; so shall *r* be the true place of the fashion piece upon that ribband; that part of the perpendicular betwixt *t* and *s* is not drawn in the plate to avoid the confusion of too many lines. The reason of this will be very evident, if we suppose the whole plane of the ribband to be turned round upon the axis W K; for then the point *r* will always be right over some point of the

 perpen-

perpendicular *r t s*; and when the ribband is in its proper inclination, a perpendicular from *r* will fall into the point *t*, and the plane of the fashion piece will intersect the floor plane in the line *t* F, and the plane of the diagonal ribband in a strait line drawn from *r* to F: For it must be observed that when the ribband is in its proper place, the line W K will be in the sheer plane, in a line parallel to the keel; the heighth of which may be had from the body plane. In this case it will be the distance betwixt K and *r*, but it will be needless to draw this line in the plate.

Having now found the place of the fashion piece on the fourth diagonal ribband, we must by the same method find its place on the other diagonals, as in the plate, where lines perpendicular to W K, are drawn to the points *o*, *o*, *o*, in the diagonal ribbands, from the points where the line F P intersects the corresponding horizontal ribbands.

These points being now found, we may take the nearest distance of each point to the line W K, and set off those distances on the proper diagonals in the body plane. Thus, for the fourth ribband, place one foot of the compasses in the point *r*, and the other in the point *s* in the floor plane; and set off that distance from *r* to S, on the fourth diagonal in the body plane: Do the same by all the rest of the diagonals; and a curve intersecting the diagonals in these points would be the projection of the fashion piece in the body plane, but we have not drawn this in the plate; for as the plane of the fashion piece is not parallel to that of the body plane, its projection will be less than the original: However this may be found by *Prob.* 6. *Chap.* 1. *Part* 2. by the following method.

1st. Draw a perpendicular to the line K O, in the body plane, to pass thro' the point S to F.

2d. Take the distance from *r* to F, in the floor plane, and set it off from *r* to F, in the body plane. In like manner draw perpendiculars to the line K O, in the body plane, thro' the points before found on the diagonals, as in the plate, where only that part of the perpendicular is drawn which lies without the diagonal; and take the several distances betwixt the points *o* and F, in the floor plane, and set them off from the intersections of their corresponding diagonals, with the line K O, to the points *o*, *o*, in the body plane: So we have the points *o*, *o*, F, thro' which the curve must pass.

3d. To find the point P in the body plane, thro' which the curve must pass. Transfer the point P in the floor plane, to the point P, in the sheer plane, by a perpendicular to the line W K, to intersect the heighth of breadth line in the point P; and set off this heighth upon the line K O, in the body plane, which will be a little above W, the heighth of the

the wing tranſom: Draw a perpendicular at this point, to the line K O; take the line F P, in the floor plane, and ſet it off upon this perpendicular, from the line K O, to the point P: So ſhall the curve P F *o o*, be the form of the faſhion piece.

Theſe points may all be found without the diagonal ribbands, by half breadth lines and water lines, formed on the floor plane, as for inſtance: To find the point F; place one foot of the compaſſes in *t*, the point where the horizontal ribband interſects the plane of the faſhion piece, and the other in *s*; *t s* being perpendicular to W K: With that extent, move the compaſſes with one point in the line K O, and the other point perpendicular to it, till it interſect the fourth diagonal, in the point S; thro' which draw the perpendicular *t* F. Then take the diſtance from *t* to F, in the the floor plane, which ſet off from *t* to F, in the body plane; ſo ſhall F be the point required, as before: In like manner the points *o*, *o*, may be found: But this, as well as the other method, requires two operations; whereas, if ſeveral water lines were formed, with their planes parallel to the keel, we might find the points by one operation. Thus, ſuppoſe it was required to find a point in the level line, that repreſents the plane of the water line formed in the floor plane, which is marked W *a t* L. Fix one foot of the compaſſes in the point *f*, where the line F P interſects the water line in the floor plane, and the other foot in the point F. Set off this upon the level line in the body plane, from the line K O to *f*, which will be the point required. All the other cant timbers, both in the fore and after body, are formed after the ſame manner as the faſhion piece. We have formed but one more in the plate, which is abaft the faſhion piece, to aſſiſt us in forming the tranſoms.

S E C T. II.

Of the TRANSOMS.

THE tranſoms are faſtened to the ſtern poſt, in the ſame manner that the floor timbers are to the keel; and as the floor timbers have a riſing, ſo likewiſe have the tranſoms, which is called the flight; and beſides this flight, the wing tranſom has a round aſt, and a round up, both which are arbitrary: The deck tranſom has a round up, the ſame with that of the beams: But in forming the tranſoms, there is no regard had

had to the round up; for that may be done by the beam mould, after the tranſom is properly hewed the moulding way.

In forming the tranſoms, the firſt thing to be done, is to aſſign each its proper place upon the poſt, and then to determine the poſition of their planes with reſpect to the floor plane; for their planes are always perpendicular to the ſheer plane. In the plate there are five tranſoms: Their upper ſides upon the poſt are in the points W, D^k, 1^{st}, 2^d, and 3^d: The planes of the wing, deck, firſt and ſecond tranſoms, are ſuppoſed parallel to the floor plane, and repreſented in the ſheer plane by lines drawn parallel to the keel from the poſt, till they interſect the lower heighth of breadth line; and the plane of the third is repreſented by a line perpendicular to the poſt, as in the plate, So it will not be parallel to the floor plane.

The heighth and poſition of the tranſoms being determined, we have no more to do, but to form water lines for each. That for the third we have already formed: The reſt being ſuppoſed parallel to the floor plane, may be formed in the ſame manner as the water line there laid down: The only difficulty will be to find a ſufficient number of points to determine their forms; becauſe in the deck and firſt tranſoms, their planes interſect the breadth; ſo that we could only have a point in timber 8, if the faſhion piece and a timber abaſt it, had not been formed by the ribbands; but now they are formed, we may have likewiſe a point in each of their planes, thro' which the curves of the water lines ſhall paſs.

We ſhall begin with the wing tranſom. Firſt determine the round aft which ſuppoſe the line W T, in the floor plane: Take its heighth from the ſheer plane, and ſet it up in the body plane from K to W, and draw the line W T: Then take this line W T, and ſet it off on the floor plane, on the line F P, which will reach to the point *n*. A curve drawn thro' the point *n*, to break in fair with the breadth line, as in the plate, will interſect the line W T in T; ſo ſhall W T *n*, be the aft ſide of the wing tranſom. Next for the deck tranſom, draw a level line in the body plane at the point D^k to timber 8. Set off this diſtance upon timber 8, in the floor plane, from the line W K; which will give us a point thro' which the curve muſt paſs: Then take the diſtance in the level line, betwixt the line K O, and the curve of the faſhion piece; which ſet off from the point F, upon the line F P, in the floor plane; and this will give another point thro' which the curve muſt paſs: Again, take the diſtance in the ſame level line, betwixt the line K O, and the curve of the timber abaſt the faſhion piece; which ſet off from the point G upon the line G *g*, in the floor plane; and we ſhall have a third point thro' which the curve muſt

muſt paſs. Laſtly, let fall a perpendicular to the line W K, from the point D^k upon the poſt, and produce it into the floor plane, upon which ſet off half the thickneſs of the poſt, allowing for the rabbet, which will limit the end of the water line that forms the deck tranſom. After the ſame manner are the firſt and ſecond tranſoms formed, by drawing level lines in the body plane, at their heighths upon the line K O.

Now ſome are apt to miſtake theſe level lines for the lengths of the tranſoms: The reaſon, as was before obſerved, is becauſe they imagine the line K O to be the ſtern poſt; whereas it is the perpendicular in which the plane of the faſhion piece interſects the ſheer plane; and ſo theſe lines are drawn upon the plane of the faſhion piece.

All that now remains, is to determine the length of each tranſom; and this is done by the line F P, in the floor plane, which interſects the wing, deck, firſt and ſecond tranſoms, to their proper lengths. But before we can find the length of the third, the plane of the faſhion piece muſt be projected upon the ſheer plane: Thus, take the neareſt diſtance betwixt any perpendicular in the floor plane, and the point where the line F P interſects the water line; and ſet that off from the ſame perpendicular upon the line that repreſents the ſame water line in the ſheer plane. Now the curve P F will be found to be the projection of the aft ſide of the faſhion piece upon the ſheer plane: For the diſtance betwixt the perpendicular of timber 8, and the point *f*, where the line F P interſects the lower water line in the floor plane, is equal to the diſtance betwixt the ſame perpendicular and the curve P F, taken in the line M N. The diſtance betwixt the perpendicular of timber 8, and the point where the line F P interſects the ſecond water line in the floor plane, is equal to the diſtance betwixt the ſame perpendicular and the curve P F, taken in the line that repreſents the ſecond tranſom in the ſheer plane. And by the ſame method, we find points in the lines that repreſent the deck and firſt tranſoms. The point P is transferred from the half breadth line in the floor plane plane, to the heighth of breadth line in the ſheer plane. The curve being thus drawn, will interſect the line that repreſents the plane of the third tranſom, in the point *z*: From which point draw the perpendicular *z* F, to the curve of the dotted water line; ſo ſhall 3^d *z*, F, be the true form of the third tranſom; and a line drawn from F to P, will be the plane of the faſhion piece. It muſt be obſerved that the ends of the tranſoms are let into the faſhion piece; for which there muſt be a proper allowance left without the lengths found by the line F P. We have in the plate only laid down the half of each tranſom. Thoſe who incline

incline to lay down the whole tranſoms, may eaſily transfer the halfs already deſcribed to the other ſide of the line W F.

Having now formed all the timbers, both ſquare and cant, in the after body; we ſhall proceed to the fore body. The cant timbers are laid down in the ſame manner as thoſe in the after body, by the diagonal and horizontal ribbands; where the dotted line K T repreſents the plane of the knuckle timber, canted upon the floor plane; from whence it is transferred to the body plane, and repreſented by the dotted curve betwixt timber H and G.

The hawſe pieces are ſeldom laid down in the loft; it being the general practice to make moulds for them after the other timbers are put up, and the harpins are brought about; but they may be formed in the following manner.

Let P H repreſent the plane of the hawſe piece on the floor plane, which may be produced to K T, the plane of the knuckle timber: In the plate let H be ſuppoſed the heel of the hawſe piece; from which point erect the dotted perpendicular *b l* into the ſheer plane, and draw the dotted level lines *a l*, *c l*, in the body plane; by which, form the water lines *a l*, *c l*, in the floor plane, and draw the dotted lines *a l*, *c l*, perpendicular to the line *b l*, in the ſheer plane; which will repreſent the planes of the water lines. Draw alſo the dotted perpendicular *b l*, to the point where the line *b l* interſects the upper heighth of breadth line. Upon the line *c l* in the ſheer plane, ſet off the diſtance H P, taken from the floor plane, P being the point where the plane of the hawſe piece interſects the water line *c l*. Then take the diſtance from H, to the point where the line H P interſects the M $\frac{:}{r}$ Bth line, and ſet it off upon the line *b l* to *t*; or, rather find the point upon the lower heighth of breadth line, where the hawſe piece comes to; from which draw a perpendicular to the line *b l*, and upon this ſet off the diſtance, as before: Then take the diſtance from H to the point where the line H P interſects the water line *a l* in the floor plane, and ſet it off upon the line *a l*, in the ſheer plane to *p*. Laſtly, to find the heighth of the heel, becauſe we have not formed a timber at the point H, produce the line H P, to interſect the plane of the knuckle timber K T, in the floor plane, at the point *r*: Take *r* K, with a pair of compaſſes, and placing one foot in the curve of the knuckle timber in the body plane; ſo as that the other foot touch the line K O, or rather ſet off *r* K from the line K O to *k* upon the baſe line; at which point erect a perpendicular to interſect the curve of the knuckle timber in the point *k*; ſo ſhall *k k* be the heighth of the heel, if the plane of the hawſe be produced to interſect the plane of knuckle

knuckle timber: But in the plate the heel of the plane of the hawſe piece is ſuppoſed to be at the point H: Therefore a perpendicular muſt be erected from the point *r*, into the ſheer plane, upon which ſetting up the heighth *k k*, we ſhall have the point *k*. We may by the ſame method find a point in the plane of timber H, in the ſheer plane; through which the curve of the hawſe piece muſt paſs; and if produced to *k*, it will interſect the perpedicular *h l*, in the point *h*; which is the heighth of the heel.

Tho' the hawſe pieces are ſeldom laid down, yet by forming them on the ſheer plane, we ſhall thereby diſcover if there be any faults in the half breadth lines or water lines: For if the timbers that are formed by theſe lines are not fair, ſome of thoſe lines from which they are formed muſt certainly be the occaſion of it; which therefore muſt be rectified before we can find the true form of the harpins; which is the next thing to be done.

S E C T. III.

To form the Harpins and Rails of the Head.

AS the harpins are level'd acroſs, they will be formed by the ſection of a plane perpendicular to the ſheer plane: But there is no neceſſity for theſe ſections to be parallel to the keel. In the plate we have drawn only a ſtrait line to repreſent the plane of the harpin above the wale. It is drawn from the ſtem to timber E, and marked harpin. Now in order to form the curve of this harpin, it would be proper to form timber F, in the body plane: Alſo to draw perpendiculars to the ſeveral points where the plane of the harpin interſects the planes of the timbers E, F, G and H, in the ſheer plane; and upon theſe to ſet off the half breadths correſponding to each, taken from the body plane. This would give us the points thro' which the curve muſt paſs, which would be a water line. But as this is performed exactly in the ſame manner as the water line that repreſents the third tranſom, we judge it unneceſſary to form it in the plate.

The rails of the head are projected on the ſheer plane, according to their true hangings; and in order to find their true lengths, draw the dotted line S T, parallel to the keel at the heighth of the rails, upon the head. We muſt then determine the ſtation of the cat-head upon the floor plane; and likewiſe the thickneſs of the head at the rail; and let fall a perpendicular from the point T, where the line S T interſects the cat-head in the ſheer plane, to the point T in the floor plane; and likewiſe a perpendicular from S in the ſheer, to S in the floor plane; and draw the line T S: *s* S in the floor plane being half the thickneſs of the head of the figure at the rail; ſo ſhall T S, in the floor plane, be the true length of the rail. Let the line T S, in the ſheer plane, be divided into any number of equal parts: Suppoſe into the points *x*, *y*, *z*; from which points draw perpendiculars to the line T S, to be limited by the rail. Divide the line T S, in the floor plane, into the ſame number of equal parts, in the points *x*, *y*, *z*. Draw perpendiculars to theſe points, and make them equal to the correſponding ones in the ſheer plane; ſo we ſhall have the points thro' which the curve of the rail muſt paſs.

We have now ſhewn different ways of forming all the timbers; where it muſt be obſerved that we have always ſuppoſed every timber to be one intire piece of wood from the keel to the top of the ſide; whereas in reality, they are in ſeveral different pieces; the head of the lower piece being cut ſquare to join to the heel of the next above it: And in order to ſupport theſe joinings, another ſett of pieces are cut, and joined together in ſuch a manner, that if both the ſetts were faſtened together, the joinings in one ſett, would be nearly againſt the middle of the pieces in the other ſett. In this manner are all the frames faſtened and erected, as if each was one piece of wood. The pieces laid acroſs the keel, to which they are faſtened, are called floor timbers: The other pieces are called futtocks, except that which goes to the top of the ſide, which is called a top timber. Hence it is plain that the mould which ſerves for the floor timber, will ſerve for the lower part of the correſponding futtock. The mould for the upper part of the firſt futtock, will be the ſame with that for the lower part of the ſecond; and the mould for the lower part of the top timber will be the ſame with that of the upper part of the correſponding futtock. It is of great importance in building, to give proper ſcarph to the timbers; for which we refer our readers to the table of ſcantlings at the end of this part.

CHAP.

CHAP. IV. SECT. I.

Of Bevelling the TIMBERS.

IN the preceeding chapters we have conſidered the timbers as plain ſurfaces, without any regard to their thickneſs or breadth; whereas every timber conſiſts of two planes, and the ſpace contained betwixt them is the breadth of the timber. We have already ſhewn how to find the form of one of theſe planes, which is called the moulding ſide of the timber. The form of the other ſide will be different from the moulding ſide, except in midſhips. Now if the timber be properly hewed from the moulded ſide, we ſhall have the form of the other ſide: This is what is called bevelling the timbers; a term ſo well known that it needs no explication, We ſhall only remark that the bevelling is the angle made by the meeting of two planes limiting a ſolid; and as this angle cannot be meaſured by ſcale and compaſſes, without cutting the ſolid by another plane perpendicular to both; it is done by an inſtrument called a bevel. When the angle is a right one, the timber is ſaid to be ſquare, and is meaſured by an inſtrument of that name.

In order to hew any piece of timber to its proper bevel, it will be very proper firſt to make one ſide fair, and out of winding; a term uſed to ſignify that the ſide of the timber ſhould be a plane. Now if this ſide be uppermoſt, and placed horizontally, or upon a level; it is plain if the timber is to be hewed ſquare, it may be done by a plummet and line; but if the timber is not hewed ſquare, the line will not touch both the upper and lower edge of the piece; or if a ſquare be applied to it, there will be wood wanting either at the upper or lower ſide. This is called within or without a ſquare. When the wood is deficient at the under ſide, it is called under bevelling; and when it is deficient in the upper ſide, it is called ſtanding bevelling; and this deficiency will be more or leſs, according to the depth of the piece; ſo that before the proper bevellings of the timbers are found, it will be ſometimes very convenient to aſſign the breadth of the timber; nay in moſt caſes it will be abſolutely neceſſary, eſpecially afore and abaft; tho' the breadth of two timbers, or the timber and room, which, as was before obſerved, includes the two timbers, and the ſpace betwixt them, may be taken without any ſenſible error; as far as the ſquare body goes. For as one line repreſents the moulding

moulding ſide of two timbers, the foreſide of the one being ſuppoſed to unite with the aft ſide of the other; the two may be conſidered as one intire piece of timber.

Notwithſtanding it is uſual in draughts to lay down only every third or fourth timber; yet in the loft it will be neceſſary to lay down all the timbers: But as our plate will not admit of this, let us ſuppoſe the line *a a e*, betwixt the timbers 5 and 7, in the floor plane, to repreſent the moulding ſide of two timbers; and the lines *m n* and *r s*, the moulding ſides of other two timbers. Draw the lines *b c* and *k l*, the one in the middle betwixt *e a* and *m n*, and the other in the middle betwixt *e a* and *r s*; ſo ſhall the diſtance betwixt the lines *b c* and *k l*, be the breadth of two timbers, together with the ſpace betwixt them: The portion *b k* of the ribband may be taken for a ſtrait line, and then the angle that is made by the line *b c* and *b k*, or the angle made by the line *k l* and *k b*, will be the bevelling according to the ſide on which the timber is moulded; the one being as much ſtanding as the other is under bevelling. In order to find how much this is from a ſquare, draw the lines 1, 2, 3, 4, perpendicular to the lines *b c* and *k l*; and the portions of the line *e a* intercepted betwixt the ribbands, and theſe perpendiculars will be what the timber is either within or without a ſquare; ſo 4 *e* will be that at the fourth ribband: And becauſe the line *e a* repreſents the moulding ſide of both timbers, the timber before it will be ſtanding, and the timber abaft it, under bevelling.

It is very neceſſary to obſerve that the planes of the ribbands ſhould be perpendicular to the planes of the timbers, which is the caſe in all the ſquare timbers: But the planes of the cant timbers are inclined to the planes of the ribbands; therefore their bevelling cannot be had by the ribband lines in the ſame manner as theſe of the ſquare, becauſe when the ſtock of the bevel is laid upon the moulding ſide of the timber, the tongue of the bevel will be out of the plane of the ribband.

Another thing to be carefully obſerved, is in what direction the ſtock of the bevel is to be laid upon the moulding ſide of the timber. This is found in the body plane: If we bevel by ribband lines, the diagonals will give the line of direction; but if the bevellings are taken by water lines, the level lines in the body plane will give the direction in which the ſtock of the bevel is to be laid upon the timber: When theſe lines in the body plane are very oblique to the curves of the timbers, if the bevel is not kept exactly in the ſame direction, it will occaſion a very great error; and only the very ſharp edge of the tongue will touch the timber. For this reaſon, the beſt way to take the bevellings will be, ſo that

that both ſtock and tongue may be ſquare to the timber; but this will alter the bevelling, and bring it likewiſe nearer to a ſquare, which is another advantage we ſhall gain by altering the direction of the ſtock; and the true bevelling may be found by the following method.

Let the diſtance betwixt timber 7 and timber 8, in the floor plane, be ſuppoſed the breadth of a timber; then the perpendicular at 8 will repreſent the plane of the aft ſide; and the perpendicular at 7, the plane of the fore ſide of the timber in the floor plane: The curve of timber 8 in the body plane, will be the form of the aft ſide; and the curve of timber 7, the form of the fore ſide of the timber: So that the neareſt diſtance betwixt theſe two curves, will certainly be what the bevelling differs from a ſquare; for if the timber were ſquare, the ſame curve would repreſent both ſides of it.

Now if it were required to find the bevelling of this timber by the water line, (W *a t'* L in the floor plane) it is evident it will be the angle 8 *i u*, if the moulded ſide be aft; and *x u* will be what it is without a ſquare. This will be in the direction of the level line in the body plane, where it is *x u*; but *x v* being the neareſt diſtance betwixt the curves taken from the point *x*, that muſt be ſet off from *x* to *v*, on the floor plane; ſo ſhall 8 *i v*, be the true bevelling, when the bevel is ſet ſquare to the timber at the point *v*, where the firmark muſt be placed. But if the moulded ſide be forward, the angle *x u i*, will be the bevelling, and *x u* what it is within a ſquare; the ſame as that which was without a ſquare when the moulding ſide was aft. Here *u z* is the neareſt diſtance betwixt the curves, taken from the point *u*; and when this is ſet off from *x* to *z*, in the floor plane, the angle *x z i*, will be the true bevelling at the point *z*, in the body plane, where the firmark muſt be placed. This method will be very uſeful for the cant timbers, when they are bevelled by water lines, and may be done by the workman, if the bevelling is given in the direction of the plane of the water line, by obſerving the following directions, which are in effect the ſame with theſe now preſcribed.

1ſt. Apply a ſquare to the bevelling board, at the point where the line that determines the bevelling of the timber interſects the ſide of the board; and the diſtance of the other end of the line from the ſquare upon the oppoſite ſide of the board, will be what it is within or without a ſquare.

2d. If the timber has an under bevelling, take the quantity of it, found by the ſquare, with a pair of compaſſes, and ſet it off upon the line of direction, on the timber, from the point where the line interſects the moulded ſide, which is raſed in upon the timber: One foot of the compaſſes being fixed in this point, let the other foot reſt in a point in the line

line of direction: From this last point take the nearest distance to the outside of the timber, and mark the firmark at that place.

3*d*. Take the nearest distance found on the timber, and set it off from the square upon the same side of the bevelling board, from which the distance set off upon the line of direction, was taken; and mark that place upon the board. A line drawn from that point, to the point where the square was applied on the opposite side of the board, will give the true bevelling, to be taken square to the timber, observing to set it to the proper firmark.

If the timber has a standing bevelling, we must apply a strait edged batten to the line of direction upon the timber, upon which we must set off what the bevelling is, without a square; and proceed in the same manner as before.

Note. If the bevelling board is not exactly the breadth of the timber, the bevelling must be transferred from the board to two parallel lines, the breadth of the timber being the distance betwixt them.

But if it be required to bevel the cant timbers by the diagonal ribbands, the angle *F o b*, will be that which the fashion piece will then make with the third ribband: For *o* being the point where the plane of the fashion piece intersects the third ribband; a line drawn from *o* to F, will be that in which the plane of the timber intersects the plane of the ribband: But then as these planes are not perpendicular to one another, the angle F *o b* will not be the true bevelling, unless the bevel be so applied that the tongue may be in the direction of the ribband, and then the stock cannot lay flat upon the side of the timber: For which reason this method will not do for practice; for the surest way to take any bevelling, is when both the stock and the tongue of the bevel are square to the timber.

In order then to find the true bevelling upon a square, the direction in which the ribband intersects the timber must be given, as well as the angle F *o b*; and likewise the breadth of the timber: Now if these three be given, the angle upon the square may with certainty be found by the following method.

Let the distance betwixt the parallel lines A B and E F, be the breadth of the timber; B *a* the direction of the ribband; and *a b* what the bevelling is without a square. (*See the* Fig. *under the* Scale, Plate 7.)

Now, that we may the easier conceive how this bevelling may be found, let us suppose the timber to be quite strait, and first trimmed square. Then, because *a b* is what it is without a square, it is plain there must be so much lined off the aft side of the timber, and when this is hewed

hewed off; the line *a* B will be the breadth of the outside of the timber; and if B D be made equal to B *a*, and D *d* equal to *a b*, and the angle at D a right one; then it is plain the angle A B *d*, will be the bevelling, if the tongue of the bevel can be kept in the direction of the line B *a*: But when the stock of the bevel is laid flat on the side of the timber, the tongue will naturally be perpendicular to the plane of the timber, which will be in the line B F; and if F *f* be made equal and parallel to D *d*; then will the angle A B *f*, be the true bevelling upon a square: But if the outside of the timber is a curve, the stock must be placed at the point *t*, and *t* B made equal to F *a*; and the tongue will come to the point *a*.

To apply this to find the bevelling of the fashion piece at the third ribband, the angle F *o b*, is given in the floor plane; and to find the direction in which the plane of the ribband intersects the plane of the timber; we must find the angle, or the inclination of these two planes to one another: For tho' the plane of the timber is perpendicular to the plane of the water lines; it will not be so to the planes of the ribbands: And what was asserted in *Prob.* 7. in regard to the angle at the top and bottom of the chest, *viz.* that they would be equal, must be understood so, as that both the stock and tongue of the bevel be kept parallel to the back side of the chest; which might be easily done, when the partition is properly bevelled to the backside of the chest: But here the case is different, therefore we must find it by the following method.

1st. From the point *o* draw the line *e o* perpendicular to F *o*; the line in which the plane of the timber intersects the plane of the ribband on the floor plane.

2d. Thro' the point *o* in the body plane, draw the line *e o* perpendicular to K O, *o* being the point where the ribband intersects the fashion piece.

3d. Take the line *o e* from the floor plane, and set it off from the point 3, where the third diagonal intersects the line K O in the body plane to the point *e* in the line *o e*.

And lastly draw the line 3 *e*; so shall O 3 *e*, be the angle the plane of the ribband makes with the plane of the fashion piece.

Now let the distance betwixt the line K O and *p f*, be supposed the breadth of the timber; then will 3 *q* be the breadth of it upon the plane of the ribband; which set off upon the floor plane from *o* to *m*, and draw the line *b m* B, parallel to F *o*; so *b m* will be what the bevelling is without a square, when taken in the direction of the ribband; and the angle F *o b*, the bevelling. In order to find the bevelling upon a square,

set

ſet off the breadth of the timber from *o* to *l*, and make *l* 2 equal to *m b*; ſo ſhall F *o* 2, be the true bevelling on a ſquare. The line 2 *o* is omitted in the plate, becauſe it would be too near to the ribband line *b o*.

We have now ſhewn how to bevel the cant timbers either by water or ribband lines; and much after the ſame method, may the faſhion piece of a ſquare tuck be bevelled. It both rakes and cants, and of conſequence will be inclined to the planes of the water lines, if they are parallel to the keel; as that of the long-boat. (*See Plate* 5.)

Now, in a ſhip the planes of the water lines may be repreſented by lines perpendicular to the plane of the faſhion piece, in the ſheer plane; and formed in the ſame manner as that of the third tranſom in the ſhip (*Plate* 7.): And then we ſhall have the true bevelling in the ſame manner as that of the cant timbers. It may likewiſe be done by water lines parallel to the keel, as in the long-boat; where we cannot form all the neceſſary water lines perpendicular to the plane of the faſhion piece, becauſe there are no top timbers. We may therefore find the angle the planes of the water lines make with the plane of the faſhion piece; and from thence find the true bevellings, by the method directed for finding the bevellings by the ribband lines, when the plane of the faſhion piece is perpendicular to the floor plane: So that all that ſeems now neceſſary, is to ſhew how to find the angle the plane of the faſhion piece makes with the floor plane, to which the planes of the water lines are ſuppoſed parallel; in order to which;

1*ſt*. Produce the line W *k* to the point *m*, in the line *g* L, the common ſection of the ſheer and floor planes.

2*d*. Thro' the point *m* draw the line *d n* parallel to *g* G.

3*d*. Let fall the perpendicular *k y*, and thro' the point *y* draw the line *l n* perpendicular to *d n*. Draw alſo the line *y v* perpendicular to *l n*.

Note. The point *k* may be aſſumed at pleaſure.

4*th*. From the center *m*, with the radius *m k*, interſect the line *l n* in the point *l*. We may alſo draw the line *m l*; ſo *k m l*, will be the angle formed upon the plane of the faſhion piece by its interſection with the ſheer and floor planes.

Laſtly. With the radius *n l*, from the center *n*, interſect the line *y v* in *v*; ſo ſhall *l n v*, be the angle which the plane of the faſhion piece makes with the plane of the water lines; as in *Prob*. 7. *Chap*. 1. *Part* 2.

Having thus found the angle, let *d m* be the breadth of the faſhion piece: Thro' *d* draw a line parallel to *n l*, to interſect the line *n v* in the point *c*; ſo ſhall *c n* be the breadth of the faſhion piece upon the plane of the ribband. If then lines be drawn on the floor plane, parallel to

g G,

g G, *e* 3, *o* 2, *n* 1; as *x f* parallel to *g* G, and if *x* G be equal to *n c*, and perpendicular to *g* G; then the angle *g* G *f*, will be the bevelling upon the plane of the ribband, and *x f* what it is without a ſquare. Again, if from the point G we ſet off *d n*, the breadth of the timber, and draw the dotted line *i i*, equal and parallel to *x f*; we ſhall have the angle *g* G *i*, the true bevelling upon a ſquare.

After the bevellings of the timbers are found, they are put on a board provided for that purpoſe, called a bevelling board. This board ſhould be made the exact breadth of the timber, which ſuppoſe *x, z, s, t*, in the figure bevel (*Plate* 5.). If upon one edge of the board we ſet off as many points, as we intend it ſhall contain timbers, and place them at any convenient diſtance from one another, whether equal or unequal is indifferent, and diſtinguiſh them all by their proper names; we may then lay the graduated edge of the board to the line that repreſents the moulding ſide of the timber, ſo that the proper point be at the interſection of the plane of the ribband, and the plane of the timber; and when in this poſition, if we mark the other edge of the board where it croſſes the ribband; a line drawn acroſs the board from theſe two points, will be the true bevelling of the timber at that place. So when the board is applied to timber 7 in the floor plane, we ſhall find the bevelling to be as much from a ſquare as is expreſſed by the dotted ſquare line drawn acroſs the board at that place. It muſt be obſerved that the perpendicular at timber 7. repreſents the foreſide of the floor timber, and aftſide of its correſponding futtock: So that the floors will be under, and the futtocks ſtanding bevellings, and where the ſpace of the ribband, containing theſe two timbers, is ſtrait, the one will be as much ſtanding as the other is under bevelling; as in the plate, where the perpendiculars drawn to timber 7, from the points in which the aftſide of the floor, and foreſide of the futtock interſect the ribband, are parallel to that drawn thro' the point, where the line that repreſents the moulding ſide of both timbers, interſects the ribband: This method will anſwer for all the timbers, whether we bevel by half breadth lines, water lines, or ribbands, only obſerving the directions given before, to transfer oblique to ſquare bevellings. We ſhall likewiſe hereby find, that in ſome caſes, where the ribbands are very round, one bevelling will not do for two timbers, and even when it is only taken for one timber an allowance ought to be made for the round of the timber; for which purpoſe it will be neceſſary in ſome caſes to make a mould to fit the round or hollow, and faſten this to a ſtrait edged batten in the proper direction.

Another method practiſed to find the bevellings for the ſquare timbers

bers, is by the diagonals in the body plane; without regarding the curves of the ribbands in the floor plane: But this cannot be used, unless we first form all the timbers in the body plane.

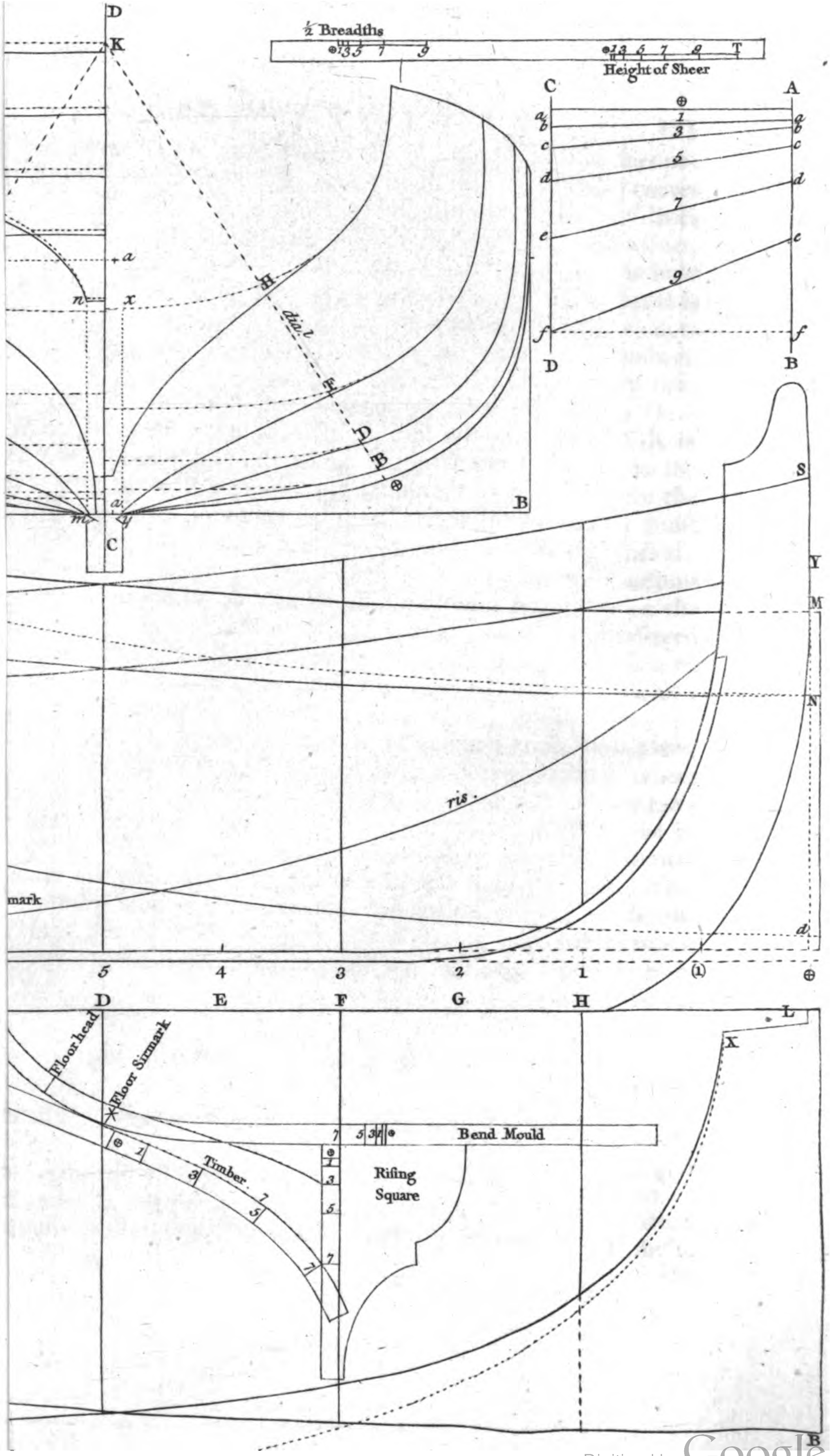
Let then the distance betwixt the parallel lines A B and C D, be the timber and room, that is, as before observed, the breadth of two timbers, and the space betwixt them, and if this be equal to the distance betwixt the perpendiculars representing the planes of the timbers, we have all the timbers ready formed in the body plane in *Plate* 5. and may find the bevellings in the following manner.

Draw the line A C, perpendicular to A B and C D, for the bevelling of ⊕: Then take the several distances in the diagonal, from ⊕ to the points where it intersects the timbers 1, 3, 5, 7 and 9, and set them off from the point *a*, to the points *b*, *c*, *d*, *e* and *f*, in the lines A B and C D; so that lines drawn from *f* to *f*, from *e* to *e*, &c. would be parallel to A C. Then draw the lines *a b*, *b c*, *c d*, *d e*, *e f*; which will give the bevellings of the timbers 1, 3, 5, 7 and 9: That is to say, of two timbers, *viz.* a floor timber and a futtock. So *f e* will be what the bevelling of timber 9 is within a square, as may be seen in the floor plane, by producing the line that represents the plane of timber 9, till it is equal in length to that which represents the plane of timber 7: But it must be observed that the perpendicular at 9, represents the foreside of the floor timber; therefore the futtock corresponding to it, is before the perpendicular at 9; the futtock corresponding to timber 7, will likewise be before its perpendicular: So that tho' this method may give us nearly the bevellings of two timbers, yet these are not the two that are to be fastened together. Therefore this method ought to be rejected, unless we set off the breadth of the timbers on each side of the line that represents the moulding edge, and draw perpendiculars betwixt each on the floor plane. We might then indeed find points in the diagonal, betwixt the timbers formed, which would give the bevellings of each floor timber with its corresponding futtock.

SECT. II.

To find the Bevellings of the Transoms. (Plate 7.)

THere are two ways of doing this. One is by forming curves on the sheer plane, by sections of planes cutting the ship fore and aft, parallel to the sheer plane. These planes will be represented by strait lines in the



½ Breadths
Height of Sheer
dia. 1
ris
mark
Floor head
Floor Sirmark
Timber
Bend Mould
Rising Square

the floor plane, parallel to W K; and in the body plane, by ſtrait lines parallel to K O. In the plate we have only formed the two dotted curves in the ſheer plane, to timber 7; the interſections of theſe with the planes of the tranſoms, which in the ſheer plane are repreſented by ſtrait lines, will give the bevellings; ſo that all that is now neceſſary, is to ſhew how theſe curves are formed, and in what direction the ſtock of the bevel is to be placed upon the tranſom. In order to this, firſt draw the two dotted lines in the body plane, parallel to K O, to interſect the timbers. Transfer the heighths of theſe interſections to their correſponding timbers on the ſheer plane; which will give the points thro' which theſe curves muſt paſs. Secondly draw two dotted lines parallel to W K in the floor plane, to interſect all the tranſoms: Theſe transferred to the planes of the tranſoms in the ſheer plane, will give the points where the curves interſect theſe planes. Theſe dotted lines in the floor plane, muſt be the ſame diſtance from the line W K, that the correſponding ones are from the line K O, in the body plane. They will interſect the tranſoms in the direction in which the ſtock of the bevel is to be laid upon the tranſoms; and if this ſhould be judged too oblique, it may be transferred to a ſquare one, as before directed. As the third tranſom is not formed in the floor plane, a line muſt be drawn parallel to the plane of it, where it is formed, to find the proper place and direction of the bevel.

The other method is by forming more timbers abaft the faſhion piece. Their planes will be repreſented by ſtrait lines in the floor plane, where they will interſect all the tranſoms already formed. In the plate we have only drawn one G *g*, by which we have formed another cant timber in the body plane, in the ſame manner as the faſhion piece was formed. The angle formed by the curve of this timber, and the level lines that are drawn at the heighth of each tranſom in the body plane, will be the bevelling; and the ſtrait line which repreſents the plane of the timber, will give the direction in which the bevel is to be placed upon the tranſoms. This may likewiſe be transferred from an oblique to a ſquare bevelling; and if needful, a mould made for the hollow of the tranſom.

The only thing that remains in regard to bevellings, is to ſhew how the ribbands are bevelled. Now as their planes are repreſented by diagonals in the body plane, the angles that are formed by the diagonals, and the timbers in the body plane, will be the bevellings; and the perpendiculars repreſenting the planes of the timbers in the floor plane, will give the direction for the bevel. The harpins are bevelled by level lines in the body plane; but as they are not parallel to the keel, when the ſtock is laid flat upon the upper ſide of the harpin, the tongue will not be in the direction of the timber; yet as the harpins are not above four or five inches broad, this need not be regarded. Those who incline to greater exactness may use the same method as in finding the bevellings of the fashion piece for a square tuck, or form the harpins by diagonals in the body plane, which may be so contrived as to intersect the timbers nearly in the same points with the sheer.

CHAP. V.

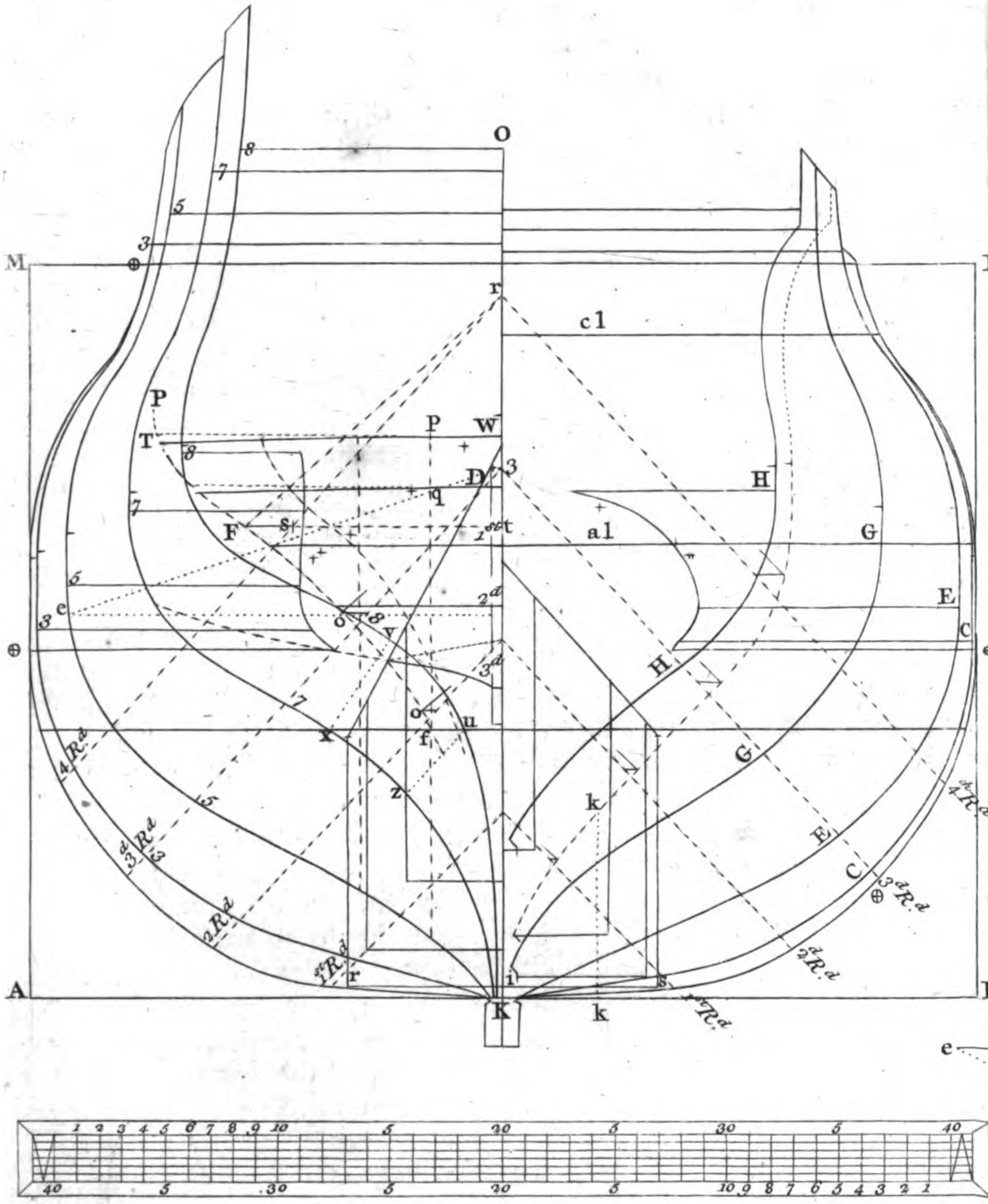
Of forming Bodies not similar to that by which the Lines on the Sector were constructed.

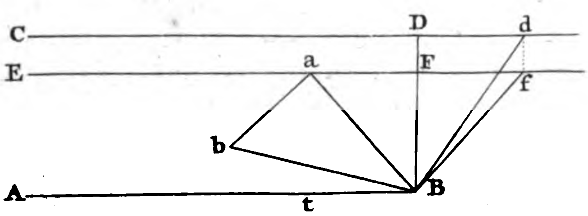
WE have, as was proposed in this second part, shewn the general methods used in drawing of ships, and how to lay down a ship by the sector, if similar to that from which the lines were constructed. We shall now shew the use of the sector in forming bodies that are not similar to one another.

The first thing to be done, is to set the sector by the proposed half breadth, and draw the diagonals as before directed. Then we must form the midship frame by the points, as in *Plate* 7. By this we shall discover, that it will be either too full or too sharp; and therefore must be altered by the artist, according to the service the ship is designed for. The foremost and aftermost timber must likewise be formed by the artist; and then the portions of the diagonals intercepted betwixt these timbers and the midship, will not be the same as that given by the sector, when set to the half breadth. In order then to find the points in the diagonals for the intermediate timbers, the sector must be set to each seperately. Thus, take with a pair of compasses, the portions of the diagonals intercepted betwixt the midship and aftermost timber, now made conformable to the service for which the ship is designed; and by these set the sector separately for each, till the distance taken by the compasses reach from the proper points in one leg, to the corresponding points in the other leg; and being thus set, we may find the proper points in that diagonal; and then set the sector for the next diagonal.

In order to illustrate this, we have in *Plate* 8, laid down the midship, foremost and aftermost timbers of two ships; the one an *East India* ship, the other a *French* privateer. Their bodies are very different from one another, and likewise from that by which the sector was formed; and if the intermediate timbers formed by the sector in both ships, will produce fair

Plate VII. facing page 174.





Thos. Mitchel del. et sculp.

fair ribbands, it may be presumed that it may be very useful in any other cases.

Now because we cannot open the sector in the plate, we have taken the several divisions of the four ribband lines upon the after body of the sector, and set them off from the point C, upon the four lines C 1st R, C 2d R, &c. intersecting one another in the point C: So that the point C is the same distance from the points 7, 5, 3 and ⊕, upon these lines, that the same points are from the center of the joint upon the sector. We have likewise drawn four other lines to intersect the former in the point C. But the angle formed by the two lines marked 1st R, is not equal to the angle formed by the two lines marked 2d R; nor to that formed by the two lines marked 3d R, or by those marked 4th R. The angle formed by each two lines of the same name, is determined in the following manner. From the center C, an arch of a circle is described to intersect the line 1st R in the point ⊕. From this point, the distance betwixt the midship and aftermost timber now formed, taken upon the first diagonal, is set off upon the arch; and the other line C 1st R is drawn thro' this point. In like manner the other three lines are drawn, by describing arches of circles from the center C, to meet each line in the point ⊕, and setting off upon each arch the distance betwixt the midship, and aftermost timber, taken upon the diagonal corresponding to each line. So the distance betwixt the points ⊕ and ⊕, in the two lines marked 1st R, is equal to the portion of the first diagonal intercepted betwixt the midship and aftermost timber; the distance betwixt these points in the lines marked 2d R, is equal to the portion of the second diagonal intercepted betwixt the aforesaid two timbers, &c. The lines being thus drawn, and each divided in the same proportion as its corresponding one of the same name is, the points on each diagonal for the timbers 3, 5 and 7, will be found upon examination to be at the same distance from the after timber, that these points are from one another, in the two lines corresponding to each diagonal.

The like process may be used for forming the intermediate timbers in the fore-body: But it must be observed, that in forming the ribbands from these timbers, their stations in the sheer plane must be determined by the sector, as in *Plate* 7. and after the ribbands are all formed, the proper stations of the timbers may be assigned.

Now tho', both the ribbands and timbers thus formed may prove fair, yet neither this method by the sector, nor any other method, which has been published, can be established as a certain invariable rule; because the curves by which they are formed have no properties peculiar to themselves to distinguish them from all other curves, as was before observed.

The

The only way to make any confiderable improvements in this art, we prefume, will be, by carefully examining the different bodies of feveral fhips that have been actually built, and whofe good or bad qualities have been difcovered by experience. We have therefore, for the fake of fuch of our readers as are not furnifhed with a fufficient number of draughts for that purpofe, collected into the following table all the dimenfions that are neceffary to determine the form of fourteen different fhips; and we may venture to affirm, that the youth will receive more benefit by delineating thefe from the dimenfions, and thereby fooner acquire the art of drawing, than by all the rules and directions that have been hitherto publifhed on that fubject.

The dimenfions in the following tables are taken from the diagonal fcale, *Plate* 2. and the number of equal parts contained in twelve feet, is fpecified at each fhip.

PRIN-

R

♀ 3d R

−♀ 2d R

♀ 3d R

PRINCIPAL DIMENSIONS of fourteen SHIPS.

	Rates of Ships of War.							Merchant Ships.						Tons.
	N° 1.	N° 2.	N° 3.	N° 4.	N° 5.	N° 6.	N° 7.	N° 8.	N° 9.	N° 10.	N° 11.	N° 12.	N° 13.	N° 14.
	1ft.	2d.	3d.	4th.	5th.	6th.	Sloop.	630.	398.	340.	372.	162.	114.	50.
Length from the forefide of ⊕ to the fore-part of the poft on the keel	19976	19680	20619	17669	14888	12802	7558	15300	12994	11952	13104	9658	6840	10993
Ditto to the wing tranfom	21656	21272	21879	19155	16102	13752	8266	16686	14744	13414	14360	10426	7840	12614
Ditto to the touch of the ftem	11195	12105	11096	9880	8616	9129	4024	7756	6826	6808	5664	4580	2514	5426
Ditto to aftfide of ftem on the l. deck	17446	17413	16458	14270	12844	11945	6474	11710	10150	9720	8946	7378	5148	—
Ditto to ditto at the head	17556	17995	16458	14280	12844	12097	6634	11730	10266	9762	9108	7380	5012	8494
Ditto to the knuckle	16200	16485	15080	13200	12076	11256	—	10944	—	9200	8468	—	—	—
From the after timb. to the l. counter	3504	3120	2700	2816	2350	2120	1546	2646	2250	2500	2006	1478	—	3518
Ditto to the fecond counter	3950	3410	3060	3164	2688	2390	—	3010	—	2870	2270	1656	—	—
Ditto to upright of the ftern at fh^r^. rail	5370	4440	3610	3806	2952	2674	1800	3360	2370	3350	1477	1960	—	3900
Heighth of the wing tranfom	6000	6280	6060	5428	4742	3690	2194	4485	5440	3778	3571	3048	2240	3634
Heighth of the ftem	7750	9060	8000	6568	6130	5242	3472	5900	5526	5031	4776	4130	3950	4893
Heighth of the lower deck: on the poft	5408	5800	5458	4756	4318	3260	—	4036	3080	3548	3135	3048	—	—
Heighth of the lower deck: ⊕	4940	5130	4939	4278	3930	2930	2130	3500	2896	3172	2690	2541	2810	—
Heighth of the lower deck: ftem	5150	5641	5304	4342	4200	3200	2600	3766	3080	3120	2834	2770	3090	—
Heighth of the lo. edge of the lower wale: on after tim.	5230	5840	5574	4684	4127	3371	2078	3954	3000	3260	2966	2856	1880	2690
Heighth of the lo. edge of the lower wale: ⊕	3510	4196	4080	3450	3250	2630	1675	3133	2490	2464	2123	2310	1730	2236
Heighth of the lo. edge of the lower wale: ftem	4400	5088	4782	4100	3740	3170	2200	3400	3080	2860	2726	2604	2320	3130
Heighth of the lower counter	7300	7400	7150	6355	5710	4620	2500	5510	5680	4720	4560	3596	—	4321
Ditto the upper	7990	8070	7812	6880	6510	5320	—	6100	—	5424	4970	3870	—	—
Heig. of heig. of br^h^. line on the ftem	6816	7360	6100	5916	4840	4258	2680	4385	4100	4314	4658	3371	3200	3430
Upper breadth fweep	4770	3806	2756	2532	2370	2430	1750	2710	2354	2286	2082	2140	1500	3570
Sweep of the ftem: upper	—	—	—	—	—	—	3210	4310	7080	3826	3920	2790	3190	—
Sweep of the ftem: lower	6380	5190	5370	4420	4200	2780	1416	3510	3296	2738	2928	—	2154	2950
Heighth of the upper water line: abaft	3786	4606	4250	3526	3108	—	1562	3650	2700	3050	2700	2300	1900	—
Heighth of the upper water line: afore	3596	4400	4014	3270	2950	—	1330	3380	2436	2756	2450	2100	1900	—
From fore-part of ⊕ on lower deck to cen. of fore-maft	13850	13300	12624	10808	9956	9145	4572	8936	7236	8786	5484	5155	—	—
From fore-part of ⊕ on lower deck to cen. of ma-maft	3878	3800	4384	3950	2800	2422	1968	3850	2726	2060	3500	1266	0200	1950
From fore-part of ⊕ on lower deck to cen. of miz-maft	14582	14464	15148	13213	10954	9394	0000	11596	9832	7337	9874	3374	—	—

DI-

N° 1. A first RATE 100 Guns. 2262=12 Feet.

Timbers names.	Diagonals.					Heighth of bread.			Half brea.		Distance of frames.
	1st.	2d.	3d.	4th.	5th.	low.	upp	top ti.	main	top.	fr. ⊕
⊕	1620	3114	5104	6432	7380	4178	4790	9790	5476	4030	—
8	1460	2790	4678	6094	7200	4316	4844	10080	5388	3886	7232
16	1186	2212	3930	5552	6822	4646	4998	10438	5214	3661	11862
24	0766	1532	2980	4470	5940	5290	5490	10934	4742	3374	16492
30	0311	0577	1780	2940	4636	6070	6137	11423	4110	3135	19976
D	1586	3016	4998	6322	7316	4212	4776	9822	5457	4028	4626
M	1346	2538	4398	5848	7000	4430	4777	9965	5262	3940	9270
W	0518	1386	2878	4362	5736	5148	5240	10240	4418	3728	13870
X	—	0828	2251	3668	5046	5526	5578	10325	3924	3652	15006
Heighth of the diagonals on the middle line	1178	2281	4142	6003	7904	—	—	—	—	—	—
Distance from ditto on the base produced	1184	2300	4128	5944	7780	—	—	—	—	—	—

N° 2. A second RATE 90 Guns. 2731=12 Feet.

Timbers names.	Diagonals.					Heighth of bread.			Half brea.		Distance of frames.
	1st.	2d.	3d.	4th.	5th.	low.	upp.	top. ti.	main	top.	fro. ⊕
⊕	1589	3010	4974	6220	7164	4224	5070	9750	5410	3916	0000
9	1410	2728	4700	6008	7051	4416	5130	10041	5366	3830	6820
18	0973	1140	3812	5363	6650	4837	5310	10436	5228	3652	11914
27	0405	0813	2163	3894	5517	5714	5844	11000	4704	3312	17014
32	0105	0206	0880	2414	4223	6396	6425	11388	4010	3026	19842
I	1465	2804	4750	6072	7126	4355	5060	9774	5412	3848	6556
S	1022	2012	4184	5364	6624	4960	5295	10006	5216	3740	11645
W	0743	1580	3186	4786	6083	5360	5569	10136	4867	3686	13345
Z	0265	0930	2336	3872	5132	5927	6018	10272	4116	3612	15045
Heighth of the diagonals on the middle line	1164	2214	4060	5912	7774	—	—	—	—	—	—
Distance from ditto on the base produced	1164	2214	4084	5944	7823	—	—	—	—	—	—

N° 3. A third RATE 74 Guns. 2730=12 Feet.

Timbers names.	Diagonals.					Heighth of bread.			Half brea		Distance of frames
	1st.	2d.	3d.	4th.	5th.	low.	upp	top ti.	main	top.	fro. ⊕
⊕	1533	2877	4658	5923	6864	4069	4632	7920	5166	4064	—
9	1462	2729	4533	5806	6743	4199	4717	8142	5103	3966	6840
18	1191	2199	3922	5241	6229	4580	5026	8482	4790	3762	11962
27	0586	1194	2500	3956	5182	5299	5572	8990	4228	3350	17083
33	0140	0340	1156	2512	3992	6086	6195	9430	3564	2952	20499
F	1464	2745	4547	5834	6778	4114	4662	7952	5140	4012	4812
M	1310	2426	4206	5504	6455	4270	4741	8063	4926	3867	8214
S	0962	1796	3267	4660	5722	4647	4996	8256	4385	3580	11628
Y	—	0694	1932	3210	4199	5295	5400	8468	3147	3197	14468
Heighth of the diagonals on the middle line	1152	2155	3928	5706	7462	—	—	—	—	—	—
Distance from ditto on the base produced	1144	2172	3942	5714	7484	—	—	—	—	—	—

N° 4. A fourth RATE 50 Guns, 1034 Tons, 2722=12 Feet.

Timbers names.	Diagonals.					Heighth of brea.			Half brea.		Diſtance of frames.
	1ſt.	2d.	3d.	4th.	5th.	low.	upp.	top.	main	top.	fro. ⊕
⊕	1406	2600	4154	5216	6000	3444	3950	7010	4496	3409	0000
9	1260	2316	3884	4978	5803	3620	3968	7146	4398	3355	5646
18	0918	1676	2923	4120	5118	4050	4219	7500	4068	3094	10993
24	0532	1042	2062	3206	4386	4466	4598	7857	3654	2798	14559
29	0146	0349	1004	2151	3454	5000	5056	8246	3040	2456	17505
I	1264	2322	3900	4970	5754	3666	3962	7078	4390	3318	5342
P	0967	1742	3000	4191	5149	4108	4314	7240	4056	3020	8898
S	0640	1265	2358	3466	4520	4482	4582	7360	3626	2798	10688
W	0190	0640	1507	2452	3414	5024	5024	7512	2690	2540	12460
Heighth of diagonal on the middle line	1058	1980	3480	4974	6471	—	—	—	—	—	—
Diſtance from ditto on the baſe produced	1071	1984	3482	4976	6480	—	—	—	—	—	—

N° 5. A fifth RATE 40 Guns, 706 Tons, 2736=12 Feet.

Timbers names.	Diagonals.					Heighth of brea.			Half brea.		Diſtance of frames.
	1ſt.	2d.	3d.	4th.	5th.	low.	upp.	top.	main	top.	fro. ⊕
⊕	1208	2000	3333	4400	5090	3230	3766	6404	4006	3027	0000
10	1098	1818	3136	4230	5030	3254	3778	6570	4006	3027	5954
19	0680	1218	2346	3500	4416	3628	3960	6850	3712	2828	10424
25	0339	0600	1448	2558	3560	4220	4398	7114	3198	2511	13400
28	0128	0216	0761	1796	2864	4708	4760	7262	2950	2376	14888
G	1175	1958	3300	4360	5040	3290	3778	6436	3990	3027	4210
N	0911	1534	2764	3880	4682	3514	3862	6544	3762	2998	7178
T	0356	0772	1768	2880	3710	4015	4194	6680	3066	2800	10156
Y	0000	0282	1133	2200	2910	4414	4510	6776	2298	2574	11676
Heighth of diagonal on the middle line	1020	1744	3066	4342	5564	—	—	—	—	—	—
Diſtance from ditto on the baſe produced	1016	1716	3123	4634	6230	—	—	—	—	—	—

N° 6. A ſixth RATE 20 Guns, 508 Tons, 2744=12 Feet.

Timbers names.	Diagonals.					Heighth of brea.			Half brea.		Diſtance of frames.
	1ſt.	2d.	3d.	4th.	5th.	low.	upp.	top.	main	top.	fro. ⊕
⊕	1064	1870	2988	3818	4316	2570	3126	5408	3590	3014	0000
9	0939	1640	2724	3556	4153	2680	3136	5511	3544	2950	4424
18	0610	1100	1944	2808	3600	2984	3298	5742	3222	2650	8834
24	0290	0532	1144	1964	2796	3385	3588	6034	2683	2286	11792
26	0119	0224	0680	1460	2350	3580	3729	6148	2383	2128	12802
I	0922	1620	2684	3520	4100	2722	3136	5511	3504	2864	5710
P	0642	1154	2052	2893	3526	3126	3330	5654	3094	2595	8665
S	0432	0828	1536	2254	2844	3494	3606	5784	2510	2372	10147
U	0000	0370	0894	1498	2016	3836	3900	5886	1764	2180	11136
Heighth of diagonal on the middle line	0848	1508	2528	3450	4314	—	—	—	—	—	—
Diſtance from ditto on the baſe produced	0860	1514	2720	4106	5600	—	—	—	—	—	—

N° 7. A SLOOP of 150 Tons, 2696=12 Feet.

Timbers names.	Diagonals.					Heighth of brea.			Half brea.		Diftance of frames.
	1ft.	2d.	3d.	4th.	5th.	low.	upp.	top.	main	top.	fro. ⊕
⊕	0754	1262	1830	2290	2575	1448	1622	2510	2078	1976	——
4	0678	1145	1712	2168	2512	1500	1665	2550	2062	1906	2000
8	0518	0899	1405	1898	2315	1632	1758	2654	1939	1830	4150
12	0254	0476	0910	1403	1898	1895	1939	2819	1704	1600	6268
14	0115	0220	0538	1057	1590	2074	2090	2920	1446	1362	7334
D	0714	1194	1766	2217	2538	1500	1656	2538	2078	1961	1999
F	0616	1072	1651	2134	2481	1597	1717	2608	2048	1918	3070
H	0449	0836	1376	1872	2257	1793	1848	2702	1930	1848	4144
K	0110	0412	0870	1351	1773	2106	2106	2856	1586	1668	5212
Heighth of the diagonals on the middle line	0614	1066	1616	2136	2644	——	——	——	——	——	——
Diftance from ditto on the bafe produced	0614	1046	1700	2378	3101	——	——	——	——	——	——

N° 8. The LONDON EAST INDIA Ship 630 Tons, 2730=12 Feet.

Timbers names.	Diagonals.					Heighth of brea.			Half brea.		Diftance of frames.
	1ft.	2d.	3d.	4th.	5th.	low.	upp.	top.	main	top.	fro. ⊕
⊕	1430	2588	3886	4720	5210	3230	3654	6014	3674	2980	——
4	1374	2476	3790	4636	5133	——	3754	6126	3623	2966	4152
12	1140	2040	3306	4252	4816	——	4012	6360	3461	2826	8296
20	0560	1148	2182	3270	4110	——	4396	6698	3170	2580	12444
24	0228	0546	1372	2430	3528	——	4646	6920	2974	2420	14526
D	1376	2485	3784	4632	5140	——	3694	6012	3624	2958	3894
H	1264	2276	3586	4481	5020	——	3766	6036	3566	2914	5978
M	1012	1886	3120	4116	4712	——	3900	6090	3416	2812	8060
O	0578	1340	2577	3652	4346	——	4036	6152	3186	2712	9404
Heighth of the diagonals on the middle line	1110	2020	3330	4640	5950	——	——	——	——	——	——
Diftance from ditto on the bafe produced	1106	2020	3226	4407	5571	——	——	——	——	——	——

N° 9. The BONETTA 398 Tons, 2723=12 Feet.

Timbers names.	Diagonals.					Heighth of brea.			Half brea.		Diftance of frames.
	1ft.	2d.	3d.	4th.	5th.	low.	upp.	top.	main	top.	fro. ⊕
⊕	0915	1707	2935	3934	4616	2704	3026	5232	3260	2521	——
9	0842	1548	2797	3775	4462	2790	3084	5308	3140	2474	4599
18	0516	0956	1900	2898	3690	3410	3660	5508	2733	2238	9198
24	0206	0354	0890	1666	2422	4150	4336	5740	1836	1696	12264
26	0120	0168	0498	1082	1710	4410	4584	5832	1244	1242	13254
I	0810	1524	2766	3756	4460	2830	3136	5320	3142	514	4364
P	0539	1044	2047	3060	3786	3292	3580	5550	2708	2330	7430
R	0220	0640	1550	25·0	3220	3522	3800	5678	2290	2040	8470
S	——	0362	1174	2090	2756	3677	3956	5764	1942	1748	9000
Heighth of the diagonals on the middle line	0710	1320	2490	3740	5000	——	——	——	——	——	——
Diftance from ditto on the bafe produced	0706	1315	2361	3400	4393	——	——	——	——	——	——

N° 10. The THAMES 340 Tons, 2745=12 Feet.

Timbers names.	Diagonals.					Heighth of brea.			Half brea.		Distance of frames
	1st.	2d.	3d.	4th.	5th.	low.	upp.	top.	main	top.	fro ⊕
⊕	1024	1731	2909	3962	4966	2500	2930	5211	3196	2464	—
3	0845	1441	2608	3694	4722	2722	3058	5333	306[illegible]	2378	4791
5	0510	0913	1920	3107	4249	3165	3341	5554	2846	2251	7966
7	0193	0334	0995	2113	3436	3661	3728	5872	2533	2023	11130
8	0106	0155	0653	1735	3096	3816	3854	5983	2410	1928	11952
B	0970	1647	2826	3870	4850	2624	2979	5220	3118	2400	3190
D	0722	1240	2315	3395	4382	3115	3233	5290	2876	2235	6157
E	0532	0951	1909	2986	3942	3396	3458	5354	2600	2136	7272
F	0345	0699	1561	2600	3536	3594	3622	5398	2310	2055	7950
Heighth of the diagonals on the middle line	0824	1434	2682	4068	5581	—	—	—	—	—	—
Distance from ditto on the base produced	0858	1439	2419	3317	4110	—	—	—	—	—	—

N° 11. A FRENCH Privateer of 372 Tons, 2730=12 Feet.

Timbers names.	Diagonals.					Heighth of brea.			Half brea		Distance of frames.
	1st.	2d.	3d.	4th.	5th.	low.		top.	main	top.	fr. ⊕
⊕	0807	1753	2746	3770	3686	2762		4514	3242	2734	—
8	0700	1536	2480	3174	3578	2830		4536	3216	2711	3948
16	0450	1010	1790	2530	3134	3082		4708	3030	2584	7876
24	0164	0370	1754	1300	2040	3608		5062	2600	2280	11812
27	0073	0116	0279	0598	1330	3907		5260	2364	2110	13294
H	0750	1544	2449	3114	3464	2874		4600	3096	2660	3700
M	0625	1238	1990	2628	3056	3122		4700	2810	2548	5674
O	0428	0956	1616	2248	2710	3354		4752	2526	2486	6670
Q	0106	0518	1088	1672	2146	3711		4834	2084	2398	7648
Heighth of the diagonals on the middle line	0800	1611	2472	3232	3906	—		—	—	—	—
Distance from ditto on the base produced	0806	1620	2880	4363	6176	—		—	—	—	—

N° 12. A SHIP of 162 Tons. 2702=12 Feet.

Timbers names.	Diagonals.					Heighth of brea			Half brea.		Distance of frames.
	1st.	2d.	3d.	4th.	5th.	low.	upp.	top.	main	top.	fro. ⊕
⊕	0914	1652	2500	3018	3218	2308	2590	3838	2494	2136	—
4	0814	1486	2340	2898	3138	—	2664	3926	2430	2086	3520
12	0535	0962	1590	2220	2692	—	2926	4170	2226	1910	7034
16	0260	0504	0956	1544	2090	—	3179	4340	2026	1776	8776
18	0090	0217	0510	1034	1670	—	3317	4444	1888	1682	9658
D	0800	1458	2334	2916	3175	—	2668	3898	2465	2110	3356
H	0630	1140	1892	2546	2900	—	2842	3974	2318	2010	5135
K	0298	0776	1486	2110	2506	—	3010	4026	2066	1848	6014
M	—	0100	0716	1264	1622	—	3232	4090	1361	1287	6912
Heighth of the diagonals on the middle line	0716	1300	2116	2920	3652	—	—	—	—	—	—
Distance from ditto on the base produced	0712	1300	2187	3132	4160	—	—	—	—	—	—

N° 13. A Fishing SMACK, 114 Tons, 2723=12 Feet.

Timbers names.	Diagonals.					Heighth of brea.			Half brea.		Distance of frames.
	1st.	2d.	3d.	4th	5th.	low.	upp.	top.	main	top.	fro. ⊕
⊕	0681	1316	2100	2672	3143	1920	—	3310	2400	2051	—
6	0554	1101	1916	2500	2988	2042	—	3186	2361	2000	2142
12	0321	0660	1366	2090	2698	2211	—	3200	2158	1800	4298
15	0186	0424	1026	1728	2418	2336	—	3278	1930	1608	5374
18	0098	0216	0653	1300	1960	2504	—	3386	1552	1348	6440
C	0628	1234	2031	2600	3064	1930	—	3400	2361	2024	1078
F	0566	1102	1872	2450	2918	2020	—	3508	2248	1930	2164
I	0440	0861	1522	2108	2580	2176	—	3620	1968	1684	3240
U	—	0124	0726	1345	1814	2600	—	3800	1330	1000	4312
Heighth of the diagonals on middle line	0500	1000	1820	2644	3456	—	—	—	—	—	—
Distance from ditto on the base produced	0500	1000	1820	2644	3456	—	—	—	—	—	—

N° 14. A SLOOP of 50 Tons, 5462=12 Feet.

Timbers names.	Diagonals.					Heighth of brea.			Half brea.		Distance of frames.
	1st.	2d.	3d.	4th.	5th.	low.	upp.	top.	main	top.	fr.o ⊕
⊕	1422	2547	3684	4724	5698	1956	2770	4190	3570	3303	—
4	1400	2518	3657	4710	5676	2028	2782	4164	3570	3303	2968
8	1234	2235	3354	4452	5454	2295	2900	4257	3494	3252	5984
12	0854	1648	2665	3808	4882	2770	3216	4465	3216	3022	8898
14	0536	1174	2112	3244	4424	3052	3408	4606	2985	2812	10346
B	1372	2458	3598	4670	5634	2050	2800	4296	3548	3252	2662
D	1278	2280	3400	4484	5454	2193	2915	4412	3436	3154	4154
F	1116	1993	3001	4045	4981	2444	3113	4578	3112	2882	5624
H	0558	1338	2264	3214	3980	2873	3432	4790	2388	2200	7120
Heighth of the diagonals on the middle line	1129	2024	3110	4352	5778	—	—	—	—	—	—
Distance from ditto on the base produced	1120	2020	2838	3583	4250	—	—	—	—	—	—

These tables of dimensions are so particular that one example will be sufficient to illustrate their use in laying down any of the ships.

Let it then be required to lay down the *Bonetta* pink, which in the tables is N° 9. 398 Tons.

1st. Draw the line A B, upon which erect the perpendicular C D; and because the main half breadth in the column is 3268, take that from the scale, and lay it off from C to A, and from C to B, and erect perpendiculars at A and B.

2d. Look for the lower heighth of breadth, in the column; which will be found to be 2704. Set up this from A to L, and from B to L, and draw the line L L, which will be parallel to A B. Set up also the upper heighth 3026, from A to V, and from B to V; and lay off 2354, which by by the tables is the radius of the upper breadth ſweep, from the points V and V; which will give us the center of the ſweep.

3*d*. Set up 5232, the heighth of the top timber line, from C upon the line C D; thro' which point draw a line parallel to A B, and lay off 2521. the half breadth of the top timber, both ways upon it, from the line C D; and form the top timber by a mould, to break in fair with the back of the upper breadth ſweep. This compleats the upper part of the midſhip frame.

4*th*. To form the lower part of the midſhip frame, draw the five diagonals; their heighths from the point C, upon the line C D, and their diſtance from C, upon the line A B produced, is given in the proper columns. The heighth of the fifth diagonal is 5000, which ſet up from C to 5: The diſtance from the middle line upon the baſe is 4393, which lay off from C, upon the baſe produced; and draw a line to this point from 5; which will be the upper or fifth diagonal. In the ſame manner are the other diagonals drawn, by taking the numbers from the proper columns.

5*th*. Lay off 4616 from 5 upon the fifth diagonal, 3934 from 4 upon the fourth, 2935 from 3 upon the third, 1707 from 2 upon the ſecond, and 915 from 1 upon the firſt diagonal: A curve paſſing thro' theſe points, and the main breadth at the lower heighth will form the midſhip frame to the floor head. A ſtrait line to touch the curve in this point, drawn to the upper edge of the keel, compleats the whole midſhip frame. The points in the diagonals are in the column correſponding to ⊕.

In like manner we may find points in the diagonals, for all the other timbers, and alſo for their lower, upper, and top timber heighths of breadths and half breadths, from the dimenſions in the proper columns correſponding to each; and by theſe form the timbers. We may likewiſe ſtation the timbers in the ſheer and floor planes, the diſtances of each from ⊕ being in the proper columns, and alſo form all the curves that are neceſſary upon theſe planes. But as our plate will not contain this, we ſhall only lay down the ſtem, poſt, counter and ſtern.

In order to do this, draw the line Z X, to repreſent the upper ſide of the keel; and at any convenient place erect a perpendicular for the after timber 26: The diſtance of this timber from ⊕, is 13254; and the wing tranſom from ⊕, is 14744; which therefore is 1490, abaft timber 26. The heighth of the wing tranſom is 5440. Set up this from the line Z X, upon a perpendicular erected to the point W 1490, abaft timber 26. The diſtance of ⊕ from the fore part of the poſt is 12994, which ſubtracted from 13254, remains 260. Lay this off from 26 to P, and draw

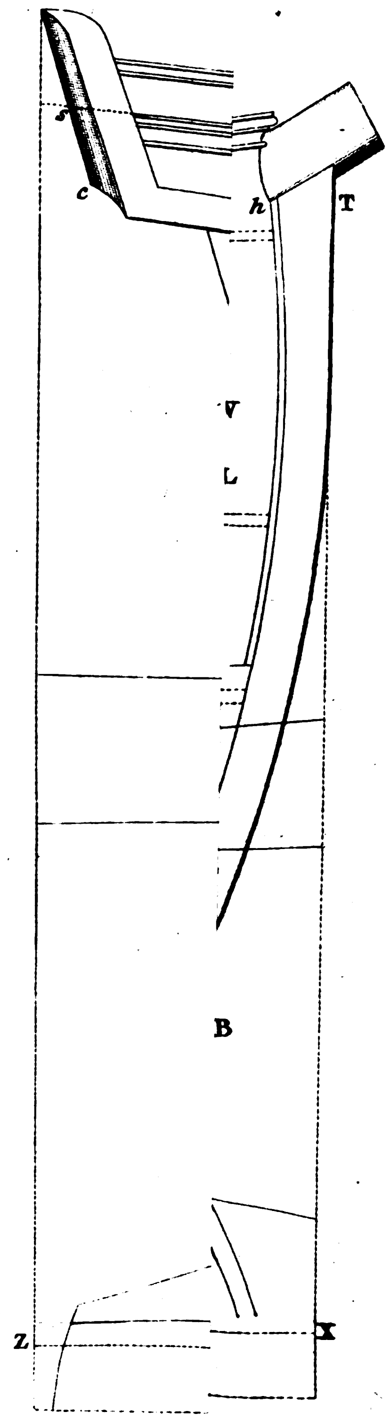
draw the line P W for the fore side of the post. The counter is 2250 abaft 26, which lay off on the line Z X, and erect a perpendicular, upon which set up 5680 to *c*, which is the heighth of the counter. The upright of the stern at the sheer rail, is 2370 abaft timber 26, which set off from timber 26 upon the line Z X, at that point erect a perpendicular to *s*, and draw the line *c s*, which may be produced to the heighth of the stern, as in the plate. We may then form the counter; where it must be observed that a pink has no transom. We have only assumed the point W to determine the rake of the post. Timber 26 is 13254 from ⊕, and timber 24 is 12264 from ⊕. Therefore the distance betwixt them is 990, which set off from 26 to 24, gives the station of that timber; and by the same manner, the stations of the other timbers may be found.

Having thus laid down the stern, we shall in the next place lay down the stem. Erect the perpendicular X T, to limit the fore part of the stem; upon which set up 5526, the heighth of the aft side of the stem from X to T, and let *b* be the aft side of the head. The head of the stem is 10266, and the touch of the stem is 6826 before ⊕, therefore the distance betwixt them is 3440, which set off upon the line Z X, from a perpendicular let fall from *b*; this will give the point *t*, the touch of the stem, where erect a perpendicular, and set up 3296 to *o*; which will give the center of the lower sweep of the stem. The radius of the upper sweep is 7080, and *n* the center; and these two sweeps will form the stem. We may now station the timbers F, I, M, P, R and S, as in the plate; for as the touch of the stem is 6826, and timber P 7430 before ⊕; P will be 604 before the touch of the stem. We have in the plate laid down the main and top timber half breadth lines, also the rising and narrowing of the floor and floor sweeps. After the same manner any of the other ships may be laid down from the tables.

Having now given the principal dimensions, we shall in the next place give the scantlings.

SCANT-

Plate IX. facing page



F. Chapman del

Scantlings of the principal Pieces of Timber. In MERCHANT SHIPS. In SHIPS of WAR.

NAMES of the PIECES. Tons	70		100		200		300		400		600		800		1000		1200		1400		1600		1730		2000	
	f.	in.	f.	in.	f.	in.	f.	in.	f.	in.	f.	in.	f.	in.	f.	in.	f.	in.	f.	in.	f.	in.	f.	in.	f.	in.
BEAMS. Lower deck { Sided or broad	0	9	0	11	0	11½	1	0	1	1	1	2	1	1¾	1	2¾	1	3½	1	4½	1	4¾	1	5½	1	6
Lower deck { Moulded or deep	0	8	0	10	0	10½	0	11	1	0	1	0½	1	1½	1	2¼	1	3	1	4½	1	4¾	1	5½	1	6
Upper deck { Sided	0	0	0	8	0	9	0	10½	0	11	1	0	0	10½	1	0½	1	1	1	2	1	0¼	1	1	1	1½
Upper deck { Moulded	0	0	0	6½	0	8½	0	9	0	10	0	11	0	9¾	0	10¼	0	11	1	0	0	10½	0	11	0	11½
Quarter deck { Sided } Two thirds of the beams, and 1 inch more the other way											0	8	0	6½	0	8	0	9	0	9½	0	9	0	10	0	10
Quarter deck { Moulded }											0	6	0	5½	0	6⅝	0	7	0	8	0	7	0	7¾	0	8¼
Fore castle { Sided }											0	8	0	6½	0	8½	0	9	0	9½	0	9	0	10	0	10
Fore castle { Moulded }											0	6	0	5½	0	6⅝	0	7	0	8	0	7	0	7¾	0	8¼
BREAST HOOKS. Sided			0	8	0	9	0	10	0	11	0	11½	1	0	1	1	1	1	1	2	1	2¼	1	3	1	4
CARLINGS. Two thirds of the beams, and one inch more the other way													0	6½	0	7	0	7¾	0	8½	0	9	0	9¼	0	9½
CLAMPS. { Lower deck	0	3	0	4	0	4	0	4½	0	5	0	6	0	5½	0	6	0	7	0	8	0	8½	0	9	0	9
CLAMPS. { Upper deck			0	3½	0	3½	0	3½	0	4	0	4	0	4	0	4	0	5	0	5½	0	5¼	0	5½	0	6
KEEL. Sided at the stem	0	9	0	11	0	11½	1	0	1	1	1	2	1	0¾	1	2	1	3	1	4	1	4	1	4	1	4
KEEL. Ditto in the midships and deep	0	9	0	11	0	11½	1	0	1	1	1	2	1	1½	1	2½	1	3	1	4½	1	5½	1	6	1	7
KEEL. Ditto at the post	0	8	0	11	0	11	0	11½	1	0	1	1	0	9	0	9½	0	10½	0	11	0	11½	1	0	1	0½
KEEL. False			0	2	0	2½	0	3	0	3	0	4	0	4	0	4	0	4	0	4	0	4½	0	5	0	5
KEELSON. { Deep	0	8	0	11	0	11½	1	0	1	1	1	2	1	1½	1	2½	1	3½	1	4½	1	5	1	6	1	7
KEELSON. { Thwartships	0	8½	0	10	0	10½	0	11	0	11½	1	2	1	1½	1	2½	1	3½	1	4½	1	5	1	6	1	7
KNEES. Lower deck { Hanging sided			0	7	0	7½	0	8	0	8½	0	9	0	8	0	8½	0	9	0	9½	0	10	0	10½	1	0½
KNEES. Lower deck { Lodging one inch less	0	5																								
KNEES. Upper deck hanging			0	5½	0	6½	0	6	0	7	0	7½	0	6	0	6⅓	0	7	0	7	0	7½	0	8	0	8½
KNEES. Transom one inch less than the transoms													0	9½	0	10	0	11	1	0	1	1	1	1½	1	2
PLANK. In the bottom	0	2	0	2	0	2½	0	3	0	3	0	3	0	3	0	3	0	4	0	4	0	4	0	4	0	4
PLANK. Under the wale	0	3	0	3	0	3½	0	4	0	4	0	4	0	4	0	4	0	5½	0	6	0	6½	0	7	0	8
PLANK. Above ditto	0	3	0	3	0	3½	0	4	0	4	0	4	0	4	0	4	0	5½	0	6	0	6½	0	7	0	8
PLANK. At the floor heads within and without, two or three strakes to be one, or one and half, inch thicker than the bottom. Also two or three strakes under the clamps, and next the limber boards of thicker stuff, and a strake or two of thicker planks under the thick stuff below the wale, to diminish gradually till they are the thickness of the plank in the bottom																										
SCARPHS. { Of the keel long 3 times their breadth	3	8	4	0	4	4	4	6	5	0	5	6														
SCARPHS. { Of the timbers	4	6	4	8	5	0	5	6	5	8	6	0	5	4	6	6	6	10	7	2	7	6	7	9	8	0

STEM.

Scantlings of the principal Pieces of Timber. In MERCHANT SHIPS. In SHIPS of WAR.

NAMES of the PIECES.	Tons.	70		100		200		300		400		600		800		1000		1200		1400		1600		1730		2000	
		f.	in.	f.	in.	f.	in.	f.	in.	f.	in.	f.	in.	f.	in.	f.	in.	f.	in.	f.	in.	f.	in.	f.	in.	f.	in.
STEM.	Fore and aft below	0	9	0	11	0	11½	1	0	1	1	1	2	1	1½	1	2½	1	3	1	4	1	5	1	6	1	7
	At the head	0	10	1	1	1	3	1	4	1	5	1	6	1	2½	1	3	1	4	1	5	1	6	1	7½	1	9
	Falſe			0	6	0	6½	0	6½	0	7	0	7½	0	8½	0	9	0	10	0	10½	0	11	0	11½	1	0
STEPS.	Main (Sided to the diameter of their reſpective maſts in ſhips of war)			1	6	1	8	1	10	2	0	2	1														
	Fore			1	3	1	4	1	6	1	8	1	10														
	Mizen			1	0	1	1	1	2	1	3	1	4														
STERN POST.	Fore and aft — At the keel	0	11	1	2	1	4	1	6	1	8	1	9½	2	3	2	4	2	5	2	6	2	7	2	8	2	9
	Fore and aft — At the head	0	10	0	11	1	0	1	4	1	2	1	3	1	4	1	6	1	8	1	9	1	10	1	11	2	0
	Thwartſhips — At the keel	0	6½	0	6½	0	7	0	7½	0	8	0	9														
	Thwartſhips — At the head	0	9	0	10½	0	11	0	11½	1	2	1	4														
TRANSOMS.	Wing	0	9	0	10	0	11	0	11½	1	0	1	1	0	11½	1	0	1	0½	1	1	1	1½	1	2	1	3
	Second			0	10	0	10½	0	11	0	11½	1	0	0	10	0	11	0	11½	1	0	1	0½	1	1	1	2
	Third			0	8½	0	9	0	9½	0	10	0	11	0	9	0	10	0	10½	0	11	0	11½	1	0	1	1
TIMBERS.	Sided — Lower futtocks	0	9	0	10	0	10½	0	11	1	0	1	0½	1	0	1	1	1	1½	1	2	1	2½	1	2½	1	3
	Sided — Middle ditto	0	8	0	9	0	9½	0	10	0	11	0	11½	0	11¾	1	0½	1	0¾	1	1½	1	2	1	2	1	2
	Sided — Upper ditto			0	8	0	8½	0	9	0	10	0	10½	0	11	1	0	1	0½	1	1	1	1½	1	1½	1	1½
	Sided — Top timbers at the heel	0	6½	0	7½	0	8	0	8½	0	9½	0	10	0	11	1	0	1	0½	1	1	1	1½	1	1½	1	1½
	Sided — Ditto at the head	0	5½	0	5½	0	5½	0	6	0	6½	0	7	0	8¼	0	8¾	0	9¼	0	9½	0	9¾	0	10½	0	10¾
	In and out — At the floor heads	0	6½	0	7	0	7½	0	8	0	8½	0	9½	0	11	1	0	1	1	1	1½	1	2	1	2¼	1	2½
	In and out — At the breadth	0	5	0	5	0	5½	0	6	0	7	0	7½	0	9	0	10	0	10¾	0	11½	1	0	1	0½	1	0½
	In and out — At the top of the ſide	0	3	0	3	0	3	0	3½	0	4	0	4½	0	4¼	0	4½	0	4¾	0	5	0	5	0	5½	0	5½
WALES.	Main broad	0	10	1	6	2	2	2	6	2	10	3	0	3	2	3	6	3	10	4	2	4	6	4	10½	5	2
	Ditto thick	0	4	0	5½	0	6	0	6½	0	7½	0	7½	0	5	0	5½	0	7	0	7½	0	8½	0	9	0	10
	Channel broad			0	6	0	8	0	9	0	9½	0	10	2	1½	2	2½	2	4	2	5	2	6½	2	8	2	9
	Ditto thick			0	3	0	3½	0	4	0	4½	0	4½	0	3½	0	4	0	4	0	5	0	5¼	0	5½	0	5½

N. B. The ſtern poſt is the ſame thwartſhips with the keel below, and the head ſquare, if to be had.

Of the ROTHER.

THE general method is to make the breadth of the rother, at the lower end in large ſhips ⅛ or ⅑, and in ſmall ſhips ⅒ of the extreme breadth of the ſhip; but this cannot be eſtabliſhed as an invariable rule, for a full built will require more rother than a ſharp-built ſhip, and it will be neceſſary ſometimes to have a piece, fayed to the aftſide of the rother, called a back.

As to the ſcantlings, the head of the rother, if it can be had, may be 3 or 4 inches bigger than the head of the ſtern poſt thwartſhips, and two inches bigger afore and aft than thwartſhips.

For the ſake of ſuch of our readers as are not acquainted with the terms relating to ſhipbuilding, we ſhall conclude this ſecond part with the following gloſſary.

A

GLOSSARY,

OR

EXPLICATION,

Of TERMS relating to

SHIPBUILDING.

BEAMS, are the large pieces of timber, which are laid acrofs the fhip; their ends are lodged on the clamps, and being bound by knees to the fide, keep the fhip to her breadth.

Bow, is the round part of the fhip, forward. That on the right hand, with one's face forward, is called the Starboard, and that on the left the Larboard-bow; they both unite at the ftem.

Breast-hooks, are large knees fayed acrofs the ftem to both bows, into which they are bolted.

Carlings, are fquare pieces of timber, lying fore and aft from one beam to another, into which they are fcored.

Cathead, is a large fquare piece of timber; one end of it is faftened upon the forecaftle, the other end projects without the bow fo faras to keep the anchor clear of the fhip when it is heaving up by a tackle, the block of which is called the Cat-block: The rope which paffes thro' the feveral fhivers of the block, and extremity of the cathead, is called the Cat-fall.

CLAMPS, are thick planks, which ſupport the ends of the beams.

COUNTER. The hollow part of the ſtern above the wing-tranſom is caled the lower, and that part betwixt it and the lower part of the cabbin-lights is called the upper or ſecond Counter.

DEAD WOOD, conſiſts of large pieces of timber laid one upon another, upon the keel, afore and abaft, where the ſhip is ſo thin as not to admit of ſufficient ſubſtance for the two half timbers, which are therefore ſcored into this dead wood. The half timbers are uſed when, by reaſon of the ſharpneſs of the floor, one piece of timber cannot be had which will make a floor-timber.

DECKS, are the ſame in a ſhip that floors are in a houſe, and are denominated, according to their heighth, lower, middle, and upper: Beſides which, there is a deck which covers the cabbin, and reaches from the ſtern near to the main-maſt; this is called the Quarter-deck. In ſome ſhips there is an apartment above the great cabbin, called the Round-houſe; the deck which covers it is called the Poop. Another deck covers the forecaſtle, which is an apartment in the fore-part of the ſhip, in which is the cook-room.

FAY, is to fitt two pieces of wood ſo as to join cloſe together. The plank is ſaid to fay to the timbers when it bears, or lies cloſe to all the timbers.

HARPINS, are the fore-part of the wales which go round the bow and are faſtened to the ſtem.

HAWSE-PIECES, are broad timbers in the bow of the ſhip, thro' which there are holes cut for the cables to paſs.

HEAD, is ſome figure, often that of a lion, carved as an ornament for the fore-part of the ſhip. There is a large piece of timber fayed to the ſtem upon which the figure reſts; this is called the Knee of the head, and by reaſon of the great breadth at the the upper part, it is compoſed of ſeveral pieces: It is let into the head, and faſtened to the bow on each ſide by knees, called the Cheeks of the head. The head is ſupported by rails, which extend from the crown of the figure to the cathead.

HEEL. The lower part of a maſt, or any timber, is called the heel, and the upper part the head.

KEEL

Keel, is the principal piece of timber firſt laid upon the blocks, which ſupports the whole ſtructure. When this cannot be had of a ſufficient depth in one piece, there is a plank faſtened to the bottom, called the Falſe Keel, which ſerves likewiſe to ſave the bottom of the main keel.

Keelson, is fayed over the floor-timbers, and bolted thro' them into the keel.

Knees, are crooked pieces of timber. One leg or arm is bolted to the beams, and the other to the ſhip's ſide. They are either lodging or hanging. The hanging knees are fayed up and down, and the others fore and aft the ſide, and reſt upon the clamps.

Limber-boards, are ſhort pieces of plank, fayed next to the keelſon, which may be taken out to clear the limber-holes, that are left either below or above the floor-timbers, for a paſſage for the water to the pump.

Rabbet. When a plank is to be faſtened to any piece of timber, ſuch as the ſtem or poſt, there is ſo much wood cut out of the piece as the plank is thick, which is called the Rabbet; and when the plank is let into this rabbet, it will be even with the outſide of the piece, as at the after-end of the keel, and lower end of the ſtern-poſt.

Rails, are narrow planks, generally of fir, upon which there is a moulding ſtuck. They are for ornament, and nailed acroſs the ſtern above the wing-tranſom and counters, &c. They are likewiſe nailed upon ſeveral planks along the ſides; one in particular is called the Sheer-rail, which limits the heighth of the ſide from the forecaſtle to the quarter-deck, and runs aft to the ſtern and forward to the cathead. The wales are nearly parallel to this.

Rother, is a piece of timber, or ſeveral pieces faſtened together, and fitted to the ſtern-poſt, to which it is hung by irons, whereon it moves, and thereby the ſhip is ſteered.

Scantling, is the breadth or thickneſs of a piece of timber.

Scarphs. When two pieces of timber are joined together, ſo that the end of the one goes over the end of the other, being tapered ſo that the one may be let into the other and become even, they are ſaid to be ſcarphed; ſuch are the keel-pieces. But when the ends of the two pieces are cut square and put together, they are said to butt to one another; and when another piece is laid upon, and fastened to both, as is the case in all the frame-timbers, this is called scarphing the timbers; and half the piece which fastens the two timbers together is reckoned the length of the scarph.

STEM, is that circular piece of timber where both the sides of the ship unite forward. The lower end of it is scarphed into the keel, and the bowsprit rests upon the upper end of it.

STEPS, are large pieces of timber fayed across the keelson, into which the heels of the masts are fitted.

STERN, is the after-part of the ship, in which are all the cabbin-lights. It likewise includes the stern-frame, which consists of the stern-post, transoms, and fashion-pieces, all fastened together.

STERN-POST, is that strait piece of timber at the after-end of the ship, which unites both the sides. The heel of it is tenanted into the keel, and the wing-transom fastened at the head of it.

TIMBERS, in a ship, are as the ribs in the body, and serve to support the sides, the planks being all fastened to them; the two aftermost are called Fashion-pieces; they support the ends of transoms. The two timbers, forward, at the cathead, are called Knuckle-timbers. [For the names of the other timbers, see Chap. III. Sect. 3. Part II.

TUCK-square, is, when the heels of the fashion-pieces are let in upon the post, at which place the heighth of the tuck is fixed.

WALES, are planks, thicker than the rest, brought about the outside of the ship, in the wake of the decks.

THE

THE THEORY OF

SHIPBUILDING and NAVIGATION.

PART III.

Of NAVIGATION.

CHAP. I. SECT. I.

Of Trigonometry by tabular Calculation from a Table of natural Sines, Tangents and Secants.

WE have in the first part explained the doctrine of trigonometry, so far as to give a solution to all the varieties geometrically by scale and compasses. We come now to shew how to perform the same arithmetically by a table of natural sines, tangents and secants; in order to which, it will be absolutely necessary to shew how this table may be constructed.

To Construct a Table of natural Sines, Tangents and Secants, to a Radius of 10000 equal Parts.

Describe a quarter of a circle, and let the radius be 10000 equal parts. Divide the arch into 90 degrees, and draw sines, tangents and secants to every degree, as directed in making the plain scale. Measure each off these seperately, by the same line of equal parts that the radius was taken from, and set down the numbers contained in each, in proper columns corresponding to every degree in the quadrant, as in the following table, where it is done only to every fifth degree, being sufficient to shew the nature of the table, which has been calculated to great exactness, by numbers for

for every minute to a radius of 100,000; but in practice the logarithms of those sines, tangents and secants are used.

A Table of natural SINES, TANGENTS *and* SECANTS.

Deg.	Sines.		Tang.		Seca.		
5	871	9962	875	11430	10038	11473	85
10	1736	9848	1763	56713	10154	57588	80
15	2588	9659	2679	37320	10353	38637	75
20	3420	9397	3639	27474	10642	29238	70
25	4226	9063	4663	21445	11034	23662	65
30	5000	8660	5773	17320	11547	20000	60
35	5735	8191	7002	14281	12208	17434	55
40	6427	7660	8390	11917	13054	15557	50
45	7071	7071	10000	10000	14142	14142	45
		Sines.		Tang.		Seca	Deg.

This table gives by inspection, the sine, tangent, or secant of any arch, or number of degrees therein expressed, if the radius of the circle be 10000; and because, as the radius of a circle is to the sine, tangent, or secant of any arch of the same circle, so is the radius of any other circle; to the sine, tangent, or secant of a similar arch of this other circle, as proved in *Prop.* 8. *Chap.* 3. *Sect.* 2. *Part* 1.

Therefore we may find the sine, tangent, or secant of any arch, in any circle, provided the radius be known, by the following proportion.

As the radius in the table.

Is to the sine, tangent, or secant of any arch in the table.

So is the radius of any other circle.

To the sine, tangent, or secant of a similar arch of this other circle.

E X A M P L E.

Required how many feet in length is the sine of 35 degrees, supposing the radius to be 130 feet.

The tabular radius is 10000, the sine of 35 degrees in the table is 5735; therefore by the rule of three.

$$10000 : 5735 :: 130 : 74\tfrac{555}{1000}, \text{ or } 74,555.$$

```
      5735
       130
    172050
     5735
1|000)74|5550
```

If

If the length of the radius of any circle, and the length of a ſine, tangent, or ſecant of the ſame circle be given, and it be required to find the arch; the proportion will be,

As the given radius of any circle
Is to the given ſine, tangent, or ſecant of the ſame circle:
So is the radius in the table
To a ſine, tangent, or ſecant in the table;

which muſt be found in the table; and correſponding thereto in the column of degrees, is the quantity of the arch required.

EXAMPLE.

Let the radius be 500, and the given ſine 383; then

500 : 383 : : 10000 : 7600
383
5|00)38300|00(7660, and this found in the column of

ſines in the table, correſponding thereto, is 50 degrees the quantity of the arch required.

It will be needleſs to give any more examples, as in practice we ſhall uſe the table of logarithms.

SECT. II.

Of artificial SINES, TANGENTS *and* SECANTS.

WE obſerved in the firſt part, that the ſides of a right angled triangle were diſtinguiſhed by different names, *viz.* hypothenuſe, perpendicular and baſe; and that by the angle, is underſtood that oppoſite to the baſe, which is ſuppoſed to be drawn acroſs the paper; and the perpendicular up and down, making a right angle with the baſe, to which angle the hypothenuſe is always oppoſite.

The ſides may be conſidered likewiſe as ſines, tangents, or ſecants, by which means, beſides the aforeſaid proper names, they will acquire another, which we ſhall call their ſirnames; and theſe will vary according to the ſide made radius.

If the hypothenuſe be radius, the baſe will be the ſine; and the perpendicular, the ſine complement of the angle.

If

If the base be made the radius, the hypothenuse will be the secant complement; and the perpendicular, the tangent complement of the same angle as above.

If the perpendicular be made the radius, the base will be the tangent, and the hypothenuse the secant of the angle; all which has been demonstrated in the first part.

Hence, the whole business of trigonometry, may be said to consist, either in finding a sine, tangent, or secant, to a given arch, the radius being known; or if the radius, and either the sine, tangent, or secant be known, to find the arch; both which may be performed by a due attention to the two foregoing proportions: And as any side may be made radius, there may be different operations for each case.

All the various cases of right angled triangles, are express'd in the following table.

The

The PROPORTIONS *for the ſeveral Solutions of the ſix Caſes of Plane right-angled Triangles.*

Given	Requir.	PROPORTIONS.	Radius.	Caſes.
Hypo.	Baſe	R : ſine of the angle : : H : B R : ſine comp. of the angle : : H : P	Hypo.	
and	and	Sec. comp. of the angle : R : : H : B Sec. co. of the angle : tan. co. of the ang. : : H : P	Baſe.	*1ſt.*
Angle	Perpen.	Sec. of the angle : tang. of the angle : : H : B Sec. of the angle : R : : H : P	Perpen.	
Baſe	Hypo.	Sine of the angle : R : : B : H Sine of the ang. : ſine comp. of the ang : : B : P	Hypo.	
and	and	R : ſec. comp. of the angle : : B : H R : tang. comp. of the angle : : B : P	Baſe.	*2d.*
Angle	Perpen.	Tang. of the angle : ſec. of the angle : : B : H Tang. of the angle : R : : B : P	Perpen	
Perpen.	Hypo.	Sine comp. of the angle : R : : P : H Sine comp. of the angle : ſine of the angle : : P : B	Hypo	
and	and	Tan. co. of the ang. : ſec. co. of the ang. : : P : H Tang. comp. of the angle : R : P : B	Baſe.	*3d.*
Angle	Baſe	R : ſec. of the angle : : P : H R : tang. of the angle : : P : B	Perpen	
Hypo.	Angle	H : B : : R : ſine of the angle required	Hypo.	
and	and	B : H : : R : ſec. comp. of the angle required	Baſe.	*4th.*
Baſe.	Perpen.	After finding the angles the perpendicular is found by caſe *1ſt.* or *2d.*		
Baſe	Angle	B : P : : R : tang. comp. of the angle required	Baſe.	
and	and	P : B : : R : tang. of the angle required	Perpen	*5th.*
Perpen.	Hypo.	After finding the angles the hypothenuſe is found by caſe the *2d.* or *3d.*		
Perpen.	Angle	P : H : : R : ſec. of the angle required	Perpen.	
and	and	H : P : : R : ſine comp. of the angle required	Hypo.	*6th.*
Hypo.	Baſe.	After finding the angles the baſe is found by caſe *1ſt.* or *3d.*		

It is evident from what has been ſaid, that the firſt thing to be done, in order to give a ſolution to any of the caſes, is, to make one of the ſides radius; now if the thing required be a ſide, any of the three may be made radius; for there is no neceſſity, for making the given ſide radius, but after the radius is fixed, then both the given and required ſides, will have particular ſirnames; the proportion for finding a ſide will always be,

As the firname of the given fide,
Is to the firname of the required fide:
So is the given fide
To the required fide.

If the thing required be an angle, one of the given fides muft be made radius, which will determine the firname of the other fide; and the proportion will be

As one of the given fides, *viz.* that made radius
Is to the other given fide:
So is the radius
To the firname of the fecond fide.

And when this is found in the table, the quantity of the requir'd angle will be found in degrees and minutes in their proper columns.

Note. If the thing requir'd be a fide, the two firft terms will be either fines, tangents, or fecants, to be taken out of the logarithmick table of fines, tangents and fecants; the third term will be a natural number, as miles, yards, or any other meafure, the logarithm of which, muft be taken out of the table of logarithms; and, when the logarithms of the fecond and third terms are added together, if from this fum be fubtracted the logarithm of the firft term, look for the remainder in the table of logarithms; and correfponding thereto in the proper column, will be the natural number, expreffing the length of the required fide, taken by the fame meafure with the given fide.

When an angle is required, we muft not work for the angle itfelf, but for the fine, tangent, or fecant of it; the two firft terms of the proportion will be natural numbers, and their logarithms muft be taken out of the table of logarithms; the third term will be a fine, tangent, or fecant, and its logarithm muft be taken out of the table of artificial fines, tangents and fecants; and then the fecond and third terms muft be added, and the firft fubtracted from their fum as before; the remainder muft be found in the table of artificial fines, tangents and fecants; and the quantity of the angle required will be found in degrees and minutes correfponding thereto.

We fhall illuftrate the whole by an example in each cafe, by the tables, and alfo by *Gunter*'s fcale.

The general rule by the pen, is the fame as in any other queftion in the rule of three; and if we ufe the natural fines, it will be performed, by multiplying the fecond term by the third, and dividing the product by the firft term, the quotient will be the fourth term required; obferving to make the firft term according to the aforefaid proportions, but it will be indif-

indifferent which of the other two is made the ſecond term, as they are to be multiplied into each other.

But as the natural ſines, &c. are calculated to ſeven places, this would make the operations very tedious, upon which account their logarithms are uſed, and are had by the table of artificial ſines, tangents and ſecants; if the table of logarithms, was made for natural numbers to ſeven places, we could find the logarithms of natural ſines, &c. as eaſily as of any other number; but even in that caſe, we muſt have a table of natural ſines, &c. and afterwards have recourſe to the table of logarithms, whereas by the table of artificial ſines, &c. we have the logarithm at once.

This table contains every degree, and minute, of the quadrant; if the number of degrees be leſs than 45, look for it at the head of the table; and for the minutes under Min. increaſing downwards on the left hand of the page; but if the degrees exceed 45, look for them at the bottom of the page, and the minutes in the right hand column above M, increaſing upwards; and when the degrees and minutes are thus found, the logarithmick ſine, tangent, or ſecant, will be found in its proper column; obſerving if the degrees be found at the top, the word ſine, tangent, or ſecant, muſt be found at the top, underneath which, right againſt the minutes, which muſt be found under Min. is the thing requir'd; but if the degrees are at bottom, theſe words muſt be found at the bottom, above which, and right againſt the minutes, which muſt be alſo found above M; is the thing required.

EXAMPLE.

Let it be required, to find the ſine, tangent and ſecant of an arch, or angle of 33° 45′. Here the degrees are leſs than 45; therefore look for them at the top, and the minutes under M; right againſt which, and under the word ſine, is 9.744739; under tangent, is 9.824893; under ſecant, is 10.080154; but if the ſine, tangent and ſecant of 56° 15′ were required, look for 56 degrees at the bottom, and 15 minutes over M; right againſt which, and above the word ſine, is 9.919846; above tangent, is 10.175107; above ſecant 10.255261.

The degrees at the top begin at *o*, and increaſe to 44; the degrees at the bottom, in the firſt page, are 89, and decreaſe to 45; the one including the minutes, is always the complement of the other; ſo that if it was required to find the ſine complement of any arch, look for the ſine, and in the ſame line, you'll find the ſine of the complement.

Thus to find the ſine complement of 33°; 45′; look for the degrees 33° at the top, and 45′ in the left hand column under Min. the ſine will be under the word ſine as before, 9.744739, and the ſine complement 9.919846 in the ſame line above the word Sine; the like may be ſaid of the tangent complement and the ſecant complement.

Having the logarithmick ſine, tangent, or ſecant, to find the degrees and minutes correſponding thereto.

This is only the reverſe of the former, for you muſt look over the table, till it is found, and if in a column that has the word ſine, tangent, or ſecant, at the top; the degrees are at the top; and the minutes in the left hand column under Min. but if it is in a column, which has theſe words at the bottom, the degrees are at the bottom, and the minutes above M in the right hand column.

E X A M P L E.

Let it be required to find the degrees and minutes, anſwering to the tangent, 10,346337; this will be found over the word tangent, therefore the degrees muſt be at the bottom, *viz.* 65, and right againſt it above M, is 45; ſo 65°, 45′; is the arch required: But if the complement was required, the degrees would be at the top, and the minutes under Min. *viz.* 24°, 15′. Sometimes the exact number cannot be found in the table, in which caſe, all that can be done, is, to take the neareſt to it; ſo we can never err a whole minute.

To work theſe by ſcale and compaſſes, there is a line of logarithmick ſines, and a line of logarithmick tangents, upon *Gunter*'s ſcales, conſtructed in the ſame manner as the line of numbers; for againſt 30 degrees, in the line of ſines, is 5000 in the line of numbers, the natural ſine correſponding thereto; and againſt 30 degrees, in the tangents, is 5773, the natural tangent; the line of ſines is continued to 90 degrees, but the tangents to 45 degrees; the tangents above 45 degrees, are the ſame with their complements, for the radius, which is equal to the tangent of 45 degrees, is a mean proportional betwixt the tangent of any arch, and the tangent of its complement to 90 degrees, which is the reaſon, that the line of tangents is numbered 1 and 89, 5 and 85, 10 and 80, *&c.*

The tables being thus explained, we ſhall in the next place ſhew their uſe in the reſolution of the ſix caſes, of right angled triangles.

CASE I.

CASE I.

Given the hypothenuse 60 miles, the angle 56°, 15′; required the base, and perpendicular.

For the BASE.

As the radius, or sine of 90° 0	- - - -	10.000000
Is to the sine of 56° 15′	- - - - - -	9.919846
So is the hypothenuse 60 miles	- - - - -	1.778151
To the base 49.9 miles	- - - - - -	+.697997

For the PERPENDICULAR.

As the radius, or sine of 90°	- - - - -	10.000000
Is to the sine complement of the angle, or sine of 33° 45′		9.744739
So is the hypothenuse 60 miles	- - - - -	1.778151
To the perpendicular 33.3	- - - - -	+1.522890

By GUNTER's *Scale.*

Extend the compasses from 90 on the line of sines, to 56° 15′; the same extent will reach in the line of numbers, from 60 to 50 nearly, for the base; and the extent from 90 to 33° 45′ on the line of sines, will reach in the line of numbers, from 60 to 33 ½ nearly for the perpendicular.

CASE II.

Given the perpendicular 33.3 miles, and the angle 56°, 15′; required the hypothenuse, and base.

To avoid working by the secants, let the hypothenuse be radius, and it will be,

For the HYPOTHENUSE.

As the sine complement of the angle, or sine of 33° 45′ com. arith.		0.255261
Is to the radius, or sine of 90° 0′	- - - -	10.000000
So is the perpendicular 33.3 miles	- - -	1.522890
To the hypothenuse 60 miles	- - - - -	+1.778151

In this, and such like cases, where the first term is not radius, instead of the logarithm of the first term, use the complement arithmetical of it, which is, what any logarithm wants of the logarithm of the radius; now this being always 10.000000, the complement arithmetical, will be found, by subtracting each figure in the logarithm, from 9 excepting, the first

first towards the right hand, which must be subtracted from 10; as in this example, the logarithm of the sine of 33° 45′, by the table, is 9.744739, and by subtracting, as above directed, the complement arithmetical will be 255261; this is so plain and easy, that by a little practice, the complement arithmetical, will be as readily taken out of the table, as the logarithm itself, and then it must be added to the other two logarithms; and when the logarithm of the radius is subtracted from this sum, the remainder will be the logarithm of the fourth term required; and this will be the same thing, as if the operation was performed by the common method, *viz.* by subtracting the logarithm of the first term, from the sum of the logarithms of the second and third terms; in this example the sum of the two is - - - - - - 11.522890

Sine 33° 45′	9.744739
Radius	10.000000
Perpen. 33.3 miles	1.522890
	11.522890
	9.744739
	1.778151

Logar. of the first is	9.744739
Remainder	1.778151

Now in subtraction as a lesser number is taken from a greater; it is plain if any number be added to both, and then the lesser subtracted from the greater, it will make no alteration in the remainder; and this is the very case here, for 9.744739, is to be subtracted from 11.522890; if to each of these be added the complement arithmetical, *viz.* 0.255261; the sum of the greatest will be 11.778151; the sum of the least will be 10.000000; and as all the figures in the lesser number are cyphers, except the first to the left hand; the subtraction is performed, only by cancelling the first figure to the left hand in the greatest number.

For the BASE.

As the sine complement of the angle or sine of 33° 45′ com. arith.	0.255261
Is to the sine of the angle 56° 15′ - - - - -	9.919846
So is the perpendicular 33.3 miles - - - - -	1.522890
To the base 49.9 miles - - - - - - -	+1.697997

By the GUNTER's *Scale.*

The extent from 33° 45′ to 90° in the line of sines, will reach in the line of numbers from 33.3 to 60, for the hypothenuse; and the extent from 56° 15′ to 33° 45′ in the line of sines, will reach in the line of numbers, from 33.3 to 49.9 for the base.

CASE III.

CASE III.

Given the base 49.9 miles, and the angle 56° 15′ required the hypothenuse, and perpendicular making the hypothenuse radius.

For the HYPOTHENUSE.

As sine of the angle 56° 15′ comp. arith. - - -	0.080154
Is to the radius - - - - - - - -	10.000000
So is the base 49.9 miles - - - - - -	1.697997
To the hypothenuse 60 miles - - - - -	+1.778151

For the PERPENDICULAR.

As the sine of the angle 56° 15′ comp. arith. - -	0.080154
Is to the sine complement 33° 45′ - - - -	9.744739
So is the base 49.9 miles - - - - -	1.697997
To the perpendicular 33.3 - - - - -	+1.522890

By GUNTER's *Scale*.

The extent from 56° 15′ to 90 in the line of sines, will reach from 49.9 to 60 on the line of numbers; for the hypothenuse, and the extent from 56° 15′, in the line of sines, will reach from 49.9 to 33.3 on the line of numbers for the perpendicular.

CASE IV. and V.

Are exactly the same, only changing the names of the base and perpendicular.

Given the hypothenuse 60 miles, and base 49.9 miles; required the angles and perpendicular.

As the hypothenuse 60 miles complement arithmetick	8.221849
Is to the base 49.9 miles - - - - - -	1.697997
So is the radius - - - - - - -	10.000000
To the sine of the angle 56° 15′ - - - -	+9.919846

By GUNTER's *Scale*.

The extent in the line of numbers from 60, to 49.9, will reach in the line of sines, from 90° to 56°, 15′; and when the angle is found, the other side, may be found by *Case* 1. or 3.

CASE

CASE VI.

Given the baſe 49.9 miles, and perpendicular 33.3; required the angles and hypothenuſe:

Making the perpendicular radius, the baſe will be the tangent, and the proportion will be,

As the perpendicular 33.3 miles complement arithmetical	8.477110
Is to the baſe 49.9 - - - - - -	1.697997
So is the radius, or tangent of 45° - - - -	10.000000
To the tangent of the angle 56°, 15′ - - -	20.175107

In this caſe, the firſt figure in the logarithm of 33.3 is a cypher; therefore the next to it muſt be ſubtracted from 10, and all the reſt from 9 as before, to find the complement arithmetical; and when the three are added, the characteriſtick will be 20, the characteriſtick of the radius being ſubtracted, there will remain 10; and this found in the table of artificial ſines, tangents and ſecants, againſt it is 56° 15′ the tangent of the angle required.

By GUNTER's *Scale.*

The extent in the line of numbers from 33.3 to 49.9; will reach in the line of tangents, from 45° to a point in the ſame line, which is either 33°, 45′ or 56°, 15′; and to know which of the two it is; I find the extent in the line of numbers, is from a leſs to a greater, and therefore it muſt be ſo in the line of tangents; ſo the point muſt be more than 45° 0′, and of conſequence muſt be 56°, 15′.

Theſe are all the caſes in right angled triangles. We might here likewiſe ſhew how to ſolve all the caſes in oblique triangles, but as the whole buſineſs of navigation may be performed without them, we ſhall apply the doctrine of right angled triangles to navigation.

Now as navigation is a ſcience which teaches us how to direct a ſhip's way from one port to another; it will from thence follow, that the ſituation, and diſtances of the places muſt be known. Our firſt buſineſs then ſhall be to ſhew how this may be attained. In order to this, it will be abſolutely neceſſary to underſtand the principles of geography, which ſhall be the ſubject of the next chapter.

CHAP.

CHAP. II. *Of GEOGRAPHY.*

SECT. I. *Of SURVEYING of LAND.*

GEography is that ſcience by which we learn how to lay down, either all the places in any particular country, called a chart, or map; or all the habitable parts of the world upon a globe. The firſt is commonly called ſurveying of land, and the latter is what is generally underſtood by geography; we ſhall explain both.

The chief deſign of ſurveying, is to lay down all the places, in any particular piece of ground, upon paper; by which means, their diſtances, and poſitions, from one another, may be had, with as much certainty, as if they were to be meaſured on the very ſpot of ground where they are ſituated.

Let it be required to lay down all the places in the plane A B D E, *viz.* F, G, H, I, K, L, M.

The inſtruments neceſſary for this purpoſe, are, a well graduated circle, with an index, and ſights, to take angles by; and a chain, line, or ſtaff, to meaſure diſtances by.

Aſſume, (*See Plate* 10.) at any convenient diſtance from one another, any two points B and D, from whence, all the points in the plane, may be ſeen, place the circle with its center over the point B, and the index right over the diameter; move the circle, upon a pin provided for that purpoſe, till the point D may be ſeen thro' the ſights, and faſten the circle in that poſition; then move the index, till all the points, may be ſeen ſucceſſively, thro' the ſights, noting the degrees cut by the index, correſponding to each point; then move the inſtrument, from the point B, to the point D, placing it ſo, that the index being laid upon the diameter, the point B may be ſeen thro' the ſights; and then faſten the inſtrument, with its center right over the point D; move the index, till all the points, ſucceſſively, may be ſeen thro' the ſights, noting the degrees cut by the index, as before; then meaſure the diſtance betwixt B and D, which ſuppoſe 40 fathoms, or yards, &c.

To lay this down upon paper, take 40 from any ſcale of equal parts, which lay off upon any ſtrait line from *b* to *d*, at the point *b*, make the ſame number of angles, equal to thoſe taken in the field; do the ſame at the point *d*, the interſections of the lines drawn from *b*, with their corresponding lines drawn from *d*, will give the points *f*, *g*, *h*, *i*, *k*, *l*, *m*; and if we meaſure their diſtances from one another, by the ſame ſcale that the diſtance of the points *b* and *d* was taken from, we ſhall have the true ſtance of the points in the field.

For the triangles D E B, and *d f b*, are equiangular, by conſtruction; therefore *b d* : *b f* :: B D : B F, but *b d* contains as many equal parts of the ſcale, as B D does fathoms; therefore *b f* muſt contain as many equal parts of the ſcale as B F does fathoms.

As theſe are all the places that are ſuppoſed can be ſeen from the points B and D, the adjacent places cannot be laid down without altering the two ſtations: Let the next two ſtations be K and M, from whence taking the angles as before; all the places that can be ſeen from K and M, may be laid down in the ſame manner as thoſe ſeen from B and D. We may proceed in the ſame manner to lay down all the places in an iſland, or country, by continuing to alter our ſtation to places already laid down, 'till we have gone over the whole; this gives us the neareſt diſtance betwixt any two places, but by the intervention of valleys, it will be impoſſible to go the neareſt way in a mountainous country.

Another method is by actually travelling over the country, in a direct line, and meaſuring the diſtances by a wheel provided for that purpoſe. There muſt alſo be a contrivance to keep in a direct line, which may be done by ſetting a ſtake, at ſuch a diſtance from the place we ſet out from, that it may from thence be diſtinctly ſeen; we may then, by the help of the ſights, ſet ſeveral intermediate ſtakes in a ſtrait line between them. After we have travelled to the fartheſt ſtake, we may then place another ſtake at ſuch a diſtance, that from it, at leaſt two of the former ſtakes may be ſeen, by which it may be ſet in a direct line with them; and placing ſeveral intermediate ſtakes betwixt theſe two laſt ones, we may by them, produce the line to another ſtake, and proceed thro' the whole extent of the country. We may likewiſe keep in a direct line by the mariner's compaſs, of which, we ſhall only here remark, that wherever it is carried to, the needle will always point to the north, or its variation may be found. We do not propoſe this method as the moſt expeditious, neither is it ſtrictly true, or practicable, unleſs upon a plane; but as it agrees exactly with the plain ſea chart, we ſhall inſert it.

Being now provided with a wheel, and mariner's compaſs, with ſights properly fitted to it, let us ſet out from the point C, directly as the needle points, ſetting a ſtake at every mile, till we arrive at the point A, ſo from C to A, is due north: in travelling along, if we ſee any places either to the right or left, thro' the ſights placed at angles to the needle, they will

will be either due east, or due west of us; we must measure their distance from the line C A, and also the distance we are then from the point C, both which must be noted in a book provided for that purpose. When we arrive at A, the sights being at right angles with the needle, we may set a stake at X, and move directly to it, setting stakes at every mile as we go, and when we arrive at X, we may travel directly south again to S, measuring the distances of all the places from the line X S, as soon as they can be seen thro' the sights; we may proceed in the same manner, first going north or south, then east or west, till we have gone over the whole country to be laid down. The lines A C and X S, will be so nearly parallel to each other, that they may without any sensible error be taken as such.

Now in order to lay down this in a map (*See Plate* 10.) let the extent from south to north be 90 miles, and likewise that from east to west 90 miles; draw the line A C, and perpendicular to it, the lines A E and C F; draw also the line E F parallel to A C; so these four lines will limit the map, let each be graduated into 90 equal parts, and at every tenth draw lines parallel to A C, and also to E F; so the whole map will be divided into nine equal squares.

We may now lay down all the places in the map from the notes taken off in the field, as for instance, if it were required to lay down the point Y. I find by my notes, I travelled 28 $\frac{1}{2}$ miles due north from C, and 14 $\frac{1}{2}$ miles due east before I arrived at Y; therefore lay a ruler across from 28 $\frac{1}{2}$ upon the line A C, to 28 $\frac{1}{2}$ on the line E F; and take 14 $\frac{1}{2}$ with a pair of compasses, which lay off by the edge of the ruler, from the line A C, and this will give the point Y.

It is very plain that no place can be laid down in the map, unless the distance and position of it, with respect to some other place be known; and when this cannot be measured by reason of its being inaccessible by the intervention of seas, or otherwise we must have recourse to cælestial observation; and tho' two places be so remote, that they cannot be seen from one another; yet we may by observing the sun, or some star, at both places, find how far the one is to the northward, or southward of the other; we must also find some way to know how far the one is to the eastward, or westward of the other, and when those two are found, their distances and situation may with certainty be found by trigonometry.

That we may comprehend, the manner, by which the situation, and distances of places, have been found, we must explain some principles of geography, which science, consists, in giving a true description, of all the habitable parts of the whole world, as was before observed.

SECT. II.

Of the GLOBE.

THE firſt thing we ſhall obſerve is, that this earth, and ſea together, is ſuppoſed to compoſe a globe; the firſt geographers found out this, by obſerving, that in whatſoever place of the earth, they were; their ſight was always terminated, by a circle, unleſs intercepted by hills or otherwiſe, and the obſerver in the center of it; the firmament at the ſame time, forming the half of a concave ſphere over his head, and tho' he moved his ſituation, for thouſands of miles, he ſtill found himſelf in the center of a circle, and the point over his head continue at the ſame diſtance, which could not be, if the earth was flat; for ſuppoſing the obſerver placed at C, in the center of the circle H Z O N; (*Plate* 11.) then Z would be the point in the firmament over his head, but if he moved upon the diameter, from O to G, he would then be under the point D, which is much nearer to him, than the point Z was, when in C; but if the earth be allowed to be round, and the obſerver at *l*; Z will be the point over his head, but when he has moved to G, the point H will be over him, and at the ſame diſtance from him, that the point Z was, when in C: Upon this account, they reſolved, to deſcribe the earth, not upon a plane, but upon a globe: For which purpoſe, an artificial one was made, to repreſent the natural one. As the earth, was ſuppoſed a ſolid globe, the firmament, which every where ſurrounded it, was ſuppoſed to be a concave ſphere, in which the ſun and ſtars ſeemed to move, for they were continually ſhifting their ſituation with reſpect to us, ſo that either the ſun and ſtars, or the earth muſt be in a continual motion, and tho' it is certain that the earth has the motion, we ſhall ſuppoſe the ſun, moon and ſtars to move round, and the earth immoveable in the center, being apparently ſo to our ſenſes.

They likewiſe found, that the ſun, by his daily motion from eaſt to weſt, deſcribed an arch of a circle, aſcending one half, and deſcending the other, the ſtars, alſo deſcribing arches by their motion; amongſt them, they obſerved one, ſeemingly not to change its ſituation, but always at or near the ſame diſtance, from the point over the obſerver's head, while he continued his ſtation, or moved either eaſt or weſt: But when he moved, either ſouth, or north, the ſtar appeared either nearer to, or further from, that point.

They

They likewiſe found that the obſerver might move ſo far to the ſouthward till the ſtar would juſt diſappear, and the ſun, which at his firſt ſetting out, was a great diſtance from the point over his head at noon, was now right over him; from thence, they ſuppoſed the heavens, with all the ſtars, ſun and moon, to be carried round the earth once in 24 hours, two points only being immoveable, one near the aforeſaid ſtar, and the other diametrically oppoſite to it; by which motion, every point in the heavens, excepting thoſe two, would by every revolution, deſcribe a circle.

GEOGRAPHICAL DEFINITIONS.

Def. 1. A globe, or ſphere, is a ſolid body, which has a point within it, equally diſtant from every point of its ſurface; and may be conceived to be formed by the revolution of a ſemi-circle, round the diameter, which is ſuppoſed to continue immoveable. (As was obſerved in the firſt part).

Def. 2. The axis of the globe, is that line paſſing thro' the center, round which the whole globe is moved, and may be termed a diameter.

Def. 3. The poles, are the two points in the globe's ſurface, through which the axis is ſuppoſed to paſs; theſe have no motion, one is called the north, and the other the ſouth pole.

There are two globes uſed; upon one, all the kingdoms of the earth are deſcribed, this is called the terreſtrial, or terraqueous globe, containing all the land, and water: The other is called the celeſtial, containing all the conſtellations; and tho' the firmament be ſuppoſed a concave, they may be duly repreſented on the convex ſuperficies of a globe; and ſuppoſing it tranſparent, and the obſerver in the center, they would appear to him, in the ſame manner as they really are in the heavens; the ſame circles are deſcribed upon both, and are either great, or ſmall; the planes of the great circles go thro' the center of the earth, and their diameters are the ſame with the diameter of the globe; the planes of the ſmall circles do not paſs thro' the center, they are parallel to the plane of ſome great circle, their diameters are leſs than the diameter of the globe, and continually decreaſe, according to their diſtance from the plane of the great circle, to which they are parallel.

Def. 4. Meridians, are great circles, interſecting one another in both poles, of which there may be an infinite number.

Def. 5. The equator, or equinoctial line, is a great circle, cutting all the meridians at right angles exactly in the middle, being equally diſtant from both poles.

Def. 6. The eclipᵗick, is a great circle croſſing the equinoctial in two oppoſite

oppoſite points in ſuch a manner, that the planes of thoſe circles form an angle of 23° 30′.

Def. 7. The horizon, is that circle which terminates the ſight; ſuppoſing the obſerver at ſea, he finds himſelf in the center, and the ſea and ſky uniting: It is plain, if he alters his ſtation, this circle will change likewiſe; ſo that it cannot be deſcribed upon the ſuperficies of the globe; it is called the viſible, or ſenſible horizon.

Def. 8. The zenith, is that point in the heavens, which is right over the obſerver's head; the oppoſite point in the other hemiſphere, is called the nadir; theſe two points vary continually as the obſerver changes his ſtation.

In order to repreſent the horizon, and zenith by the globe, the axis is fitted in a braſs circle, in which it turns round, this circle then will repreſent any meridian, becauſe any place upon the ſuperficies of the globe, may be brought right under it; this, with the globe within it, is placed in a broad wooden circle, the inſide of it is the ſame diameter, with the inſide of the brazen meridian; ſo the half of the globe will always be under, and the other half over this circle; and the meridian may be ſo moved in the notches, as to bring any part of it to this wooden circle; which is therefore called the real, or rational horizon, and is always parallel to the viſible, before deſcribed, as in the artificial globes they are both the ſame, becauſe the eye may be ſo placed as to ſee the whole half of the globe at one view; by the help of this circle, we may repreſent the horizon of any place, for it is only turning the globe upon its axis, 'till the place is under the brazen meridian; and then moving the meridian in the notches, 'till the point is 90 degrees diſtant from the horizon; for that point will be the zenith, and is equally diſtant from every point in the horizon.

Def. 9. Azimuth circles, are ſuppoſed to paſs thro' the zenith, and nadir points, all divided into two equal parts by the horizon; theſe cannot be deſcribed upon the globe, but are repreſented by a quadrant of altitude, which is a thin piece of braſs, that may be ſcrewed to any part of the brazen meridian; and when this is brought to the zenith, the other end will move round upon the horizon; all theſe five circles, here defined, biſect each other, and the globe into two equal parts.

Def. 10. Parallel circles, are ſuch as divide the globe into two unequal parts, and are drawn parallel to ſome great circle; thoſe parallel to the horizon are called parallels of altitude or almicanters, but are not deſcribed on the globe: Thoſe parallel to the equator, on the terreſtrial globe, are called parallels of latitude, and are actually drawn thro' every tenth degree of the meridian.

The

The equinoctial is divided into 360 degrees, and a meridian drawn thro' every tenth: One of these meridians is graduated, every quarter of it into 90 degrees; beginning at the equinoctial, and ending at each pole, each pole being 90 degrees distant from the equinoctial.

Tho' there are no more meridians, nor parallels, actually drawn upon the globe; there may be an infinite number of both; for every place on the earth's surface, is supposed to be at the interfection of a meridian, and a parallel of latitude, which point may be found by the brazen meridian; it is by these interfections, that the situations of places are determined.

Def. 11. Latitude of a place, is an arch of the meridian, intercepted between the place, and the equinoctial; and is either south, or north, according as it lies to the southward or northward of the equinoctial.

Difference of latitude, is an arch of the meridian, intercepted between the parallels of two places.

Def. 12. Longitude of a place, is an arch of the equinoctial, intercepted betwixt the graduated meridian, and a meridian drawn thro' the place, and sometimes is accounted quite round the globe, and at other times it is accounted both ways, east and west.

Difference of longitude, is an arch of the equinoctial, intercepted betwixt the meridians, of two places.

Def. 13. Departure, is the distance of any place from the meridian, taken upon the parallel of latitude of the place, and therefore is always less than the difference of longitude; it is sometimes called the meridional distance; of which in another place.

Def. 14. Declination, is an arch of the meridian, intercepted betwixt the sun, or star, and the equinoxial.

Def. 15. Tropicks, are two circles, parallel to the equinoctial 23° 30′ distant from it; that to the northward, is called the tropick of cancer; that to the southward, the tropick of capricorn, these two limit the eclipticк.

Def. 16. Polar circles, are 23° 30′ distant from the pole, that at the north, is called the artick, and that at the south antartick.

From these definitions, the following inferences may be deduced.

Inf. 1. Every point on the earth's surface, has a corresponding point in the heavens for its zenith, from which, if a line be drawn thro' the point assumed on the earth's surface, it will pass to the earth's center: And the latitude of any place, is always equal to the distance of the zenith from the equinoctial.

Inf. 2. Those inhabitants of the earth, who have the pole in their zenith, have the equinoctial in their horizon, and are in 90 degrees of latitude;

titude; and those who have the poles in the horizon, have the equinoctial in their zenith, and are in no latitude.

Inf. 3. The elevation of the pole, above the horizon, is equal to the latitude of the place. (*Plate* 11. *Fig.* 1.)

DEMONSTRATION.

Let P represent the pole, Z the zenith, Æ Q the equinoctial, H O the horizon; the arch Æ Z, is the latitude of the place by *Inf.* 1. if to the arch Z P, be added the arch Z Æ, their sum will be 90 degrees, because the pole is 90 degrees from the equinoctial; but if to the same arch Z P, be added the arch P O; their sum will also be 90 degrees, because the zenith is 90 degrees from the horizon; therefore the arch P O, is equal to the arch Æ Z.

The globe being thus prepared, with the aforesaid circles delineated upon it, their next business was to find the latitudes and longitudes of places which they effected in the following manner, by celestial observations.

As to the latitude, it is plain, that if there was a star in the pole, there would be no more required, but to take its altitude or heighth above the horizon, with a good instrument, which would be the latitude of the place: But as there is no star in the pole, they were forced to take the least and greatest altitude of any star near the pole, which by making an entire revolution round the pole in 24 hours, would be twice in the observer's meridian: Suppose then its least altitude 48 degrees, and greatest 52 degrees, the difference betwixt these is 4 degrees, the half of which must be the star's distance from the pole; and this being added to the least, or subtracted from the greatest, gives the latitude; for in the first, the pole is 2 degrees higher above the horizon than the star, which must therefore be added to the altitude 48, which gives 50 the latitude, but when the star is 52 degrees of altitude, it will then be elevated 2 degrees above the pole; which being subtracted from the altitude, there remains 50, the latitude as before. This way of finding the latitude would require 12 hours difference betwixt the two observations, which therefore cannot be done at sea, because a ship in that time, may alter her latitude considerably; neither is it practicable when the pole is near the horizon: But after they had found the declination of some stars, or their distances from the pole; it was then enough to find one of the altitudes, and when below the pole, the distance of the star from the pole, added to the altitude, gives the latitude; but if above the pole, it must be subtracted from the altitude, the remainder is the latitude; one example

ple will be ſufficient to illuſtrate this: Let the ſtar be at 10 degrees from the pole, and *b* O, the altitude, 40 degrees; the arch P O is the latitude by *Inf.* 3. but this is the ſum of the ſtar's altitude, and diſtance from the pole, which muſt therefore be 50 degrees the latitude of the place.

But if the ſtar is in *r*, then *r* O, the altitude, is 60 degrees, and P O the latitude as before; therefore ſubtracting the ſtars diſtance from the pole, *viz.* 10 degrees, from the altitude 60, there will remain 50, the latitude as before. But as theſe obſervations are made in the night, they cannot be depended upon at ſea; we ſhall therefore ſhew how to find the latitude by the ſun's zenith diſtance, or altitude when in the meridian of the obſerver.

If the ſun was always in the equinoctial, his diſtance from the zenith, would be the latitude of the place, by *Inf.* 1. but that it is not ſo, is evident, becauſe then the ſun would always riſe and ſet in the ſame points of the horizon, and his meridian altitude, would be the ſame every day to an obſerver while he continues in the ſame place; therefore we muſt likewiſe know the declination at the ſame time that we have the zenith diſtance.

The geographers, by a daily obſervation of the ſun's greateſt altitude, which always happens when he is upon the meridian of the place, that is at mid-day, found that an obſerver, who was above 23° 30′ diſtant from the line, had the ſun approaching his zenith, for one half of the year, and the other half receding from it, and after his neareſt approach, it was a whole year before he came back to the ſame place; they likewiſe found the greateſt diſtance from the zenith was juſt 47 degrees, more than the neareſt approach; and from thence his greateſt diſtance from the line was 23° 30′, and by keeping an exact account of his zenith diſtance every day, they formed tables of his daily declination, or diſtance from the line; ſo that we have now by inſpection, the ſun's declination on any day of the month.

The table being thus made, the latitude will always be found, either by adding the zenith diſtance and declination together, or ſubtracting the one from the other, all the various caſes that can happen, ſhall be explained in the next ſection; and we ſhall now proceed to ſhew how they found the longitude.

To attain this, they conſidered that the ſun, and ſtars in the ſpace of 24 hours, return'd near to the ſame points in the heavens, in which they were at that time the day before, and conſequently moved 360 degrees in the 24 hours, which in one hour would make 15 degrees; ſo that if two places were 15 degrees to the eaſtward or weſtward of one another, those in the eastermost place, would see the sun, a whole hour before those in the westermost place; and these last would see the sun an hour after it disappeared to the others.

Now if the whole earth, should be, at the same instant of time, deprived of the light of the sun, by the intervention of some opake body, it would happen just at the sun's rising in some places, whereas in other places the sun would be upon the meridian, and in other places 15 degrees from the meridian; in short it would happen at different times of the day, to all the inhabitants of the enlightened hemisphere, that lie under different meridians; and therefore, if the hour were exactly known at all those places, from thence their difference of longitude might with certainty be found, for the difference of the hours multiplied by 15, will give the degrees; as supposing three places C, B, A, to those at B, let it be noon; to those at A, let it be 10 in the forenoon; and at C, 3 in the afternoon; C would be the eastermost, for the sun passed the meridian of that place, and got to the meridian of B, in three hours, which makes 45 degrees difference of longitude that B is to the westward of C, but then the sun must move two hours more to come to the meridian of A, it being only 10 in the forenoon at that place, which makes 30 degrees difference of longitude, A is to the westward of B, and 75 degrees to the westward of C.

As such cases of total darkness seldom happen, the longitudes of places are not much to be depended upon except where good observations, of the sun, moon, or other heavenly bodies, have been made; sometimes the longitude has been found by actual mensuration, or by good sea journals; and by frequently going to the same ports at last, tables of the latitudes and longitudes of the most remarkable places in the whole habitable world have been made; and by these tables, all the places may be laid down, according to their latitudes and longitudes upon the globe; as suppose it were required to lay down a place in 10 degrees east longitude, and 20 degrees north latitude, or which is the same thing, to find a place upon the globe, in that latitude and longitude. Move the globe upon its axis till the brazen meridian cuts the 10th degree upon the equator, on the east side of the first meridian; then look for the 20th degree upon the brazen meridian, and right under that is the place required.

SECT.

S E C T. III.

The Description and Use of the A N A L E M M A.

IT would be foreign to our design to shew how to solve all the geographical problems by the globe; it will be of greater use in navigation, to shew how to delineate all the circles of the sphere upon a plane, which may be done several ways; but as we have already explained the principles of the orthographick projection of solids, we shall now make use of that way. (*Plate* 12.)

It will be very easy to conceive that if the globe were cut by a plane passing thro' the center; the section would be a circle, and when this plane passes thro' the poles, the circle will be a meridian, as in the plate; P Æ S Q, may represent the meridian of the place, P the north, and S the south pole; P S the axis; Æ Q the equinoctial; for tho' this is only the diameter of it, yet because the plane of the equinoctial is perpendicular to the plane of the meridian, it will be represented by its diameter, and for the same reason all the parallels of latitude, which upon the globe, are circles parallel to the equinoctial, will in this projection, be represented by their respective diameters, and drawn parallel to the line Æ Q; each quarter of the meridian is divided into 90 degrees, beginning at the equinoctial, which will be all equal; but the degrees of the equinoctial, tho' upon the globe they are all equal, will here be unequal, and are represented by their respective sines, beginning at the center: P S will represent a meridian described upon the globe, at right angles to the meridian, thro' which the globe is supposed to be cut; every other meridian will be represented by an elipsis, and may be drawn thro' every 10th degree of the equinoctial. The most expeditious way of describing them, is by finding the points in every parallel of latitude, thro' which the meridians will pass, which may be done by a sector, or as readily without it; thus, First divide the equinoctial into degrees, which being only a line of sines, will be the same as the construction of the line of sines on the plain scale; and because there are as many degrees in every parallel, as in the equinoctial, they must likewise be divided into the same number of parts; and in the same proportion that the line Æ Q is divided into: Now let it be required to divide the parallel of 23° 30′: Draw the line E C K, which will represent the ecliptick, divide it into degrees, by fixing one point of the compasses in

C, and with the other transfer all the divisions of the equinoctial to this line, and thro' each division, draw a line parallel to the line P S, to intersect the parallel of 23° 30′, which will divide it into the same number of parts, and in the same proportion with the equinoctial; for E C *m*, is a right angled triangle, of which E C, the radius, or semi-diameter of the globe, is the hypothenuse; E *m* the radius of the parallel, or sine complement of the latitude, is the base; and C *m*, the sine of the latitude, is the perpendicular. Now the lines drawn thro' the several divisions of the line E C, will all be parallel to the perpendicular C *m*; and therefore constitute so many right angled triangles, all similar to E C *m*. Therefore E C : E *m* :: E *o* : E *n*, that is as the radius is to the sine comp. of the latitude, so is a degree, or any part of the equinoctial, to a degree, or proportional part of the parallel. After the same manner all the other parallels may be divided: and the meridians drawn as in the plate; there will be no occasion to draw lines from the divisions of E C to the parallel, only lay a ruler parallel to P S, and mark its intersection with the parallel. After the meridians and parallels are all drawn, the places may be laid down according to their latitudes and longitudes, in the same manner as on the globe, but their distances cannot be measured so easily; we shall therefore shew how to do this without laying down the places, and also how to solve all the geographical problems necessary in navigation, by scale and compasses; and by this plate, with as much certainty as with the globe itself.

The meridians, the equinoctial, with its parallels, and the eclipticks being thus delineated upon brass, wood, or pasteboard, are made to move about the center, within the circle Z H N O, which may represent a meridian in the heavens; Z is the zenith, and because the meridian Z H N O, passes thro' the zenith, Z H, and Z O will be azimuth circles of 90 degrees each; they are graduated both ways, to shew the altitude and zenith distance. H O is the horizon, the meridian's drawn thro' each 15th degree, are the hour circles, and because the sun is the same distance from the meridian of the place at any hour in the afternoon, that he is in the forenoon, at the hour which is as much before as the other is after 12; the meridian of 11 in the forenoon, will be that of 1 in the afternoon, as they are numbered in the plate. When the pole is in the zenith, the meridians will all be azimuths, and the equinoctial will be the horizon.

The observer is always in that point of the circle P Æ S Q, which is directly under Z, the points of the compass are marked on the horizon; and the sun's place in the ecliptick to every fifth day of the month, is marked on the line E K: This projection is called the analemma; when the north pole is above the horizon, the right hand quarter, *viz.* Z C O, will be eaſt, and the hours before ſix in the morning, and after ſix in the evening, will be on the right hand ſide of the meridian of ſix, which will always be repreſented by the line P S; the hours from ſix in the morning to noon, and from noon to ſix in the evening, will be in the left hand quarter, *viz.* Z C H, which is the weſt: When the ſouth pole is elevated above the horizon, that which was eaſt, will now be weſt, and the forenoon hours, will be the afternoon hours; the ſun will riſe in the left, and ſet in the right hand quarters. Tho' in this plate, the heaven's and earth coincide, yet in reality, they are at ſuch a diſtance, that the earth is accounted only a point; ſo that in all obſervations, the obſerver is ſuppoſed at the center of the earth.

Geographical Problems ſolved by the Analemma, and by Scale and Compaſſes.

P R O B. I.

Given the ſun's declination and zenith diſtance at mid-day, or meridian altitude; to find the latitude of the place.

This admits of three varieties.

C A S E I.

The latitude and declination both north, and the ſun to the northward of the obſerver, or both ſouth, and the ſun to the ſouthward of the obſerver.

Note. This can only happen to thoſe within the tropicks.

Rule. Subtract the zenith diſtance from the declination, the remainder will be the latitude.

E X A M P L E.

Declination	20° 00′ north	Latitude north.
Zenith diſtance	10° 00′ north	10° 00′

But if the declination and latitude were both ſouth, and the ſun to the ſouthward of the obſerver, the latitude would be 10 degrees ſouth.

To delineate this by ſcale and compaſſes.

Deſcribe the circle Z H N O, from Z ſet off towards O, the given zenith diſtance 10° 0′ to ⊕, and from ⊕ ſet off 20° 0′ the given declination to

a,

a, then *a* Z is the latitude, by *Inf* 1. or from ⊕ set off the complement of the declination 70° 0′ to *p*, then *p* O is the latitude by *Inf*. 3. By the analemma, set 20° 0′ from Æ towards P, to 10′ 0° from Z counted towards O, then P will be against 10′ 0° from O, the required latitude.

CASE II.

The latitude and declination both north, and sun south, or both south, and sun north.

Rule. Add the declination to the zenith distance; their sum will be the latitude of the place.

EXAMPLE.

Declination	20′ 00° north	Latitude.
Zenith distance	30′ 00° south	50′ 00°

But if the declination were south, and the sun north, the latitude would be 50° 00 south.

By scale and compasses, set off the given zenith distance 30° 00′ from Z to 30 towards H, and the given declination from 30 to Æ; Z Æ will be the latitude.

By the analemma, set 20° 00′ counted from Æ towards P, to 30° 00′ counted from Z towards H; P will be against 50° 00′ accounted from O, the required latitude.

CASE III.

The latitude and declination of different names, that is the one north, and the other south.

Rule. Subtract the declination (which in this case, will always be the least) from the zenith distance; the remainder will be the latitude.

EXAMPLE.

Declination	15° 00′ south	Latitude.
Zenith distance	65 00 south	50° 00′

But if the declination was north, the latitude would be south.

By scale and compasses, describe the circle Z H N O, from Z set off the given zenith distance 65 degrees towards H, if in the north latitude, but towards O, if in south latitude, from 65, set off the given declination 15 to Æ; then Æ Z is the required latitude.

By

By the analemma, set 50° 00′ counted from Æ towards S, to 65° 00′ from Z towards H, and P will be against 50° the latitude.

It is by this problem, that the latitude is found at sea, when the sun's zenith distance is taken at noon; for which we may take the following general rule. If the sun and the equinoctial be both on the same side of the observer, consider which is nearest the zenith; and if it be the sun, the declination and zenith distance must be added; but if it be the equinoctial, the declination must be subtracted from the zenith distance,

Those that live within the tropicks, will sometimes have the sun on one side, and the equinoctial on the other side of their zenith, and then the zenith distance must be subtracted from the declination.

Those that are so remote from the equinoctial, that the sun performs the diurnal revolution above the horizon, will have him twice in their meridian every 24 hours: In this case, instead of the declination, take the complement of it, or the sun's distance from the pole; when the observation is at 12 at night, or more properly when the sun is nearest the horizon, the latitude will be found in the same manner, as if the altitude of a star was taken, which was sufficiently explained in the last section: But when the observation is made at noon, it will be the same as in the second case of this problem.

PROB. II.

Given the latitude and declination, to find the sun's amplitude, and the hour of his rising or setting.

CASE I.

In order to solve this, it must be observed, that the sun by his apparent diurnal motion, describes circles nearly parallel to the equinoctial, which may be called parallels of declination, being the same as the parallels of latitude betwixt the two tropicks. The sun performs his revolution in 24 hours thro' all the meridians, and when in the meridian of the place, it is either mid-day, or mid-night; in the analemma, this meridian is that which limits the projection; the sun will be all that day somewhere in the parallel of declination, and if a meridian be drawn thro' the point where the parallel of declination intersects the horizon, it will give the hour of his rising or setting, and the degrees of the horizon intercepted between the sun, and east or west point of the horizon, is the sun's amplitude.

This

This being premised, the following general rule will solve all the varieties of this problem by the analemma.

Rule. Set the pole to the given latitude, and we have the amplitudes, and hour of the sun's rising and setting, for any day in the year in that latitude, by inspection.

For the day of the month being given, the declination may be found by the table, or by looking for the sun's place in the ecliptick, on the analemma; the degree of the horizon cut by the parallel of declination is the required amplitude, and the meridian passing thro' the sun, when in the horizon, gives the hour.

By scale and compasses, describe the circle Z H N O as before. Let the given latitude be 50° 00′ north, and declination 23° 30′ north. Set off by the line of chords 50° 00′ from O to P, and draw the line P S, and at right angles to it, the line Æ Q, which will represent the equinoctial; draw the parallel of latitude 23° 30′, to intersect the horizon in *x*; thro' *x* draw a meridian, to intersect the equinoctial in *y*. C*x* is the amplitude measured on the line of sines, and C*y* measured on the same line, gives the degrees and minutes of the equinoctial before or after six, which converted into time, gives the hour of the sun's rising or setting.

EXAMPLE.

In 50° 00′ north latitude.

Decl. north.		Amp.		Sun rise.		Sun set.	
Deg.		Deg.	M.	H.	M.	H.	M.
5		7	48	5	36	6	24
10		15	40	5	11	6	49
15		23	45	4	46	7	14
20		32	08	4	17	7	43
33	30	38	20	3	55	8	05

The amplitudes will be the same in south declination, but the hours of the sun's rising, will be the same with the hours of his setting, when the declination was north.

PROB. III.

Given the latitude, sun's altitude and declination, to find the azimuth and hour.

By the analemma, set the pole to the latitude, lay a ruler, or strait slip of paper, across the given altitude; this will intersect the given parallel of declination. The meridian passing thro' this point of intersection, gives

gives the hour; at this point make a mark with a pencil upon the ruler, then move the pole to the zenith, so as not to move the ruler; the meredian that passes thro' the pencil mark, will be the azimuth circle passing thro' the sun, and it will cut the horizon in the degrees and minutes of the required azimuth. It will be proper to take the distance with a pair of compasses, between the sun, and the line T O, or T H; in case, the ruler should slip by moving the pole to the zenith.

By scale and compasses; let the given latitude be 50° 0′, declination 23° 30′ both north, and altitude 20° 0′. Describe the circle Z H N O, and draw the equinoctial parallel of declination, and axis as before. Draw also the parallel of altitude *a l t q* by setting off 20° 0′ of the line of chords from O to *q*, and from H to *a*; this will intersect the parallel of declination in *t*, thro' which draw an azimuth circle, and a meridian, and these two will give the things required. There will be no occasion to draw the whole circles, only to find the intersection of the azimuth circle with the horizon, and of the meridian with the equinoctial, which is only dividing the horizon in the same proportion with the parallel of altitude; and the equinoctial in the same proportion with the parallel of declination: Thus, for the azimuth, draw the radius C *q*, and C *l q*, will be a right angled triangle; thro' *t* draw *t f* parallel to C *l*; transfer C *f* to the horizon in *s*, and draw *s v* parallel to C N; so *v* O will be the azimuth. Or it may be done thus, with *l q*, the radius of the parallel of altitude, describe from the center C, an arch, *i g* produce the line *t f* to *g*; thro' *g* draw a line from C, which will give the point *v* as before. For the hour, draw the line *t* 1 parallel to P S, to intersect C E in 1, transfer C 1 to the equinoctial in 2; so shall C 2, measured on the line of sines, and converted into time, give the hour after 6.

Those are the problems that are absolutely necessary to be known at sea; the first for finding the latitude, the last two for finding the variation of the compass, of which in its proper place.

PROB. IV.

Given the latitude and longitude of two places; to find their distance and angle of position.

The nearest distance between any two places will be in a plane passing thro' the two places, and the center of the earth, which upon the surface of the earth, will be an arch of a great circle; and the angle which this circle makes with the meridian of each place, is the angle of position, which will not be the same in both places.

By the analemma. Let the two places be A and B; A in 50° 0′, B in 13° 30′, both north latitude; their difference of longitude 52° 58′; B the weftmoft: Firft fet the pole to the latitude of A, then look for 37° 2′ (the complement of the difference of longitude) upon the equinoctial; the meridian paffing thro' that point, will interfect the parallel of latitude of 13° 30′ in the point B; thro' B lay a ruler acrofs parallel to the horizon; its interfection with the graduated meridian, will give the diftance of B from A, counting the degrees from the zenith, and will be about 56° 15′.

For the angle of pofition from A to B. The ruler being laid acrofs as before, mark the point B upon it; move the pole to the zenith, the meridian paffing thro' the point, will be the azimuth circle required; but if it were required to find the angle of pofition from B to A, fet the pole to the latitude of B, and proceed as before.

By fcale and compaffes defcribe the circle, and quarter it as before. Draw the axis and equinoctial to the latitude of A, alfo the parallel of latitude of 13° 30′: Now to find the point B in this parallel, it is only to draw a meridian, which fhall make an angle of 52° 58′, with the meridian of A. To do this by the line of chords, fet off 52° 58′ from Æ both ways to *d* and *d*, from which two points, lay a ruler acrofs to interfect the equinoctial in *b*, which will be the point in the equinoctial, thro' which the meridian of B muft pafs; or by the line of fines, take the complement of the difference of longitude 37° 2′, which fet off from the center C, and this will give the point *b*; thro' which draw a meridian as before directed, to interfect the parallel of latitude of 13° 30′ in the point B, and thro' this point draw L T, parallel to the horizon; Z L or Z T, meafured on the line of chords, will give the diftance nearly 56° 15′: Draw alfo the azimuth circle Z B G; C G meafured on the line of fines, nearly 21 degrees, will give the complement of the angle of pofition with the meridian of A; or thro' G, draw a line parallel to Z C N, to interfect the circle in D and F; H D or H F, meafured on the line of chords, will be the angle of pofition, that the meridian of A makes with the great circle paffing thro' A and B, and will be nearly 69 degrees; but to find the angle it makes with the meridian of B, we muft draw the equinoctial and axis to the latitude of B, and proceed as before.

This problem can be of little ufe to the mariner, becaufe it gives the neareft diftance betwixt two places, and the compafs, which is the only guide he has to conduct him from one port to another, leads him out of the direct road; for the path which the fhip defcribes, will be a curve, making equal angles with all the meridians fhe croffes; whereas the arch of the great circle which is the neareft diftance, makes unequal angles with all the meridians; and therefore in practice, we cannot go the neareft

way,

way, by the compass, unless the places lie under the same meridian, or under the equinoctal.

The compass is so well known that it needs no description, being only a card, upon which a circle is described; it is divided into 32 equal parts called points, each consisting of 11° 15′; these are subdivided into quarters, without which there is another circle divided into 360 degrees; a needle, touch'd with the load-stone, by which it acquires that surprising quality of pointing to the north pole, is fixed under the card, upon which there is a flour de luce, which will point out the meridian; for though it sometimes varies from it, we may discover the quantity by celestial observation of which in another place: The lines that the ship forms in steering by the compass, are called rumbs; and that these make equal angles with all the meridians is evident, because the needle always lies in the meridian of the place, and the rumbs all meet at the center of the circle upon the card. The needle is the diameter of this circle, so that moving the card can never alter any angles, that are made by the intersection of any two lines upon it.

It will be very difficult, if not impossible, to describe the rumb lines, either upon the globe, or upon any plane, where the meridians intersect in the pole, for they must be so described, that if a ship sail upon any rumb to any port, and then sail back the same distance upon the opposite rumb, she will then return back to the same port from which she sail'd; and that this is really true, where there is no variation or currents, or the ship is not forced out of her direct course by the sea, wind, or bad steerage, must be allowed; and agreeable to the nature of the triangles formed by the rumb lines, meridians, and parallels of latitude upon the globe; tho' strictly speaking, they cannot be called triangles, for the mathematicians have reduced all triangles either to plane or spherick, but the properties of neither agree to these. However as they have three sides, and three angles, a small distance may be allowed to be a strait line; we shall therefore consider them as right angled triangles.

Let there be two ports A and B, A in 50° 0′, B in 13° 30′, and the difference of longitude 52° 58′ as before, that is the distance of the two meridians upon the equinoctial in miles, will be 3178; but in the parallel of latitude of B, the distance of the meridians will be 3090 miles, and in the parallel of A 2043 miles; the difference of latitude betwixt A and B, is 36° 30′, or 2190 miles.

Now if a ship sail directly south 2190 miles from A, and then 3090 directly west, she will certainly arrive at B; and if she sails back again the same distance upon the opposite rumbs, that is first 3090 miles east,

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and then 2190 miles north, ſhe will certainly arrive at A; this would make the whole diſtance ſailed 5280 miles. But if in ſailing from A, ſhe firſt ſails 2043 miles weſt, and then 2190 miles ſouth, ſhe would likewiſe arrive at B; and by ſailing the ſame diſtances upon the oppoſite rumbs, ſhe would return back to A. Her diſtance ſailed would be 4233 miles, but the neareſt diſtance is 3376; ſuppoſing it poſſible to ſteer upon the arch of a great circle. The direct courſe by the compaſs from A to B, will be ſouth 50° 6′ weſt, and the diſtance to be ſailed in that courſe, will be 3414 miles, as ſhall be ſhewn in *Mercator*'s ſailing; what we are here to prove is, that if a ſhip ſails from B, north 50° 6′ eaſt 3414 miles, ſhe will arrive at A. The only difficulty will be to reconcile this to the properties of a right angled triangle; for in ſailing from B to A, the diſtance 3414, is the hypothenuſe; the difference of latitude 2190, is the perpendicular; and the meridian diſtance in the parallel of latitude of A, *viz.* 2043 will be the baſe. Now in ſailing back from A to B, upon the oppoſite rumb, the diſtance, difference of latitude, and angle, will be the ſame as before, from whence it may be argued, that the baſe will be the ſame as before, *viz.* 2043; ſo that when the ſhip arrives in the parallel of latitude of B, ſhe will be 1047 miles to the eaſtward of it.

But it muſt be conſidered that the whole difference of longitude to be run down from B to A, is 52° 58′, which cannot be done by ſailing only 2043 eaſt, unleſs it be all in the parallel of A. Now in ſailing directly upon one rumb, there is no eaſting made in the parallel of A, and of conſequence, all the eaſting that is neceſſary to be made, in order to run down the logitude, muſt be made before ſhe arrives at the parallel of latitude of A; and the nearer to B the eaſting is made, the more it will require to run down the longitude: Again, neither the whole eaſting, nor ſtrictly ſpeaking, any part of it, can be ſaid to be run down in any parallel of latitude betwixt B and A; for if ſhe ſail from B, in the direct courſe towards A any aſſignable diſtance, ſhe muſt make a ſmall part of it northing, and a ſmall part of it eaſting; and if ſhe ſail back upon the oppoſite rumb, ſhe will make the ſame weſtings, betwixt the ſame parallels ſhe made the eaſtings in; and becauſe the ſame eaſtings, are made in the ſame parallels of latitude that the weſtings are made; the difference of longitude in both, will be the ſame; and the ſhip in ſailing back upon the oppoſite rumb, will certainly arrive at B.

Or let us ſuppoſe 2190 parallels of latitude actually deſcribed betwixt A and B, likewiſe 2190 meridians. Now in ſailing from B to A, inſtead of ſailing the direct courſe, let us ſail firſt directly eaſt till we come to the firſt of theſe meridians; then due north again to the firſt parallel, and due eaſt

east to the second meridian; and so proceed sailing, first north, and then east, till the last north course brings us to the parallel of A, and then an east course will bring us to A. By this means she forms 2190 small right angled triangles; the perpendiculars will all be equal, *viz.* one mile each, but the bases will all be unequal, still decreasing the nearer we come to A. In sailing back, if we go upon the opposite rumbs, we must first sail as much west in the parallel of A, as we did east before; and then south to the next parallel, in which we must sail as much west as we did east before to come at the next meridian, and so proceed sailing first south and then west, till we come to B; where it must be observed, that in every parallel of latitude, we make as much easting as we did westing, which will infallibly bring us back to the same port.

Now let us suppose an infinite number of parallels of latitude, and an infinite number of meridians; and let us steer an infinite number of courses, first north, and then east, we shall have an infinite number of small triangles, and their perpendiculars and bases being infinitely small, both may be said to vanish, and leave nothing but the hypothenuse, which upon the globe, is represented by a curve, making equal angles with all the meridians, being the path a ship describes, which is led by the direction of the compass from B to A; and in sailing back from A to B, the triangles will be the same as before, and so quite vanish, leaving only the distance, which will certainly bring the ship back to B.

We shall not here examine how these rumb lines are described upon the globe, for as we observed before, though in theory they may be conceived to be drawn, yet it will be scarce possible to draw them true, with inclin'd meridians; this shews the necessity of a projection of the sphere upon a plane, where all the meridians may be parallel to each other; in this case the rumbs will all be strait lines. How to construct such a projection, shall be the subject of the next chapter.

We shall only here remark that in navigation, the departure, and meridian distance, may be considered as one thing, *viz.* The whole easting, or westing that a ship must make in steering upon a direct course from one port to another; and this will always be equal to the sum of all the departures made every 24 hours, but will not agree with the common definition of departure and meridian distance, as in *Def.* 10. for by that, we can assign no proper departure betwixt any two places, that are not in the same parallel of latitude; for let the two places be A and B, the distance betwixt their meridians, in the parallel of A will be 2043 miles, and in the parallel of B 3090 so that neither of these can properly be called the meridian distance, or departure betwixt these two places; but if we sail

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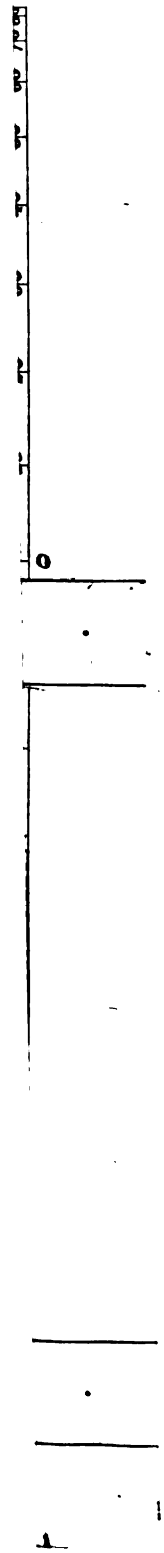
in a direct courſe, *viz.* ſouth 50° 6′ weſt from A 3414 miles; we ſhall make 2619 weſting when we arrive at A, which we ſhall call the departure, or meridian diſtance betwixt A and B, and in ſailing back from B, the courſe will be north 50° 6′ eaſt; the diſtance 3414, and the eaſting 2619, the ſame as the weſting before.

From this we may infer that the ſame courſe and diſtance, whether we ſteer from or towards the equator, will always give the ſame difference of latitude and departure; which is the reaſon that ſeveral experienced artiſts keep their account by their meridian diſtance, without regarding their longitude; for we muſt not ſuppoſe, as ſome have alledged, that they are ignorant of the cauſe why they make more weſting in going towards the equator, than they make eaſting in returning back, for they well know that (outward bound) they get into the parallel of their port, long before they run down their difference of longitude, which therefore muſt require more departure than in coming home; becauſe that it is poſſible they may have 6 or 8 degrees of longitude to run down in the parallel of 13° 30′ when ſailing to B; and the ſame number of degrees of longitude to run down in the parallel of 50° 0′, when ſailing from B to A. So they do not return near the oppoſite rumb to that on which they ſailed out; but as they conſtantly ſteer the ſame courſes out every voyage, as near as the wind will permit, in the courſe of many voyages, they find the meridian diſtance nearly the ſame every voyage outward bound; and if in ſeveral voyages they can ſteer the ſame courſes home, that they ſteered home the preceding voyages, they will find their departures nearly the ſame as in their former voyages, tho' always leſs than the departures out, for the reaſons before aſſigned: But it happens ſo rarely, that they can keep the ſame courſes two voyages; that very few have any regard to the meridian diſtance, but keep their account by the difference of longitude, which will always be the ſame out and home, whatever courſes they ſteer.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CHAP. III.

Of PLAIN SAILING.

SECT. I.

The CONSTRUCTION and USE of SEA CHARTS.

The LONG-LINE, and HALF MINUTE GLASS.

PLAIN sailing is that where the plain chart is uſed, all the various caſes of which will be exactly the ſame with thoſe of right angled triangles; ſo that we are only to ſhew how thoſe triangles are formed upon the chart; and the names of the ſides.

In this chart the meridians are ſtrait lines drawn parallel to one another; the equinoctial, and all the parallels of latitude are likewiſe ſtrait lines parallel to one another; by this means the rumbs will be ſtrait lines, and make equal angles with all the meridians, but then all the parallels of latitude will be equal to the equinoctial, which is a very great error, at any conſiderable diſtance from the equinoctial: it is called the plain chart, and conſtructed in the ſame manner as that of ſurveying land in *Plate* 10. All the lines drawn parallel to A B, may be called meridians, and thoſe parallel to B E, parallels of latitude; which being all equal to the equinoctial, will make the departure and difference of longitude the very ſame thing, though in the parallel of latitude of 60 degrees, it is only one half of the difference of longitude; and if nearer the pole, the error will ſtill be greater, ſo that this chart muſt be very erroneous.

The only true ſea chart is *Mercator*'s, which retaining the paralleliſm of the meridians, will occaſion the degrees of longitude in any parallel, to be equal to the ſame degrees in the equinoctial, as in the plain chart; but to remedy this, the degrees of the meridian in this chart, are enlarged in the ſame proportion that the degrees of the parallels of latitude are. Before we ſhew the conſtruction of this chart, we ſhall ſhew all the uſes of the plain chart; for as there are two ſea charts, from hence ariſes the diviſion of navigation in two parts, *viz.* plain, and *Mercator*'s ſailing; we ſhall here treat of the firſt.

Every place is ſuppoſed to have a meridian, and parallel of latitude, and if actually drawn, and likewiſe a ſtrait line from any place to another, that differ both in latitude and longitude; we ſhall then have a right angled

angled triangle, whereof the meridian will be the perpendicular, the departure the base; and the distance the hypothenuse: Let the two places be A and Z, A *a* or Z *z*, will be the perpendicular; *a* Z or A *z*, the base; and A Z, the distance upon the rumb line; and that A *a* is the difference of latitude, and *a* Z the departure; betwixt A and Z, is evident by the definition of those terms: The angle that is formed by the rumb line, and the meridian, is called the course, which corresponds to the angle opposite to the base in trigonometry.

As in trigonometry there are six cases, so there are the same in plain sailing, and the solutions the same in both, only changing the names of the parts. The hypothenuse, is called the distance; the perpendicular is called the difference of latitude; the base is called the departure; and the angle opposite to the base, is called the course.

Now here, as in trigonometry, there are four things, any two of which being given, the other two may be found.

The mariner has two things given, *viz.* the course and distance; the former by the compass, and the latter by the log-line and half minute glass.

To the end of this line there is fastened a piece of wood, with as much lead at the lower end as will serve to make it swim upright in the water: It is divided into knots, the distance betwixt the knots must be exactly the 120th part of a mile; and there must be a sufficient quantity of line betwixt the log, and the mark from which the line begins to be dived-ed; so that when the mark is at the ship's stern, the log may be clear of the eddy of the ship; and then the half minute glass is turned, and the line vereed of the reel, which is stopped as soon as the glass is out: this will give the exact distance the ship has sailed in the half minute; and if she continues at the same rate for a whole hour, her distance run in the hour is also known; for she will go as many miles in the hour, as there are knots run out in the half minute; this is the only method they have to measure the distance, and may be liable to great errors unless the line be very carefully divided, and the glass actually half a minute.

SECT.

SECT. II.

The Resolution of the six Cases of PLAIN SAILING.

THE most expeditious way, and which is always used in practice, is by the table of difference of latitude and departure.

This table gives, by inspection, the difference of latitude, and departure, to any course, for any distance less than 100 miles.

In the uppermost rank, are placed the courses from 1 degree to 45, including points and quarter points; and in the lower rank, the courses from 45 to 90; each course is divided into two columns; under lat. is the difference of latitude; and under dep. is the departure; and under dist. is the distance corresponding to them; but when the course is more than 45 degrees, it will be found in the lower rank; and the difference of latitude over lat. and the departure over dep.

We shall work an example in each case, by the logarithms, by scale and compasses, and by these tables which will sufficiently explain their nature: We shall likewise shew how to delineate them by the line of rumbs.

CASE I.

Given the course and distance, to find the difference of latitude and departure.

EXAMPLE.

A ship sails S W b W 60 miles. I demand the latitude come to, and departure.

This is exactly the same with the first case of trigonometry, and the triangle delineated in the same manner, only we make use of the line of rumbs to set off the angle by.

The line of rumbs is constructed in the same manner as the line of chords, from a quarter of a circle, divided into 8 equal parts, for points of the compass, and those sub-divided into quarters; the use of this line is to set off the course, which is always given in points of the compass, and not in degrees; as in this example the course is five points from the meridian; therefore upon the line R T, from the point R, describe an arch with the chord of 60 degrees, upon which set off five points taken from

from the line of rumbs; this might be taken off the line of chords, tho' not so exactly, because of the odd minutes; being 56° 15', so that in all the cases in navigation, we shall make use of the line of rumbs, observing that the arch must be described by the chord of 60 degrees, to which that line of rumbs is adapted. We shall give no further directions for delineating the following examples, only refer to the like cases in trigonometry. In order to work them by the logarithms, the points must be turned into degrees, for which there is a table to find them by inspection; 5 points is 56° 15', then it will be,

As the radius or sine of 90	- -	10.000000
Is to the distance 60 miles	- -	1.778151
So is the sine of the course 56° 15'	-	9.919846
To the departure 49.9 miles	- -	+1.697997

As radius	- - -	10.000000
Is to the distance 60 miles	- -	1.778151
So is S. C. of the course 33° 45'	-	9.744739
To difference of latitude 33.3 miles	-	+1.522880

By the table of difference of latitude and departure, the course is more than four points, therefore it will be found in the bottom, that is 5 points; and in the column over lat. is 33.3, for the difference of latitude; and 49.9 in the column over dep. for the departure, both right against 60 in the column dist.

By extending on *Gunter's* scale, if the course be turned into degrees, it will be just the same as was in right angled triangles.

But there are two lines on *Gunter's* scale by which it may be done, one is marked S R, and the other T R; that is the sines and tangents of the rumbs; and as the sine of 90 degrees, and the tangent of 45 degrees are equal to the radius, so 8 points is for the radius on the line sine rumb, and 4 points for the radius on the line tangent rumb.

For the departure R : S 5 points : : 60 : 50 nearly.
For the difference of R : S 3 points : : 60 : 33.3.

The extent from 8 points to 5 points on the line S R, will be the same as from 90 to 56° 15' on the logarithmick line of sines, and will reach from 60 to 50, on the line of numbers for the departure; the extent from 8 points to 3 points, will be the same as from 90 to 33° 45' on the logarithmick sines, and will reach from 60 to 33.3 on the line of numbers for the difference of latitude.

CASE

CASE II.

Given course and difference of latitude, to find the distance and departure.

EXAMPLE.

A ship sails S E b S, till she alters her latitude 50 miles; I demand her distance and departure.

This is exactly the same triangle with the preceding, but what was the difference latitude before, is now the departure; the proportion will be,

As sine comp. of the course, *viz.* 56° 15′	9.919846	
Is to the difference of latitude 50 miles	1.697997	
So is the radius or sine of 90	10.000000	
To the distance 60 miles	1.778151	
As sine comp. of course or of 56° 15′	9.919846	
Is to the difference of latitude 50 miles	1.697997	11.442736
So is the sine of the course 33° 45′	9.744739	9.919846
To the departure 33.3 miles	1.522890	1.522890

By the table of difference of latitude and departure, the course is now three points, find it in the upper rank, and under lat. find 50 the given difference of latitude; corresponding thereto under dif. is 60, and under dep. is 33.3.

By *Gunter*'s scale, S of 5 points : sine of 8 points : : 50 : 60; the extent from 5 points to 8 points, on the line S R, will reach from 50 to 60 on the line of numbers and gives the distance.

For the departure S of 5 points : sine 3 points : : 50 : 53.3.

CASE III.

Given course and departure to find the difference of latitude and distance.

EXAMPLE.

A ship sails N W by W till she gets 50 miles to the westward; I demand the distance and difference of latitude; this is a case that can scarce ever happen, and is the same with the preceding, only calling what was difference of latitude in the former, in this the departure, and what was departure in the former, is now difference of latitude; and then the operations will, in all repects, be the same as before.

CASE IV.

Given the distance and difference of latitude; I demand the course and departure.

EXAMPLE.

A ship sails betwixt the south and west 60 miles, and finds by a good observation, she has made 50 miles southing; I demand the course and departure, delineate the triangle as in *Case* 4. right angled triangles; the proportion is

As the distance 60 miles - - -	1.778151
Is to difference of latitude 50 miles - -	1.697997
So is radius or sine of 90 - - -	10.000000
	11.697997
To the S. C. of the course 56° 15′	9.919846

So the course will be S. W. b S. the departure will be found by *Case* 1. or 2.

By the table of difference of latitude and departure, turn over till you find 50 in the difference of latitude column, and 60 in the distance column; and because I find 50 in the column that has lat. at the top, the course will be there also, *viz.* 3 points, that is S. W. b S. and in the departure column I find 33.3 westing.

By *Gunter*'s scale; the extent from 60 to 50, on the line of numbers, will reach from 8 points to 5 points on the line S R.

CASE V.

Given distance and departure, to find the course aud difference of latitude.

EXAMPLE.

A ship sails betwixt the north and east 60 miles, till her departure is 50 miles; I demand her course and difference of latitude. This is exactly the same with the preceding, and scarce can happen, so we shall proceed to the next.

CASE. VI.

Given difference of latitude and departure, to find the course and distance.

EXAMPLE.

Two islands P and T, T in 32° 41′, and P in 20° 53′ both north latitude;

titude; P lieth to the weſtward of T 6° 45′. I demand their bearing and diſtance, or which is the ſame thing, what courſe muſt be ſteer'd from P to T, and how many leagues diſtant; delineate the triangle as in *Caſe* 6· trigonometry, and find the difference of latitude by ſubtraction, which turn to leagues off 20 to one degree, as in the operation.

A	32	41		Departure.
B	20	53		6 45 by 3 15
	11	48		20
	20			120
	220 leagues			15
	16 for 48 miles			135 departure
	236 diffe. of lati. in leagues			

The proportion is,

As the difference of latitnde 236 miles	2.372912
Is to the departure 135 miles	2.13.334
	10.00000
So is the radius or tangent of 45	12.130334
To the tangent of the courſe 29° 46′	9.757422

By the table of difference of latitude and departure, theſe numbers are too large to be found in the table, therefore take the quarter of each, ſo 59 will be for the difference of latitude, and 33 ¾, the departure; find theſe in their proper columns, and correſponding thereto in the diſtance column, will be 68 or 69, which multiplied by 4, will give the whole diſtance; and becauſe the difference of latitude is greater than the departure, the courſe will be found to be near 30 degrees, and taking 68.5, which is the neareſt diſtance correſponding to the difference of latitude and departure; in the tables, the whole diſtance will be 274 leagues, and the courſe from P to T, will be S. S. W. ½ W 2 degrees weſterly.

By *Gunter*'s ſcale; the extent on the line of numbers from 236 to 135, will reach from the tangent of 45 to 30, or 60 degrees on the line of tangents: Now to know which of theſe two will be the angle, obſerve which is greateſt, the difference of latitude or departure; for if the difference of latitude be greateſt, as in this example, the angle will be leſs than 45 degrees, but if the departure be greateſt, the angle will be more than 45 degrees.

Theſe are all various caſes in plain ſailing, which being well underſtood, it will be eaſy to give a true ſolution to the following queſtions, by the

the table of difference of latitude and departure: For it will be too tedious to work them by the logarithms, or construct them geometrically; therefore in practice we always use the tables, and for the proof, let them be performed by *Gunter*'s scale.

A ship in 22° 51′ north latitude, sails N. N. $\frac{3}{4}$ W. 83 leagues the latitude come to, and departure from the meridian is required.

A ship in 0° 56′ north latitude, sails S b W $\frac{1}{2}$ W, till by a good observation she is 1.13 south latitude; her distance and departure is required.

A ship in 36° 40′ south latitude, sails betwixt the N and E, till she gets into 34° 50′ south latitude, and finds by the log, she has run 63 leagues; the course and departure is required.

Two islands A and B, A in 2° 2′ north latitude; B in 5° 16′ south latitude; B lies 250 leagues to the westward of A; the course and distance is required.

SECT. III.

Of working a Traverse, or reducing various Courses into One.

BEing now provided with a good sea chart, to find the bearing and distance of any two places, and with a compass to direct the course, also with a log line, and half minute glass, to know how far we have advanced towards our port; we need only keep an exact account of the distances, by setting down every day at noon, the number of miles sailed the preceding 24 hours, in a book provided for that purpose: Now if at any time we want to know our distance from the port we are bound to, it is only collecting the several distances we have sailed every 24 hours, and subtracting the sum from the whole distance: the remainder will be the distance we are from our port: But as it is impossible to keep a ship in her direct course, by reason of contrary winds, the intervention of lands, or various other accidents, which forces her out of the direct course; it will be absolutely necessary to keep an exact account of every particular course and distance, sailed in 24 hours; and then reduce all these different courses and distances into one: This is what is called a traverse, and is only so many different questions of the first case of plain sailing; for after we find the differences of latitude and departure, to every particular course, if they are in the same quarter of the compass, we may collect all the differences of latitude into one; which will be the whole difference

rence of latitude, from the first port sailed from, and the sum of all the departures will be the whole departure; and having the difference of latitude and departure, the course and distance is found by the last case of plain sailing. If some of the differences of latitude be northerly, and some southerly, add the several northings together, and also the several southings; if they are equal, the ship has not altered her latitude, if unequal, subtract the less from the greater, the remainder will be the whole difference of latitude; do the same by the departure when there are eastings and westings. The whole art of navigation depends upon keeping a correct and distinct account of these various courses, and reducing them to one: For which purpose it will be proper to make a table of six columns, as the following:

Course.	Distance.	North.	South.	East.	West.
S. E.	38	0.0	26.9	26.9	0.0
S. E. b E.	42	0.0	23.3	34.9	0.0
E. S. E.	25	0.0	9.6	23.1	0.0
E.	30	0.0	0.0	30.0	0.0
E. N. E.	15	5.7	0.0	13.9	0.0
N. W.	24	17.0	0.0	0.0	17.0
W. S. W.	17	0.0	6.5	0.0	15.7
		22.7	66.3	128.8	32.7
Course good.	Distance.		22.7	32.7	
S. 66° E.	106		43.6	96.1	

In the first are the several courses; in the second their corresponding distances; then by the table of difference of latitude and departure, find a difference of latitude and departure to every course and distance, and set them down in their proper columns, sum up all the northings, is 22.7, which subtract from 66.3, the sum of the southings; there remains 43.6 miles of south difference of latitude; the sum of the eastings is 128.8, from which subtract 32.7, the sum of the westings; there remains 96.1 miles east departure. As this number exceeds the limits of the table, take half of it 48; take also half the difference of latitude 21.8; the nearest I can find to these two in the tables, is 21.6 and 48.4 in the column over 66 degrees, and in the distance column is 53, which doubled, makes 106 for the whole distance.

By this way of keeping an account, we may at any time, know the course and distance to the port sailed from, which suppose in 30° 30′ north latitude, and likewise the course and distance to the port bound to, which suppose in 29° 18′ north latitude, and 72 leagues to the eastward: Let the port sailed from be A, and the port bound to B; the difference of of latitude will be 24 leagues, and the departure 72; the course will be nearly E. S. E. ½ E. about 77 leagues distant; but after steering the several courses, as in the table, the ship arrives at C. I find by the preceding calculation, the whole difference of latitude made from A to C, is 43.6 miles southing; and the whole departure 96.1 miles easting; therefore subtracting the difference of latitude made from A to C, from the whole that was to be made from A to B; the remainder will be the difference of latitude yet to be made, and subtracting the departure made, from the whole that was to be made at first setting out, the remainder will be what is yet to be made: And so having the difference of latitude and departure given, by them the course and distance is found. See the operation.

Difference of latitude from A to B	72 miles
Difference of latitude from A to C	43.6 miles.
Difference of latitude from C to B	28.4 miles is 9 ½ leagues.
Departure from A to B	216.0 miles.
Departure from A to C	96.1 miles.
Departure to be made from C to B	119.9 miles is 40 leagues.

The nearest I find to those two in the table of difference of latitnde and departure are 39.9, and 9.2; so the course will be S. 77° W. 41 leagues distant.

By this it is evident, that it is first absolutely necessary to know the whole difference of latitude and departure, that is to be made before we set out. Secondly, we must keep an exact account of the various courses and distances steered, by which we may at any time know how much of our difference of latitude is made, and consequently by subtraction, we may always tell what difference of latitude and departure we have to make; all this may be done without a chart, by a table of latitude and longitude of places.

The geometrical construction of a traverse would be too tedious for practice, but as there are several small islands, rocks, and lands, laid down in a chart, that are not in the tables of latitude and longitude; it will not be improper, every day at noon, to mark the place the ship is in upon the chart; by this means we may have a view, not only of our port, but also of any rocks or sands; this is especially necessary when we come near to the land, we shall therefore shew how this is to be done.

SECT.

SECT. IV.

To prick the PLAIN CHART. (Plate X).

THIS is to lay down the place the ſhip is in at any time, or to find the bearing and diſtance of any two places upon the chart.

Suppoſe a ſhip bound from A to O; required the courſe and diſtance. There are ſeveral compaſſes, or rumb lines upon the chart, by which the courſe may be readily found; thus, lay the ſtrait edge of a ruler ſo that it may touch the two places A and O, then take a pair of compaſſes, and placing one foot in the center of any compaſs, open the compaſſes till the other foot touches the edge of the ruler; then ſliding the compaſſes with one foot by the edge of the ruler, the other foot being perpendicular to it, will trace out the rumb line on which the ſhip muſt ſteer: Now as the rumbs are only drawn to the whole points, it may happen, as in this caſe, that the foot of the compaſſes will not fall exactly in the rumb, but we may eaſily eſtimate the quarters; ſo the courſe from A to O, will be S. E. b S. $\frac{1}{4}$ E. A C, is the whole difference of latitude; ſuppoſe 90 leagues, C O the whole departure 66 $\frac{1}{2}$ leagues. Now it is certain if we ſail 90 leagues ſouth, and then 66 $\frac{1}{2}$ leagues eaſt, we ſhall arrive at O; but I find by my account, that I have made only 14 leagues ſouthing, and 35 leagues eaſting: Now I want to know what place I am in, that I may ſteer a direct courſe for O. To find this, provide two pair of compaſſes: In one take the difference of latitude 14 leagues, which ſet off in the line A C from A to *e*; with the other pair take the departure 35, which ſet off from C to *r* in the line C F; from *r* with the difference of latitude, deſcribe an arch, and from *e*, with the departure deſcribe another arch, to cut the former in *s*, which is the point the ſhip is in at the time the calculation is made, and may be done every day at noon, or if need be at any other time. The next day at noon, after reducing the various courſes to one, I find I have made, in that time, 42 leagues ſouthing, and 28 leagues eaſting; I want to know the place the ſhip is in: If the point *s*, where the ſhip was the day before, be at the interſection of a meridian, and parallel, we may find it by the ſame method we found the point *s* the day before; if it is not, take with one pair of compaſſes, the neareſt diſtance of the point *s*, to any parallel; with another pair of compaſſes take the departure 28, with which deſcribe an arch from *s* as center; then ſlide the firſt pair of

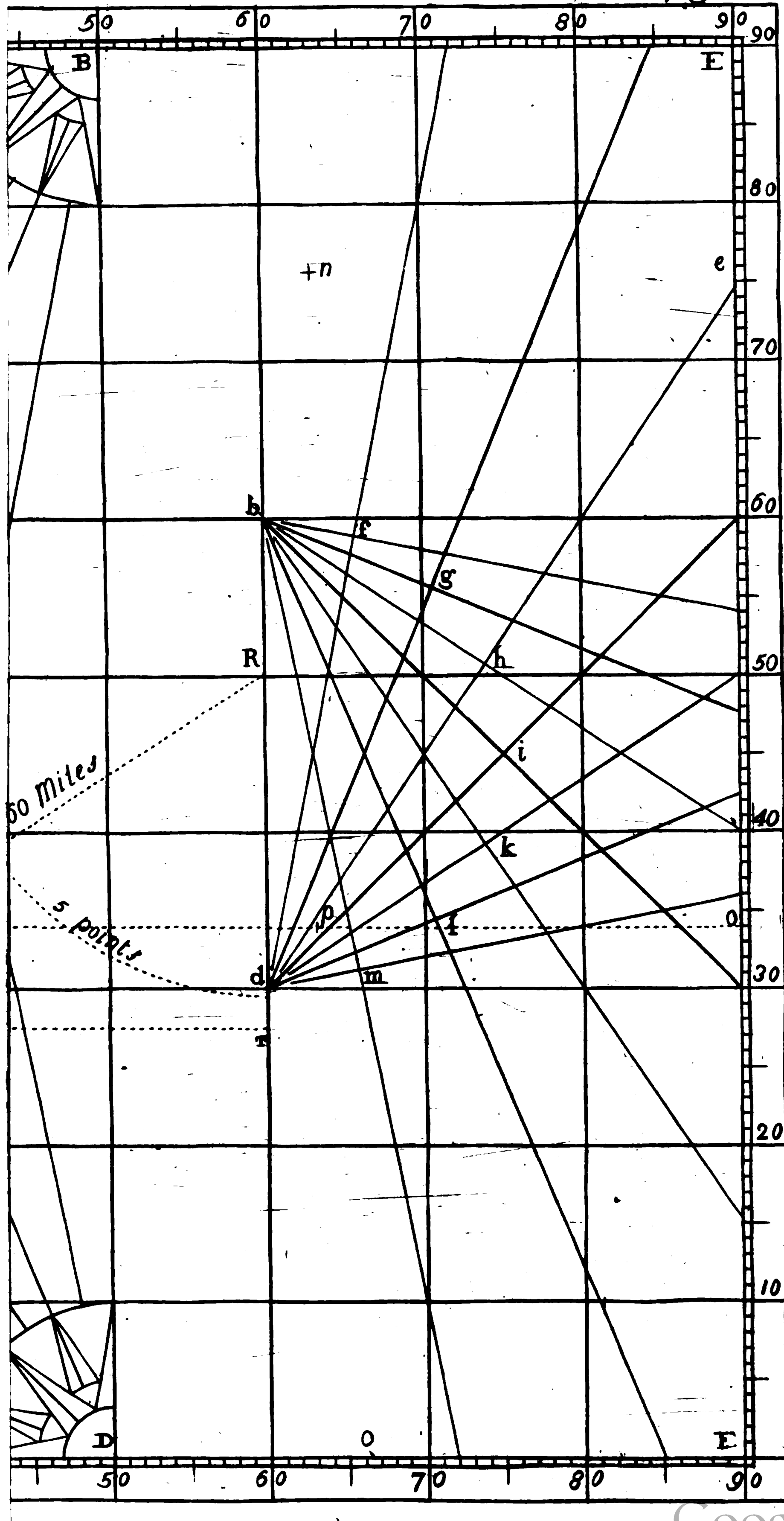
 com-

compaſſes with one foot in the parallel, and the other perpendicular to it, till it cut the arch in *n*; again with one pair of compaſſes, take the neareſt diſtance of the point *n* to a meridian, and with the other pair take the difference of latitude 42, and deſcribe an arch from the point *n* as center, and move the firſt pair upon the meridian, till it cut the arch in *p*, which is the place the ſhip is in. After the ſame manner we may find the place the ſhip is in every day at noon: When the chart is not very large, it may be done thus; let the ſhip be in *s* as before; firſt lay a ruler acroſs the chart, let the edge be parallel to ſome eaſt or weſt line, and paſſing through *s*, mark the two points *e e*, where the ruler cuts the two lines that limit the chart, then take the difference of latitude 42, and ſet it from *e* and *e* to *o* and *o*, and lay the ruler acroſs by theſe two points. It is plain the ſhip muſt be ſome where in the line *o o*; to find which, take with a pair of compaſſes, the neareſt diſtance of the point *s*, to any meridian; obſerve where that meridian cuts the edge of the ruler; from this ſet off the diſtance in the compaſſes by the edge of the ruler to *t*; then taking the eaſting 28, it will reach from *t* to *p*, which is the place the ſhip is in.

CHAP. IV.

Of Mercator, middle Latitude, and parallel Sailing.

WE have now fully explained all the varieties of ſailing by the plain chart, but it is ſubject to very great errors, for by making all the parallels of latitude equal to the equinoctial; the difference of longitude betwixt any two places will always be equal to the departure, whereas in fact the difference of longitude, may be ſometimes double, or treble, and always more than the departure, except when the two places are upon the equinoctial. To illuſtrate this, let there be two iſlands A in 32° 41′, and B in 20° 53′, both north latitude, and let their difference of longitude be 7.15, or 145 leagues, that is to ſay, if there be a meridian drawn thro' A, and one thro' B; their greateſt diſtance will be upon the equinoctial, *viz.* 145 leagues, but theſe meridians continually approach one another, till at laſt they meet in the pole, therefore their diſtance in the parallel of 20° 53′, will be leſs than 145 leagues; let it then be 135 leagues, and their diſtance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ſtance in the parallel of 32° 41′, will be ſtill leſs than their diſtance in the parallel of 20° 53′, which ſuppoſe 121 leagues; the difference of latitude from A to B, is 236 leagues, ſo if a ſhip at A ſails 236 leagues due north, and then 135 leagues due eaſt, ſhe will certainly arrive at B; again if ſhe ſails from B 236 leagues due north, and then 121 leagues due weſt, ſhe will arrive at A; but by the principles of the plain chart, ſhe muſt ſail 135 leagues, which makes an error of 14 leagues; and if the two places were more to the northward, the error would ſtill be greater.

SECT. I.

Of the Principles of MERCATOR's *Chart.*

TO remedy this error, a new chart muſt be made, in which the degrees of any parallel, ſhall have the ſame proportion to the degrees of the meridian, that they actually have upon the globe; but this muſt not be, by inclining the meridians, becauſe the rumb lines would make unequal angles with them: The meridians then muſt continue parallel to one another; this will make a degree in any parallel, equal to a degree in the equinoctial, which is a monſtrous error; for a degree in the parallel of 60, is but half a degree in the equinoctial: Therefore in this chart, the degree of the meridian that lies betwixt 59° 30′, and 60° 30′, muſt be double the degree that lies betwixt the equinoctial, and the firſt degree upon the meridian, that is ſuppoſing the parallel of latitude of one degree to be 60 miles diſtant from the equinoctial; then the diſtance betwixt the parallel of 59° 30′, and the parallel of 60° 30′ muſt be 120 miles: This will occaſion the meridian to be unequally graduated, and ſuppoſe a parallel of latitude be drawn thro' every degree of the meridian, the diſtances betwixt theſe parallels will be unequal; that betwixt the firſt and the equinoctial will be the leaſt, but it increaſeth the further the parallel is removed from the equinoctial, in the ſame proportion that the parallel is leſs than the equinoctial.

In order then to graduate the meridian, let us ſuppoſe a degree of the equinoctial, or of the meridian, which are both great circles, to be 60 miles; the parallel of 1 degree of latitude, is very near as great as the equinoctial, and ſo the firſt degree of the meridian will be 60 miles; in like manner the parallels of 2, of 3, of 4, of 5, are ſo near the equinoctial, that the diſtances betwixt them may be all equal, that is 60 miles; but then the parallels begin to decreaſe very perceptibly, and therefore the diſtance betwixt the parallel of 5 and 6 will be 61, and the diſtance betwixt the parallel of 6 and 7, will be a little more, and ſtill increaſing; ſo the only difficulty will be to graduate the meridian. The firſt thing to be done, is to calculate how many miles will make one degree in any parallel of latitude, for which take this proportion:

As the radius, is to the ſine complement of the latitude,—ſo is 60 the miles in a degree upon the equinoctial, to the number of miles that will be contained in a degree in that parallel, or which is the ſame thing.

R : S. C. latitude : : difference of longitude : departure.

DEMONSTRATION. (*Plate* 11. *Fig.* 1.)

Let Æ Z be the latitude, then will Z P be the complement; Z *z* the radius of the parallel is the ſine of the complement angle P C Z. Now the degrees of one circle are to the degrees of another, as the radius of the one is to the radius of the other, as was proved in part firſt. Therefore Æ C, the radius of the equinoctial is to Z *z*, the radius of the parallel, or ſine complement of the latitude, as a degree of the equinoctial, or difference of longitude, is to a degree of the parallel or departure.

By this calculation it will be found that a degree upon the parallel of 60 degrees, will contain but 30 miles; for the complement of 60 is 30, and the ſine of 30 degrees, is half the chord of 60 degrees, or half the radius; ſo the diameter of the parallel of 60, is half the diameter of the equinoctial.

Tho' this proportion be true, it would be a great labour to make tables by it, which Mr *Wright* has effected by another method; for he conſidering that the radius is to the ſine complement as the ſecant is to the radius, calculated a table of meridional parts, by a continual addition of the ſecants. In this chart the ſine complement is equal to the radius, becauſe the meridians are parallel to each other; ſo the degrees of the parallel are all enlarged beyond their due meaſure, and therefore the degrees of the meridian muſt be enlarged in the ſame proportion, which will be as the ſecant of the parallel is to the radius; to prove this, let C B be 60 miles, equal a degree of the equinoctial (*Plate* 11. *Fig.* 3.) with this, as radius, deſcribe a circle, and draw the parallel F E; it is plain D E would be a degree in that parallel; but upon the chart it is D G. In the triangles D E C, and A C B, the angles A C B, and D E C, are equal being alternates, to the two parallels D G, and C B; the angles at D and B, are right, therefore

fore the triangles are similar, and C E : D E : : C A : C B, but in *Mercator*'s chart D E is enlarged till it is equal to C E; therefore to keep the same proportion, C B must be enlarged till it is equal to C A, that is if C B be a degree of the meridian at the equinoctial, C A must be a degree of the meridian at the parallel F E; by this means, it will be enlarged in the same proportion that a degree of a parallel is enlarged: Now let D E be the parallel of 30°, if 60 miles be a degree of the equinoctial; then 51.96 should be a degree in this parallel, as in the following operation; but it is enlarged to 60, therefore the degree of the meridian, that lies betwixt the parallel of 29° 30′, and the parallel of 30° 30′, must be enlarged in the same proportion; that is 51.96 : 60 : : 60 : 69.28; for 60 × 60 ÷ 51.96 = 69.28. (*See the operation by the logarithms*). The complement of the parallels is 60 degrees; therefore

As radius	10.000000	60 miles	1.778151
Is to sine of 60°	9.937531		1.778151
So is 60 miles in the equator	1.778151		3.556302
To 52 miles in the par.	+1.715682	52 miles	1.715682
		69.28	1.840620

So a degree in the meridian at the parallel of 30 degrees of latitude must be 69.28 miles, and that this is equal to the secant of 30 degrees, will appear by the following operation.

As radius	10.000000
Is to the secant of 30°	10.062469
So is 60 miles	1.778151
To 69.28 miles	+1.840620

By the same manner, we may find the number of miles that will make a degree of the meridian in any latitude; but Mr *Wright* has constructed tables of meridional parts in such an excellent manner, that the meridian by them may be very easily graduated.

In this table, we find by inspection, the number of miles that any parallel of latitude is distant from the equinoctial; so the parallel of 29° 30′, will be 1854 miles from the equinoctial, and the parallel of 30° 30′ will be 1923; the difference is 69, which must therefore be a degree of the meridian at that parallel.

The manner Mr *Wright* calculated this table, was by a continual addition of the secants, which he calculated even to minutes, with the utmost care and exactness; he assumed 60 equal parts for a degree of the equinoctial, and therefore the first five degrees of the meridian would be

one

one of those equal parts each, because if the radius of a circle be 60 equal parts, the secants of 1, 2, 3, 4, 5 degrees, will be so near equal to the radius, that they may be accounted 60 each, so the sum of those secants will be 300 equal parts; these are called meridional parts, and shew the distance of the parallel of 5 degrees from the equinoctial: The sum of all the secants from the equinoctial to the parallel of 29° 30′, is 1854, which is the distance of that parallel from the equinoctial: Now if it be required to find the distance of the parallel of 30° 30′ from the equinoctial; find the secant of 30°, which will be 69, and this added to 1854, makes 1923, which exactly agrees with the meridional parts in the tables, corresponding to 30° 30′ latitude. Having thus shewn the construction of the table of meridional parts, we shall now shew its use in making *Mercator*'s chart.

SECT. II.

To make MERCATOR's *Chart.* (*Plate* 13).

THIS may be made to contain all the known parts of the whole world, which is called a general chart, or to contain only a part of it, which is called a particular chart; and as they are both projected in the same manner, we shall only shew how to make one from the latitude of 30 degrees to the latitude of 60 degrees, and to contain 44 degrees of longitude. Draw the line A B, to represent the parallel of 30 degrees, which make 2640 equal parts, being the miles in 44 degrees, upon the equinoctial, or in any parallel of latitude; then draw the perpendiculars C A and D B, to represent two meridians; and to find the parallel of 60, look for it in the table of meridional parts, from which subtract the meridional parts of the parallel of 30; the remainder will give the distance betwixt these two parallels, as in the following operation it is 2640, which set off by the same scale of equal parts,

60—4528
30—1888
2640

from A to C, and from B to D, and draw the line C D, to represent the parallel of 60 degrees, thus the chart is limited; then draw another meridian any where at pleasure, to meet the parallels A B and C D.

This

This is called the first meridian, where the longitude is supposed to begin; from which graduate the parallel into degrees both ways, and at every tenth draw a meridian, as in the draught where they are numbered 10° 20', &c. on each side of the first meridian; then graduate the first meridian into degrees; and at every tenth draw a parallel numbered 30, 40, 50, 60. Now to find the number of equal parts, or which is the same thing of equinoctial miles betwixt these parallels, proceed thus:

Lat.	Mer. parts.	Diff.
60	4528	2640
30	1888	
50	3475	1587
30	1888	
40	2623	735
30	1888	

In the first column are the degrees of latitude; in the second, the corresponding meridional parts; and in the third, the distance of each parallel from the parallel of 30 degrees. We must proceed in the same manner, to find the distance of each degree from the parallel of 30, or from one another, and where the scale will permit, the degrees may be divided into minutes; the meridians A C, and B D, may be graduated, also the parallels A B, and C D, and the places laid down according to their true latitudes and longitudes; or if the places be actually laid down their true latitudes and longitudes may be found, and also their bearings and distances by the following problems.

PROB. I.

Having the latitude and longitude of a place; to find it upon the chart.

EXAMPLE I.

I want to find the lizard, which by the tables, is in 50 degrees north latitude, and in 0 longitude.

Look on the graduated meridian for 50 degrees of latitude, where in this chart, there is actually a parallel drawn, and because the lizard has no longitude, it will be at the intersection of the first meridian with that parallel.

EXAMPLE II.

I want to find the island of *Madeira* in the chart, whose latitude is 32° 17' north, and longitude 12° 8' west.

Look on the graduated meridian for 32° 17'; then with a pair of compasses

passes take the nearest distance of that point to any parallel of latitude; find upon the graduated parallel 12° 8′, on the west side of the first meridian, and with another pair of compasses take its nearest distance from any meridian; move both compasses, keeping the foot of the first in the parallel of latitude, and the other in the meridian, till the other two feet of the compasses meet, which will be the place required; observing to move the compasses so that their points be parallel either to a meridian or parallel. Or it may be done as in the plain chart, by laying the strait edge of a ruler across the chart, parallel to some east and west line, to intersect the meridian in 32° 17′, the given latitude; then with a pair of compasses, take 12° 8′, the given longitude, which set off by the edge of the ruler, from the point where the ruler intersects the first meridian, this will give the place required.

PROB. II.

To find the latitude and longitude of any place in the chart: This is only the reverse of the former.

Let the latitude and longitude of *Madeira* be required.

With a pair of compasses take its nearest distance to any parallel of latitude, which set off from the intersection of that parallel, with the graduated meridian and it will be found to be 32° 17′.

For the longitude, take its nearest distance from any meridian, which set off from that meridian upon the graduated parallel, and it will give 12° 8′.

PROB. III.

To find the bearing and distance of any two places upon the chart.

The course is found in the same manner as upon the plain chart.

The distances in this chart cannot be measured, as on the plain chart, at once, by a scale of equal parts; for a degree of the meridian upon the globe, is at all places equal to a degree upon the equinoctial; and therefore if two places lie in the same meridian, their proper difference of latitude will be their true distance, as in the following.

EXAMPLE.

Required the distance from the *Lizard* in 50° 0′ north, and 0 longitude to an island in 32° 17′.

			Lizard	50°	0′
Lat. 50° 0′	3475	merid. parts	An island in	32	17
32° 17′	2048		Pro. dif. of lat.	17	43
Meridional difference of latitude	1427	miles		60	miles
			Distance	1063	miles

Now tho' the real diſtance betwixt theſe two parallels upon the earth be 1063 miles, yet upon the chart it is 1427; ſo in this chart the diſtances are not truly ſet down, except the places lie under the equinoctial, but there is a certain method of finding the true diſtances, by theſe laid down in the chart; which admits of four different caſes.

CASE I.

If the two places lie under the ſame meridian, then the proper difference of latitude turned into miles, is the real diſtance, as in the aforeſaid example.

CASE II.

If both places lie under the equinoctial, then the diſtance is truly meaſured upon the equinoctial.

CASE III.

To find the diſtance of two places lying in the ſame parallel of latitude.

In all *Mercator*'s charts there is a direction how this is to be found, but when the places are at any conſiderable diſtance, it will be very erroneous, as will appear in the following example.

EXAMPLE.

Let the two places be in the parallel of 50° 0', and their difference of longitude 42 degrees, which is 2520 miles, and is the real diſtance betwixt them upon the chart, when meaſured upon the equinoctial, or graduated parallel; but their true diſtance upon the parallel, is 1620 miles, which will make 42 degrees of the parallel of 50° 0', tho' it wiill take 2520 miles to make 42 degrees upon the equinoctial, for R : ſine of 40 : : 2520 : 1620; ſo the true diſtance in the parallel is 27 equinoctial degrees.

The chart directs to take the diſtance betwixt the two places with a pair of compaſſes, and apply it to the graduated meridian, in ſuch a manner, that one foot may be as many degrees above, as the other is below the parallel; the degrees intercepted between the feet of the compaſſes, allowing 60 miles to one degree, will, according to their direction be the true diſtance in the parallel; ſo that 2520 miles taken upon the equinoctial, which is the real diſtance upon the chart, ſhould reach in the graduated meridian from 36° 30', to 63° 30', the one being as much

 above

above, as the other is below the parallel of 50° 0'; for the true distance in the parallel is 27 degrees, the half of which, 13° 30', being added to 50, gives 63° 30', and subtracted from 50, gives 36° 30'; but the distance betwixt these parallels upon the chart is 2617 equinoctial miles, as by the operation, which will occasion an error of 997 miles.

63° 30'	4972 meridional parts
36° 30'	2355
	2617 difference.

To remedy this, instead of the former take the following method.

Rule. Open the compasses till one foot is half a degree upon the meridian below, and the other foot half a degree above the parallel of latitude; count how many times that extent is contained betwixt the two places, which will give the number of equinoctial degrees betwixt them, and multiplied by 60, it will give the true distance in miles betwixt these two places upon the earth, if taken in the parallel; which tho' not the nearest, as was before observed, yet is the shortest that can be made by steering upon one point of the compass: In the following operation we shall see how near this comes to the truth. A degree upon the graduated meridian at the parallel of 50° 0', should be equal to the secant of 50 degrees, supposing the radius to be 60 miles; then R : sec. 50° :: 60 : 93.34.

	Logarithm.	93.34
		27
Sec. 50°	10.191932	65338
60 miles	1.778151	18668
93.34 miles	1.970083	2520.18

CASE IV.

When the two places differ both in latitude and longitude, to find their distance.

Rule. 1st. Lay the edge of a ruler to touch the two places.

2d. Take their proper diff. of latitude, with a pair of compasses, upon the equinoctial, apply this distance to the edge of the ruler, so that when one foot is placed close to the ruler, the other foot may just touch some east and west line, crossed by that edge of the ruler; and there stay the compasses, the distance by the ruler's edge, from the place where the compasses rested; to that place where the ruler crosseth the aforesaid east and west line measured on the equinoctial, gives the true distance.

EX-

EXAMPLE. (*Plate* XIII.)

Let the two places be the *Lizard* and *Madeira*.

Lizard	50°	0′
Madeira	32°	17′
Difference of latitude	17°	43′

The ruler being laid upon the two places, take the difference of latitude 17° 43′ from the equinoctial, and when applied to the ruler as before directed, one foot of the compasses will be in the point *b*, when the other will just touch the parallel of 50 in the point *c*; so the distance from *b* to the *Lizard*, measured on the equinoctial, will be near 20 degrees, is 400 leagues, the true distance from the *Lizard* to *Madeira*; and the course S. S. W. almost half W; the distance would be the same, if the difference of latitude were taken from the point L, to touch the parallel of 60 in H; for then L M would be the distance.

It must be observed that by the distance is meant, that which is made on the rumb line from one place to another, which will not be the nearest.

PROB. IV. (*Plate* XIII).

The course and distance given, also the latitude sailed from, to find the difference of latitude, departure, and difference of longitude.

The difference of latitude and departure are found exactly as in the plain chart, which we omitted there, judging it properer to insert it here.

Let the place sailed from be A, in the latitude of 30° 0′ north, and the course N. E. b N. 192 leagues.

Rule. First thro' A, draw a meridian and parallel of latitude. Secondly, With the chord of 60 degrees, descaibe an arch, on which set off three points, the given course; and draw the line A H, which make 192 leagues, the given distance. Thirdly, From H let fall to the line A C, the perpendicular H F. A F measured on the equinoctial, or graduated parallel, will be 160 leagues, the difference of latitude; and H F measured on the same, will be nearly 107 leagues, the departure.

This may be done without delineating the triangle thus: First, lay a ruler thro' the given place A, parallel to some N. E. b. N line. Secondly, Lay off 192 leagues by the edge of the ruler from A to H. Thirdly, Thro' H lay the ruler parallel to an east and west line to intersect the graduated meridian; the distance from the ruler to 30° in the graduated meridian, will be the difference of latitude, observing to measure it upon

 the

the equinoctial, the distance must likewise be set off from A to H, in equinoctial leagues. To find the departure. First, With a pair of compasses take the nearest distance of the point A, to any meridian. Secondly, set off that distance by the edge of the ruler, (now passing thro' H, and parallel to an east and west line); from the point where the ruler intersects that meridian; the distance of that point from H, measured on the equinoctial, will be the departure.

To find the difference of longitude, it will necessary again to remark, that if a ship sail any determinate distance upon any particular rumb, the difference of latitude and departure, will always be the same in whatever latitude she is in; for if the place sailed from were in the parallel of 38°, or 46°, the course and distance the same as before; the difference of latitude would still be 160, and departure 107, as in the triangle K L M, or O P R.

Now tho' she alters her latitude equally in both places, it will not be so in respect of the longitude.

By the plain chart, when she sails from 30°, the point H at which she arrives, will be in 38°; and when she sails from 38°, the point M at which she arrives, will be in the parallel of 46° But in *Mercator*'s chart, the point H is in the latitude 36° 41', and the point M in the latitude of 44° 0'; so that neither of these points is the true place upon *Mercator*'s chart that the ship arrives at. In order to find the true place, produce the line A H, to intersect the parallel of 38° in the point K, produce also the line A F to G; K will be the place come to; K G, the difference of longitude; H F, the departure; A F, the proper difference of latitude; A G, the meridian difference of latitude: But tho' K be the place the ship is in, A K is not the true distance, but it may be found by the preceding problem: When she sails from the parallel of 38°, the true place O, at which she arrives, is found by producing the line K M to O, in the parallel of 46° 0': Produce also the line K L to N, then will N O be the difference of longitude; L M, the departure; L K, the proper difference of latitude; K N, the meridian difference of latitude. The difference of longitude in the first, will be 388 miles; in the last it will be 433, being both measured upon the equinoctial; this shews the necessity of knowing betwixt what latitudes any departure is made, before the difference of longitude can be found; and that in order thereto the first thing to be done, is to find the departure, as directed in plain sailing, and then the difference of longitude may be found as here directed; the reason of which will appear by carefully examining the principles by which *Mercator*'s chart is constructed. For tho' in this chart, the degrees of the parallels of latitude are apparently equal to the degrees of of the equinoctial. Yet if they be measured as directed in *Case* 3. of the preceding problem, they will be found to retain the same proportion to the degrees of the equinoctial in this chart, that they actually do upon the globe. Now the distance of any two places in any parallel is their departure, and the distance of their meridians on the equinoctial, is their difference of longitude equal to their apparent distance in the parallel on the chart; but when a ship alters both her latitude and longitude, the departure cannot be said to be made either in the latitude sailed from, nor in that come, at was observed before.

Let us then suppose the departure to be made in that parallel of latitude that lies in the middle between the two, *viz.* in 34°. Now a degree of the enlarged meridian at this parallel is 72.4; for 2207.8 is the meridional parts to 34° 30′, and 2135.4, the meridional parts for 33° 30′; the difference is 72° 4′, which multiplied by 5° 21′ (the degrees in the departure) gives 387 miles, or 129 leagues, the difference of longitude.

This is what is called middle latitude sailing, which tho' not strictly true, because the distances betwixt the parallels of latitude on the meridian are not in a continued geometrical proportion, yet in a short run there can be no considerable error; we shall therefore in the next section work the problems of *Mercator*'s sailing both by the meridional parts, and middle latitude.

There are two lines on *Gunter*'s scale, one of equal parts marked E P, which may serve to graduate the equinoctial; to this is adapted another line marked *Mer.* which serves to graduate the meridian; and by these *Mercator*'s chart may be constructed. Now to find the distance betwixt any two parallels by these two lines; as for instance, betwixt the parallel of 30, and the parallel of 60, extend from 30 to 60 on the line *Mer.* measure this on the line E P, is 44 degrees; makes 2640 miles.

SECT. III.

CONTAINING THE VARIOUS CASES.

Of MERCATOR's *SAILING.* (Plate XIII.)

HAving in the former section shewn how to construct *Mercator*'s chart, and the use of it in navigation; we shall now shew how to find all that is necessary for the mariner to know without the chart; this is what is called *Mercator*'s sailing, which presupposeth the knowledge of of plain sailing: The only defect of which, is that by the plain chart, the difference of longitude cannot be found, and tho' *Mercator*'s chart gives the difference of longitude; the departure must be first found by the plain chart.

PROB. I.

The latitude and longitude of two places being given to find their bearance and distance.

EXAMPLE. (*Plate* XIII.)

I demand the course and distance from the *Lizard* to *Madeira*.

	Latitude N.	Longitude W.		
Lizard	50° 0′	0° 0′	50° 0′	3474
Madeira	32 17	12 8	32 17	2048
	17 43	12 8	merid. dif. of lat.	1427
	60	60		
Proper differ. of lat.	1063	728 difference of longitude.		

To delineate this. 1st. Find by the table of meridional parts the meridional difference of latitude 1427, which set off from L to *l*. 2d. At *l* draw a perpendicular to L *m*, on which set off the difference of longitude 728, and draw the line *l m*. 3d. Set off the proper difference of latitude 1063, from L to *b*. 4th. Draw the line *b b* parallel to *m l*; *b b* will be the departure; L *m* the distance; the angle *l* L *m*, the course.

The triangles L *l m*, and L *b b*, are similar; therefore L *l* : *l m* :: L *b* : *b b*, that is the meridional difference of latitude is to the difference of longitude, as the proper difference of latitude is to the departure.

As the meridional difference of latitude	1427	3.154424
Is to the proper difference of latitude	1063	3.026533
So is the difference of longitude	728	2.862131
		5.888664
To the departure	542.3	2.734240

By *Gunter*'s scale, the extent from 1427 to 1063, will reach from 728 to 542 on the line of numbers.

The departure being thus found, the course and distance may be found by the 6th case of plain sailing; or without the departure, the course may be found by this proportion, as the meridional difference of latitude is to the difference of longitude, so is the radius to the tangent of the course. The extent from 1427 to 728, or from 1063 to 542.3; if taken on the line of numbers, will reach on the tangent line from 45 to 27° 2′. *See the Operation.* As

As meridional diff. of latitude 14727	3.154424	Prop. diff. of lat.	3.026533
Is to the radius or tangent of 45	10.000000	Radius	10.000000
So is the difference of longitude 728	2.862131	Departure	12.734240
To tang. of course 27° 2′ S.S.W. ½W.	9.707707	Tangent 27° 2′.	9.707707

The course being thus found, the distance may be found by *Case* 2. *of Plain Sailing*. Thus S. C. of 27° 2′ R : difference of latitude 1063 : : distance : 1191.

To find the departure by the middle latitude, add the two latitudes, the half of which subtract from 90, gives the complement of the middle latitude; then R : S. C. latitude : : difference of longitude : departure.

Lizard	50.0	As radius	10.000000
Maderia	32.17	Is to sine 48.52	9.876899
Sum	82.17	Diff. of long. 728	2.862131
Half	41.8	Departure 548.2	2.738920
Complement	48.52		

By *Gunter*'s scale; the extend from 90 to 49 on the line of sines, will reach from 728 to 548 on the line of numbers.

The difference betwixt the departure, by this and the preceding, is only 6 miles, which is so small, that it will make no difference in the distance and bearings.

The departure may be found in the table of difference of latitude and departure: Thus look for the complement of the middle latitude as if it were a given course, and for the difference of longitude as if it were the distance sailed on that course; the departure corresponding to that course and distance, will be the true departure required. Here the complement of the latitude is 49 nearest, which find in the table. But the difference of longitude exceeds the distance in the tables, therefore take 100 seven times, and then 28; now against 100 in distance column, is 75.7 in the departure column, which multiplied by 7, is 528.5; to which add 18.4, the departure corresponding to the distance 28, makes 546.9 for the whole departure.

The reason of this is, because, if in a right angled triangle, if the angle be made the complement of the latitude, the hypothenuse will be the radius, and the base the sine of the angle; but the hypothenuse may be called the distance, and the base the departure.

PROB. II.

Both latitudes and departure given, to find the difference of longitude.

Rule.

Rule. Find the proper and meridional difference of latitude, as in the former, and the proportion will be, as the proper difference of latitude is to the meridional difference of latitude, so is the departure to the difference of longitude.

EXAMPLE. (*Plate* XIII.)

A ship in latitude N. 49° 10′, and 15° 22′W. longitude, sails N. N. E. ¾ E. till by a good observation she is in the latitude of 52° 40′. I demand the longitude come to.

Latitude come to	52.40	3731 meridional parts
Latitude sailed from	49.10	3397
Prop. diff. of lat. 70 lea.	3.30	334 merid. dist. of lat. 111 leagues

The departure by the tables will be found to be nearly 42 leagues.

To delineate this. 1st. Make the line *a d* 70, and *a b* 111, by any scale of equal parts, and draw the perpendiculars *b c*, and *d e*. 2d. Make *d e* 42, and draw the line *a e*, which produce to *c*; then *a e* will be the distance; *d e* the departure; *b c* the difference of longitude; *a d* the proper difference of latitude; *a b* the meridional difference of latitude.

Note. This is by a larger scale than that by which the chart is made.

As 70 proper difference of latitude	1.845098
Is to 111 meridional distance of latitude	2.045323
So is the departure 42	1.623249
	3.668572
To difference of longitude 66.6 leagues	1.823474

By *Gunter*'s scale; extend from 49° 10′ to 52 40′ upon the meridional line: This upon the line E P, will be 5 degrees, and something more than a half, makes 334 miles, the meridian difference of latitude: The proper difference of latitude is 210; the departure is 126. Then

Prop. di. lat.		Depar.		Mer. di. lat.		Dif. long.	
210	:	126	: :	334	:	200	nearly.

By the middle latitude S C mid. : lat. : R : : dep. : D longitude.

52°	40′		As sine of 39° 5′	9.799651
49	10		Is to radius	10.00000
101	50	sum	So is 42 departure	11.623249
50	55	middle latitude	To dif. of long. 66° 52′	1.823598
39	5	comp. middle latitude		

By *Gunter*'s scale; the extent from the side of 39 to 90, will reach from 42, on the line of numbers, to 66 ½.

By

By the table of difference of latitude and departure find 39 degrees, and in the departure column look for 42.2, againſt which in the diſtance column is 67.

There are ſeveral other problems commonly inſerted in *Mercator*'s ſailing, which we ſhall omit; for all that is neceſſary to be known, is the latitude and longitude of the place, which may be had every day at noon; the latitude ſailed from the day before, the latitude come to at noon, and the departure made the laſt 24 hours, are found as directed in plain ſailing, and the difference of longitude by *Prob.* 2. of this; and the latitude and longitude being found, the courſe and diſtance to the port bound to, are found by *Prob.* 1.

And as to what is called parallel ſailing, it is in effect the ſame as middle latitude; for when a ſhip ſails in any parallel of latitude, the latitude ſailed from, and that come to, are the ſame, and of conſequence, the parallel the ſhip ſails in may be called the middle latitude, and the diſtance ſailed may be called the departure, by which the difference of longitude may be found as in the preceding problem; one example will be ſufficient to illuſtrate this.

Suppoſe two iſlands in the parallel of 50° 55′, and their diſtance in that parallel 42 leagues; required the difference of longitude.

It is plain that the diſtance here is the departure, which being the ſame as in the laſt problem, the operation will be exactly the ſame as in that; and if the difference of longitude were given, ſuppoſe 3° 20′, their diſtance in the parallel may be found by *Prob.* I.

In order to conſtruct this geometrically, let the globe be ſuppoſed to be cut thro' the plane of the equinoctial, and the meridians and parallels of latitude projected orthographically upon this plane; the equinoctial and parallels would be concentrick circles, of which the pole would be the center; the meridians would all be ſtrait lines interſecting one another in the center, and ſo the radius of the equinoctial would be one quarter of the meridian. The radius of any parallel of latitude would be the ſine of the complement of that parallel, or its diſtance from the pole, and the ſine of the parallel would be that part of the meridian intercepted betwixt the parallel circle and the equinoctial; this being premiſed. 1ſt. With the chord of 60, or ſine of 30, deſcribe an arch, or circle as in (*Plate* XI. *Fig.* II.) 2d. From any point H, ſet off the given difference of longitude 67 leagues, to F, and draw the chord H F, and radii C F and C H. 3d. With the ſine complement of the latitude, *viz.* 39 degrees from the center C, deſcribe the arch R M; the chord R M will be the departure required. For the triangles C H F and C R M are ſimilar, therefore C H, the radius, is to H F, the difference of longitude, as C R, the ſine complement, is to R M, the departure. If you have no line of ſines, lay off 50° 55′, the given latitude, by the chords from H, both ways upon the equinoctial, a ruler laid acroſs by theſe two points, will interſect the meridian C H in R, and C R will be the radius of the parallel.

Having the diſtance in the parallel to find the difference of longitude.

To delineate this, is only the reverſe of the former. 1ſt. With the ſine complement of the latitude from the center C, deſcribe the arch R M, making the chord R M equal 42 leagues, their given diſtance in the parallel. 2d. With the ſine of 90, or chord of 60, deſcribe from the center C another arch, and produce the lines C R and C M to interſect that arch in the points H and F, ſo ſhall H F be the difference of longitude.

We have now explained the fundamental principles of navigation, and ſhewn how to ſolve all the problems, and various caſes of plain, *Mercator*, middle latitude, and parallel ſailing, that are neceſſary for keeping a reckoning; and as to great circle ſailing, it may be ſaid to be impracticable, at leaſt by any ſea chart; for the arch of a great circle makes unequal angles with all the meridians; and how to deſcribe ſuch a curve upon a chart, wherein all the meridians are parallel to each other, ſeems if not impoſſible, at leaſt ſo difficult, that the benefit ariſing from thence would not compenſate the labour; for, ſuppoſing it actually deſcribed, there muſt be a new invention to direct a ſhip in that curve, for it cannot well be affirmed that it could be done by the compaſs. We ſhall therefore omit this, and proceed to the application of what has been ſaid to the actual keeping of a reckoning, which ſhall be ſhewn in the next chapter.

CHAP.

CHAP. V.

To find the Latitude and Variation of the Compaſs by celeſtial Obſervation, and how to keep a Reckoning at Sea.

SECT. I.

To work an Obſervation, and how to find the Zenith diſtance by DAVIS'S *Quadrant.* (Plate XI. Fig. IV. and V.)

THIS inſtrument conſiſts of two arches both drawn from one center H; to conſtruct which, upon the point H raiſe H Z perpendicular to H O; with the radius H S deſcribe an arch S F, which make 30 degrees; with the radius H G, deſcribe another arch G K, which make 60 degrees; number the great arch, beginning at its interſection with the line H O, to 30 upwards; number the little arch from its interſection with the line Z H, increaſing down to 60; ſo that both together make 90 degrees.

It has three vanes, one fixed immoveably at H, with a ſlit in it, this is called the horizon vane; another is fitted to move upon the great arch, with a hole, which muſt be put to the obſerver's eye, thro' which, and the ſlit in the horizon vane, the horizon muſt be ſeen; this is called the ſight vane; the third is fitted to move upon the ſmall arch, it is called the ſhade vane, becauſe the ſun throws its ſhadow upon the horizon vane: Theſe two vanes muſt be ſo placed, that the obſerver may ſee the ſhade exactly upon the upper ſide of the ſlit, at the ſame time that he ſees the horizon thro' the ſlit, and counting the degrees upon both arches, their ſum will be the zenith diſtance.

To prove this; from the center H deſcribe the ſemi-circle A Z M ⊕ O to repreſent an azimuth circle; A the horizon; Z the zenith; ⊕ the ſun.

Place the ſight vane at 0 degrees, on the great arch, and when the horizon A is ſeen thro' the ſlit at H, the perpendicular Z H, from the zenith, will cut the little arch at 0 degrees: Let the ſun be at ⊕; it is plain the angle Z H ⊕, is his zenith diſtance, which meaſured by the little arch, is 40 degrees, being the place where the ſhade vane is placed: But becauſe the arch will not admit of being divided into minutes, let the ſhade vane be placed at 25 degrees, the inſtrument muſt be moved till the line

H ⊕; cut the little arch at 25 degrees, and then the line H O will cut the great arch at 15 degrees: Produce the line H S to M; it is plain the angle Z H M, added to the angle M H ⊕, will be the ſun's zenith diſtance; to meaſure the angle Z H M, draw the line H G perpendicular to H S, it will cut the great arch at 0 degrees; the ſight vane muſt be placed at K, to ſee the horizon A, thro' the ſlit which will be 15 degrees on the great arch, but the angle G H O is equal to the angle Z H M, for the angles M H G and Z H O are equal, being both right. Therefore taking the angle M H O from both, the remainings Z H M and G H O will be equal; therefore the degrees on both arches being added together, will be the zenith diſtance, which being had, the latitude will be found by *Chap.* II. *Sect.* III. *Prob.* I.

The beſt inſtrument for taking the altitude at ſea is *Hadley*'s quadrant, but as there is a ſmall pamphlet explaining the nature, uſe, and the theory on which that curious and uſeful inſtrument is founded, given gratis with it wherever it is ſold; it will be needleſs to give a deſcription of it here.

After the latitude is thus found by a good obſervation, if it agrees with the latitude by the account, it may be preſumed that your longitude by account is true; but if there be any conſiderable difference, it may be feared there will likewiſe be an error in the longitude; to correct which there can be no certain rule, becauſe it is uncertain whether the error is in the courſe or diſtance; for it muſt always be ſuppoſed, that the artiſt has given all the proper allowances in caſting up the day's work, and frequently examined the log line and glaſſes, and likewiſe taken all opportunities of examining the current, and comparing this with his former journals: If after all this the obſerved latitude, and that by account do not agree, the only thing that can be done, is to let the longitude go as by his account, or make a remark what the longitude would have been, provided the error was in the courſe, and ſuppoſing the diſtance true; and likewiſe what the longitude would have been, were the error in the diſtance, and the courſe true; ſo that it may be preſumed one of theſe three may chance to hit.

If a ſhip be ſailing due north or ſouth, her difference of latitude, and diſtance, will be the ſame; and if they differ by obſervation, it is likely the error is in the half minute glaſs, or log line, but if ſhe ſails due eaſt, or due weſt, ſhe does not alter her latitude, but if by the obſervation it is found ſhe has made any difference of latitude, there certainly muſt be error in the courſe, which may be owing to the ſteerage, or the compaſs, for the needle does not always point out the meridian, but varies ſometimes to the eaſtward, and ſometimes to the weſtward of it; and this is what is called the variation of the compaſs, and muſt be found before the courſe can be corrected.

SECT.

S E C T. II.

To find the Variation of the COMPASS.

WHEN any heavenly object, as a ſtar, or the ſun, is in the horizon, the point of the compaſs that it bears upon may be had by ſights fitted to the common compaſs, and by an azimuth compaſs the degrees may be found, that the object is diſtant from the eaſt, or weſt points of the horizon; this is called the magnetick amplitude, and if this agrees with the true amplitude, there is no variation; ſo the true amplitude muſt be found either geometrically, or by calculation.

It was ſhewn in *Chap.* II. how to do this geometrically; and to do this by calculation, the following problem will ſerve.

P R O B. I.

The latitude of the place, and the ſun's declination given to find the amplitude.

E X A M P L E.

Given	Latitude	28° 16′ N.
	Declination	15 24 N.

The proportion is

As the ſine of 61° 44′ the comp. of the latitude	9.944854
Is to the radius - - - - - -	10.000000
So is the ſine of 15° 24′ the declination	+9.424156
To the ſine of the amplitude 17° 33′	9.479302

By this it appears that the ſun then riſes E. b N. ½ N. nearly, but if by the compaſs it bears E. ½ N. there will be a point variation to know whether the variation be eaſterly or weſterly, take this general rule.

When you are looking to the ſun to take the amplitude, if the magnetick be to the right hand of the true, the variation is weſterly, as in this caſe; and to rectify the courſe ſteered, there muſt be one point taken to the left hand, if the courſe ſteered be N. b E. the true courſe will be N. which ſhews that the north point of the compaſs is a point to the weſtward of the true north, but if the magnetick amplitude be to the left hand of the true, then is the variation eaſterly; and in correcting the courſe, the variation muſt be allowed to the right hand of the courſe ſteered.

DE-

DEMONSTRATION.

Of the proportion for finding the true amplitude see *Plate* XI. *Fig.* I.

Let P O be the given latitude, P Z is the complement; thro' P draw P *p*, parallel to the horizon H O, draw the parallel of declination E X, to interfect the horizon in X : C X, will be the fine of the amplitude; C *x* the fine of the declination, and P *p* the fine complement of the latitude; the triangles P *p* C, and C *x* X, are fimilar, for the angles at *p* and *x* are both right; the angle at X is equal to the angle P C *p*, being each of them the complement of the latitude; therefore P *p* the fine complement of the latitude, is to P C the radius, as C *x* the fine of the declination, is to C X the fine of the amplitude.

When the fun is rifen any confiderable heighth above the horizon, it will not be eafy to find its true bearance by the compafs, but, if it is within 10 or 12 degrees of the horizon, it may be had by an azimuth compafs.

The fun's azimuth is an arch of the horizon, contained between the fouth or north points of the horizon; and an azimuth circle paffing thro' the center of the fun to find this geometrically was fhewn in *Chap.* II.

And to do it by calculation the following problem will ferve.

PROB. II.

The latitude of the place, the fun's declination, and altitude being given, to find the azimuth.

EXAMPLE.

Latitude 28° 16′ N. declination 15° 24′ N. altitude 9° 3′.

Rule. 1ft, Take the complement of the altitude; the complement of the latitude; the complement of the declination; and add thefe three together. 2dly, Take half this fum, from which fubtract the complement of the declination. 3dly, Take the fine of this remainder, and the fine of the half fum from the logarithms. 4thly, Take the fines of the complement of the altitude, and the complement of the latitude from logarithms, and fubtract each of them from the logarithm of the radius; the remainders will be the complements arithmetical of thofe fines.

Now thefe four muft be added together, *viz.* the complement arithmetical of the fines of the complement of the altitude, and of the complement of lattitude, and the fines of the half fum, and remainder. Half the fum of thefe four logarithms is the logarithm of the fine complement of half the azimuth required.

88°

80°	30′	Comp. altitude comp. arithmetical of the sine		0.005997
61	44	Comp. latitude, comp. arithmetical of the sine		0.055146
74	36	Complement declination		
216	50	Sum of the three		
108	25	Half sum; from 180 remains 71° 35′.	Sine	9.977167
74	36	Complement declination		
33	49	Remainder - - - - -	Sine	9.745494
			Sum of the four	19.783804
38°	46′	Is the comp. of 51°14′, of which the sine is ½ the sum		9.891902
38	46			
77	32	The sun's azimuth from the north		

Note. If the declination be south, and latitude north, and the contrary, instead of taking the complement of the declination you must add 90 thereto, and proceed as before.

The demonstration of this problem depends upon the doctrine of spherick trigonometry; for here are three sides given, *viz.* the complement of the altitude, the complement of the latitude, and the sun's distance from the elevated pole; this last will be the complement of the declination, when the latitude and declination are both north, or both south; but if one be north, and the other south, 90 must be added to the declination; the three sides being given, the angles are found as in the preceding operation.

Having thus found the true azimuth, the magnetick is found by observation; the same directions as were given in the amplitudes, will serve to know whether the variation be easterly or westerly.

SECT. III.

How to keep a Reckoning at Sea in order to know at any time the Latitude and Longitude the Ship is in, and the Course and Distance to any Port.

THE regular method of doing this, is by keeping an exact account of the various courses, and distances sailed in 24 hours; for which reason the log is hove every hour, and the distance and course set down in proper columns, in a book provided for that purpose, which is called the log-book, ruled and column'd as explained in the following pages.

In the firſt column, at the head, is H for hours, under which are ſet down the hours; the ſecond has K at the head, in which are ſet down the knots run out at every hour; the third column has F at the head, in which are ſet down the odd fathoms at every hour; the fourth has courſe at the head, in which are ſet down the courſes correſponding to the hours; the fifth has winds at the head, in which are ſet down the ſhiftings of the winds; the ſixth has remarks at the head, in which is ſet down what ſail is carried, and the weather, which are two things very neceſſary to be obſerved; for if it blows hard a ſhip cannot make good the courſe ſteered by the compaſs.

In ſome ſhips the log is hove once in two hours, and ſet down upon a board, from which every one that keeps an account, tranſcribe it into his own book, and then reduces all the different courſes into one, and finds the whole difference of latitude and departure made the laſt 24 hours, as directed in plain ſailing, and the difference of longitude by *Prob.* II. of *Mercator*'s.

Tho' this account be kept by the greateſt care and exactneſs, there will often be very great errors diſcovered in the latitude, by good obſervations; and in the longitude when any known land is made.

The occaſion of theſe errors may be attributed to theſe four following cauſes. 1ſt. If the log line and glaſs be not duly proportioned to one another, there will be an error in the diſtance; for admitting the glaſs to be only 29 ſeconds, and the line right divided, the ſhip would be a head of the account the 30th part of the diſtance; for there is only an account taken of what ſhe runs in 29 ſeconds, and what ſhe runs in the 30th is omitted; but if the glaſs be true, and the line ſhort divided, the account will be a head of the ſhip; for ſuppoſing the 120th part of a mile to be 50 foot, and the ſpace between the knots to be only 45 foot, it is plain that every 50 leagues run by the account, is only 45; ſo that the ſhip muſt run the 10th part of the whole diſtance, more than by the account, before ſhe makes the land. 2d. Is owing to the variation of the magnetick needle, which does not always point out the meridian; this will occaſion a great error in the courſe, if the variation is not known, and allowed in the courſe. 3d. Is owing to the lee-way a ſhip make; for when a ſhip ſails cloſe by the wind, ſhe does not make good the courſe ſteered by the compaſs, but falls to the leeward of it, more or leſs according to the ſail ſhe carries, and the heighth of the ſea. 4th. Is when there is a current; for when that ſets upon the ſame rumb on which the ſhip is ſteered, her real diſtance will be more than by the log; and if it ſets upon the oppoſite rumb, it will be leſs; and if it ſets athwart the ſhip's way, it will occaſion an error both in the courſe and diſtance.

LOG-

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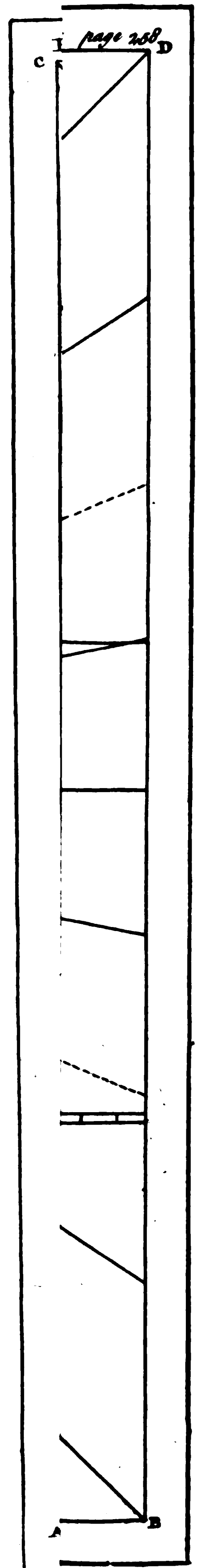

H.	K.	F.	Courses.	Winds.	REMARKS, Saturday, May 27, 1738.
1					*Lizard* 50° 0' north latitude, and 0° 0 longitude.
2					
3					*Lizard* N. W. 4 leagues distance, fine moderate gale, and pleasant fair weather.
4	5	3	S. W.	N. b E.	
5	6	4		N.	
6	7			fresh gale	Handed all our small sails, and reeft both top-sails.
7	8			N. b W.	
8	7			N. N. W.	A great sea from the N. W.
9	6	3		N. W.	Double reef M.T.S. and handed. F.TS.
10	5	4		N.W.bW.	
11	5			W. N. W.	
12	4	3			
1	4	4		W.	Drizling rain.
2	4		S. S. W.		
3	5				Set fore top-sail.
4	5	4	S. W. b S.	W. b N.	Fine clear weather, out all reefs, and set the small sails.
5	5	3		N·W. b N.	
6	5				
7	6				At noon clear weather, had a good observation.
8	7				
9	6	3			Zenith distance 25° 9'
10	6	4			Declination 22 47
11	6				Latitude come to 47 56
12	5				

Course.	Dis.	S.	W.	Lati. by account	Lati. by observa.	Diff. of longi.	Longi. come to	Variation	Meridio. distance.
S.16W.	124	118	33	48° 2'	47° 56'	51'	0° 51'	1¼ point	33 miles from the *Lizard*

Courses correc.	Dis.	S.	E.	W.
S. E. b E. ¼ E.	12	6.2	10.3	0
S. S. W. ¼ W.	46	39.5	0.0	23.6
S. b W. ¼ W.	62	58.4	0.0	20.9
S. ¼ E.	14	14.1	0.7	0.0
		118.2	11.0	44.5
				11.0
				33.5

Latitude by observation	47°	56'
Latitude sailed from	50	00
	97	56
Middle latitude	48	58
C. middle latitude	41	02
	90	00

Degrees.	Dep.	Dif. of long.
41	33.5	51

Lati. sailed from	50	00
Difference of lat.	1	58
Latitude come to	48	02

AFTER the log-book is thus copied, the next thing to be done is to reduce all the various courses into one, so that the whole difference of latitude, and the whole departure may be found; this is what

 is

is called working a day's work. In order to which, it will be proper to make a traverse table, upon a slate or waste paper, as directed in plain sailing. But it must be observed the courses must be corrected, by allowing for variation and lee-way. Now, as our account is to begin from the *Lizard*, we must suppose the ship to sail S. E. from it, to the place she was in at 3, when it bore N. W. and because there is one point, and ½ variation, the course corrected will be S. E. b E. ½ E. The next course steered is S. W. allowing variation, makes S. S. W. ½ W. but when the wind came to W. N. W. I allow 1 point lee-way, makes it S. b W. ½ W. when the wind comes to west, she lies only S. S. W. and allowing for variation and lee-way, she makes only S. ½ E. course; the last course steered is S. W. b S. the wind being large, makes no lee-way, allowing the variation; true course is S. b W. ½ W. The courses being thus corrected, against the first set down 12 miles, the distance from the *Lizard*; against the next is 46, being what the ship has run by the log from 3 to 11; against the next is 62, being what she has run from 11 to 2; and from 5 to noon; against S. ½ E. is 14 miles, what she sailed from 2 to 5: We may then find the proper difference of latitude and departure to each course, by the table; the sum of the southings is 118 miles, that is 1° 58′, and because the latitude is decreasing, this must be subtracted from the latitude of the *Lizard*, which makes the latitude by account 48° 2′, but by observation 47° 56′; the whole westing is 44.5, and easting is 11, so the departure is 33.5 west. And by this, to find the difference of longitude, add the latitude sailed from, and that come to, into one, and subtract half that sum from 90, the remainder is 41° 2′, the complement of the middle latitude. Again look in the table of difference of latitude and departure for 41°, and look for the departure 33.5, in the proper column, against which, in the distance column, is 51, the difference of longitude. To find the latitude by the observation, look for the declination corresponding to the day of the month, which is 22° 47′, added to the zenith distance 25° 9′, makes 47° 56′. After finding the whole difference of latitude and departure, because the numbers exceed those in the tables, I take half of each, which I find in the column corresponding to 16° against 62 distance, which doubled, makes 124, as in the operation under the log; the like process must be used every day at noon.

H.	K.	F.	Courses.	Winds.	REMARKS, Sunday, May 28, 1738.		
1	7		S. W. b W.	N. E.	Moderate gales and fine pleasant wea-		
2	6				ther.		
3	5						
4	5						
5	4		S. W.		Little wind with small drizling rain.		
6	3				Tried the current		
7	2				Found it S. S. W. one mile in an		
8	1				hour.		
9				Calm			
10					Fresh gales, handed all our small sails.		
11					At noon clear, zenith distance	22°	48′
12	1			S. E.	Declination - -	22	53
1	2			S. S. E.	Latitude by observation	45	41
2	4		S. S. W.		At sun rising, by a good observation,		
3	5	3			amplitude 23° 23′; in order to find		
4	6	4			the true amplitude, find what diffe-		
5	8				rence of latitude the ship made from		
6	9				sun rising till noon.		
7	9				Course corrected S. b W. distance 70,		
8	9				difference of latitude 68.7.		
9	9				Latitude at noon	45°	41′
10	9				Diffe. of lati. since sun rising	1	9
11	9				Latitude at sun rising	46	50
12	9				Complement of the latitude	43	10

Course.	Dif.	S.	W.	Lati. by account	Lati. by observa.	Diffe. of longi.	Longi. come to	Variati- on	Meridio. distance from the *Lizard*
S 18° W	144	136	45	45° 40′	45° 41′	1° 2′	1° 53′	1 point	1° 18′

Courses corrected.	Dif.	S.	W.
S. W.	23	16.3	16.3
S. W. b S.	13	10.8	7.2
S. b W.	87	85.3	17.0
Current S. b W.	24	23.5	4.7
Cour. good S. 18° W.	144	135.9	45.2

Sine of 43° 10′ comp. lati.		9.83513
Is to radius - -		10.00000
Sine of declination 22° 53′		9.58979
Sine of amplitude	34 38	9.75466
Magnetick	23 23	
Variation	11 15	

Latitude yesterday at noon	47°	56′
This day - -	45	41
	93	37
Middle latitude	46	48
Comp. of middle latitude	43	12

Degrees.	Depar.	Dif. of long.
43	45.3	62

H.	K.	F.	Courses.	Winds.
1	8		S. W.	N. b E.
2	7	3		
3	6	4		
4	6			
5	5	4		
6	5	3		N.
7	6			
8	6		S.W.bW.	N. N. W.
9	6			
10	5	4		
11	5	3		
12	6		The lati-	
1	6	4	tude at the	
2	8		time of ob-	
3	7		servation	
4	7	4	of the azi-	½ sum less
5	7	3	muth	
6	8		44° 26′	
7	9			
8	8	3		
9	8			
10	8			
11	8		Cloudy, no	
12	8		observati.	

REMARKS, Monday, May 29, 1738.

Fresh gales and cloudy most part of these 24 hours.

Found the sun's azimuth by a good observation 75° 30′ after the sun's rising; his altitude was then 9° 30′; the true amplitude is 67° 4′, which makes the variation 8° 26′, as by the following operation.

Com. alti.	80° 30′	Ar. com.	0.005997
Com. lat.	45 34	Of sines	0.146262
Com. dec.	67 2		
Sum	193 6		0.152259
Half sum	96 33		
Supple.	83 27	their sines	9.997156
Com. dec.	29 31		9 692562
Sum of the logarithms			19.841977
Half			9.920988
Sine com. half azim.			56° 28′
Complement			33 32
True azimuth from the N.			67 4
West variation			8 26
Magnetick azimuth			75 30

Course.	Dis.	S.	W.	Lati. by account	Lati. by observ.	Diffe. of longi.	Longi. come to	Variation.	Meridio. distance from the *Lizard.*
S. W.	168	118	118	43° 43′		2° 46′	4° 39′	¾ point.	3° 16′

Courses corrected.	Dist.	S.	W.	Points.	½ Depar.	½ Dif. of long.
S. W. b S. ¼ W.	45	36.1	26.8			
S. W. ¼ W.	123	82.6	91.2	4	59	83
S. W. dist.	168	118.7	118.0			2
					Dif. of long.	166
						120
						2° 46′

Latitude yesterday at noon	45° 41′
Diffe. of lati. these 24 hours	1 58
Latitude come to	43 43
Latitude sailed from	45 41
	89 24
Middle latitude	44 42
Comp. of middle latitude	45 18

The

The preceding three days will be sufficient to shew the manner of taking off and working the log, as the operations for finding the latitude by the zenith distance, and the variation of the compass by an amplitude and azimuth, are there set down at large. We have also shewn how to correct the course, by allowing for lee-way and variation, and how to account for a current. After working each day's work in the log-book, they may from thence be transferred into a journal; the form of which is hereunto annexed.

Journal of a Voyage, intended by God's assistance, in the Ship Sea-Horse, *from* London *to* Jamaica, *under the Command of* A. B. *in the Year* 1738.

Weeks day.	Months day.	Course made good.	Dist. in miles.	Latitude by account.	Latitude by observation.	Longitude.	Meridional distance from the *Lizard*.	Variation.	Winds.
	May								
♄	27	S. 16° W.	124	48° 2′	47° 56′	0° 51′	0° 33′	1¼ point	Yesterday at 3 P. W. *Lizard*, N. W. 4 leagues, the first part fresh gales and a great sea, the latter moderate and clear, N. to W.
☉	28	S. 18° W.	144	45° 40′	45° 41′	1° 53′	1° 18′	By am. 11° 15′	N. E. to S. E. fresh gales, and clear the latter part.
☽	29	S. W.	168	43° 43′		4° 39′	3° 16′	By azi. 8° 26′	N. E. E. to N. N. W. fresh gales and cloudy.

The following characters are generally used to express the days of the week.

☉ *Sunday*; ☽ *Monday*; ♂ *Tuesday*; ☿ *Wednesday*; ♃ *Thursday*; ♀ *Friday*; ♄ *Saturday*.

SECT. IV.

Of the Moon's Age, and Time of High Water.

FROM what has been said, it is plain, that if the account of the journal be true, the ship will arrive at her designed port, by steering such a course as the journal directs, and in order to sail into the harbour, if in a tide way, the mariner should know what time it will be high water; but as this is governed by the moon, it follows, that to attain this, the first thing to be done is, to find her age.

If the months all contained an equal number of days, and the change of the moon was always on the last day of every month, the day of the moon would then be the same with the day of the month, and we should have exactly twelve compleat moons every year; but it has been found by a long series of good observations, that every year contains twelve compleat moons and eleven days more, very nearly, so that if the moon happens to change any year the last day of *December*, it will be eleven days past the change on the last day of the *December* following; and twenty-two days after the change, the succeeding year, and the third year it will be thirty-three days; but as there are but 29½ days from the change of the moon till it changes again, it is plain that in three years time, which contain 36 months, we shall have 37 compleat moons, and three days more; so that the moon will be, on the last day of *December*, in the third year, 3 days after the change; and on the fourth it will be 11 days more, that is, the last day of *December* will be 14 days after the change. Now it will be very easy to find the moon's age any day of the month provided the moon's age be known the last day of the preceding year, our first business then shall be to shew how this is found.

As every year contains 12 moons and 11 days, every three years will contain one whole moon more than months, and three days more, so that 18 years will contain six whole moons more than months, and 18 days more; that is to say, if the moon changes on the last day of *December*, it will be in 18 years afterwards, 18 days past the change on the last day of *December*; and the year following, *viz.* the 19th year, it will be 11 days more, which makes 29 days, and this being a whole moon except half a day, we shall have new moon some time of the last day of *December*; so that at the expiration of 19 years the new and full moons happen on the same day of the same month they did 19 years before that.

This revolution of 19 years is called the lunar cycle, or the *Metonic* cycle, from its author *Meton* the *Athenian*. The new moon, or the change, was

was on the last day of *December*, two years before the birth of our Saviour, so that every nineteenth year from that time the moon changed on the last day of *December*, and the year of our Saviour's birth was the second year of the cycle. Hence it is manifest, that if we add 1 to any year since our Saviour's birth, and divide the sum by 19, the quotient will shew how many cycles have past since its first commencement, and the remainder will shew what year of the cycle that is, whieh is called the golden number, or prime for that year; it finds the the age of the moon on the last day of the preceding year, or the number of days past since the new moon; this number is called the epact, and since the epact of the first year is 11, of the second 22, of the thire 3, and so on constantly increasing by 11, as was before observed, it is evident that to find the epact for any year we must multiply the golden number by 11, the product if less than 30 will be the epact for that year, if it exceed 30 divide it by 30, and the remainder will be the epact.

EXAMPLE I.

Required the Epact for the Year 1730.

First find the golden number by the preceding rule.

To the year	1730	golden number	2
add	1	multiplied by	11
divide by	19)1731(91	product is the epact	22
	1729		
golden number	2		

EXAMPLE II.

Required the Epact for the Year 1744.

1744	16
1	11
19)1745(91	30)176(5
1729	150
16 golden number	26 epact

Now as the epact expresses the age of the moon on the last day of *December*, it is plain that if the moon changes on that day in any year, it will change on the 20th of *December* the next year, because on the last day of *December* it will be 11 days past the change; but if this 20th day be called the 31st, as was the case in the year 1753, when the style was altered, it will make an alteration in the epact of 11 days: Therefore to find the epact since the commencement of the new style, we must divide

divide the year without adding 1 to it, by 19, the remainder will be the golden number, which multiplied by 11 will give the epact as before.

EXAMPLE III.

Required the Epact for the Year 1754.

19)1754(92	golden number 6
171	multiplied by 11
44	product 30)66(2
38	60
6 golden number	epact 6

It is plain if the moon changes on the last day of any year, the day of the moon will be the same with the day of the month in *January* following till the change; and because the time betwixt one new moon and the next is 29 days and an half, the 30th day of *January* will be the first day of the next moon, and the first day of *February* will be the third day of the moon, and consequently if we add two to any day of the month in *February* that year, it will give us the day of the moon; now as in common years *February* has but 28 days, if to this, 2 be added, it will make 30, which is half a day more than the moon contains; so that it will change on the last day of *February*, and therefore the first day of *March* will always be the same day of the moon that the first day of *January* is, and the first day of *April*, the same as the first day of *February* (except in leap years, when one day more must be added to the first day of *March*), for if the moon changes on the last day of *February*, it will change again before the 30th of *March*, and so the first day of *April* will be the 3d day of the moon, and the 27th day of *April* the 29th day of the moon; so the 28th day of *April* will be the first day of the moon, and the first day of *May* will be the 4th day of the moon.

Now, though the moon does not change the last day of *December*, the epact gives the age of it on that day, and therefore if the epact be added to the day of the month in *January*, the sum will be moon's age; but in *February* we must add 2 to the epact and day of the month, to find the moon's age; in *March* again, except in leap years, the epact and day of the month will give the moon's age. Hence this general rule will serve to find the moon's age on any day of the month.

Rule, Add the epact to the day of the month, and the number for that month, the sum if less than 30 is the moon's age, if it exceeds 30 take 30 from it, and the remainder will give the moon's age.

The

The following numbers muſt be added in the months to which they correſpond.

January	0	*April*	2	*July*	5	*October*	8
February	2	*May*	3	*Auguſt*	6	*November*	10
March	0	*June*	4	*Sept.*	8	*December*	10

EXAMPLE I.

Required the moon's age the 24th day of *January* 1754.

The epact by the preceeding rules will be found to be 6, the number for the month is 0, now we have only 6 to add to the 24, which makes 30, which being half a day more than a whole moon, the moon changes ſome time that day.

EXAMPLE II.

Required the moon's age the 26th of *April* 1754, to 26 add 2 for the month, and the epact 6, the ſum is 34, from which ſubſtracting 30, the remainder is 4, the moon's age.

After finding the moon's age we may thereby find the time of high water from the following principles.

It has been obſerved that when it is high water in any port, the moon will always be on the ſame point of the compaſs; and as the moon in 24 hours moves through all the points of the compaſs, it is plain ſhe muſt take 45 minutes in moving from any point to the next; for 32, the points of the compaſs, multiplied by 45, gives 1440, the minutes in 24 hours; hence, if it is high water in one port at 12, and the moon then on the ſouth point of the compaſs, and high water in another port, when the moon is on the S.S.W. point, it will be 1 hour and 30 minutes after 12 when it is high water at this laſt place.

Now, if the moon and the ſun were always on the meridian at the ſame time, the moon would always come to the ſame point of the compaſs at the ſame hour of the day, and of conſequence it would be always high water at the ſame hour; but ſince the moon comes to the meridian with the ſun only on the day of the change, which happening only once in 30 days, it will from thence follow that the difference of the time of her coming to the meridian from the day of the change, or any day of her age, to the next day, will be 48 minutes; for, 30 the days in the moon, multiplied by 48 gives 1440, the minutes in 24 hours, at which time the moon will again come to the meridian with the ſun, and will then be on the ſouth ſide of the compaſs; the firſt day after the change it will be 48 minutes after 12, before the moon comes to the meridian, or ſouth point

point of the compass; the second day 1 hour 36 minutes; and as it is thus 48 minutes every day later in coming to the south point of the compass, it will be so with respect to any other point, which is the reason that it is high water in any port 48 minutes later every day than it was the preceding.

From what has been said, it is manifest, that before the time of high water can be found on any day of the moon, two things must be known; first, on what point of the compass the moon will be every day at high water; secondly, at what time the moon will come to the meridian on that day. As for the first, which is called the flowing, it must only be had by experience, and for the second, which is called the moon's southing, it will always be found by multiplying the moon's age by 48, and dividing the product by 60, the quotient will give the hour, and the remainder the minutes the moon is on that day later of coming to the meridian than the sun. Now these two being given, if to the hours and minutes of southing we add the hours and minutes corresponding to the flowing, that is, to the point of compass on which the moon is at high water, the sum is the time of high water on that day.

E X A M P L E.

Required the time of high water at *London-Bridge*, *Feb.* 27, 1754, the flowing S. W.

Because when the sun is on the south point of the compass it will be 12 hours, that is, either noon or midnight, it will be 3 hours after noon when the moon is on the S.W. point of the compass; for in 45 minutes she moves from one point to the next; S.W. is 4 points from the south, and 4 times 45 is 180 minutes, which is three hours. The age of the moon on that is, 3 multiplied by 48 is 144, divided by 60 is 2 hours and 24 minutes, the moon's southing that day, to which adding 3 hours the flowing, the sum will be 5 hours and 24 minutes, the time of high water required.

F I N I S.

TABLES *of the* SUN'S DECLINATION: *Adapted to the* New Style.

First after Leap-Year. Sun's Declination 1753, 1757, 1761, 1765, 1769.

Days	*Janua.* South	*Feb.* South	*March* South	*April* North	*May* North	*June* North	*July* North	*August* North	*Septem.* North	*Octob.* South	*Nov.* South	*Decem.* South	Days
1	23 00	16 58	07 24	04 43	15 13	22 08	23 08	17 59	08 11	03 20	14 36	21 55	1
2	22 54	16 41	07 01	05 06	15 31	22 16	23 04	17 44	07 49	03 44	14 55	22 04	2
3	22 48	16 23	06 38	05 29	15 48	22 23	22 59	17 28	07 27	04 07	15 14	22 13	3
4	22 42	16 05	06 15	05 52	16 05	22 30	22 54	17 12	07 05	04 30	15 33	22 21	4
5	22 35	15 47	05 52	06 15	16 23	22 37	22 48	16 56	06 42	04 54	15 51	22 29	5
6	22 28	15 28	05 29	06 37	16 40	22 43	22 42	16 39	06 20	05 17	16 09	22 36	6
7	22 20	15 10	05 06	07 00	16 56	22 49	22 36	16 22	05 57	05 40	16 29	22 43	7
8	22 12	14 51	04 42	07 22	17 13	22 55	22 29	16 05	05 35	06 03	16 44	22 49	8
9	22 04	14 31	04 19	07 45	17 29	23 00	22 22	15 48	05 12	06 26	17 01	22 55	9
10	21 55	14 12	03 55	08 07	17 44	23 05	22 15	15 31	04 49	06 48	17 18	23 00	10
11	21 45	13 52	03 32	08 29	18 00	23 09	22 07	15 13	04 26	07 11	17 35	23 05	11
12	21 35	13 32	03 08	08 51	18 15	23 13	21 58	14 55	04 03	07 34	17 51	23 10	12
13	21 25	13 12	02 44	09 13	18 30	23 16	21 50	14 36	03 40	07 56	18 07	23 14	13
14	21 14	12 51	02 21	09 34	18 44	23 19	21 41	14 18	03 17	08 19	18 23	23 17	14
15	21 03	12 31	01 57	09 55	18 59	23 22	21 31	13 59	02 54	08 41	18 38	23 20	15
16	20 52	12 10	01 33	10 17	19 13	23 24	21 22	13 40	02 31	09 03	18 54	23 23	16
17	20 40	11 49	01 10	10 38	19 26	23 26	21 11	13 21	02 08	09 25	19 08	23 25	17
18	20 28	11 28	00 46	10 59	19 39	23 27	21 01	13 02	01 44	09 47	19 23	23 27	18
19	20 15	11 06	00 22	11 20	19 52	23 28	20 50	12 42	01 21	10 09	19 37	23 28	19
20	20 02	10 45	Nor. 01	11 40	20 05	23 29	20 38	12 22	00 57	10 31	19 50	23 29	20
21	19 49	10 23	00 25	12 01	20 17	23 29	20 27	12 02	00 34	10 52	20 04	23 29	21
22	19 35	10 01	00 49	12 21	20 29	23 29	20 15	11 42	00 11	11 13	20 17	23 29	22
23	19 21	09 39	01 12	12 41	20 41	23 28	20 03	11 22	Sou. 13	11 34	20 29	23 28	23
24	19 06	09 17	01 36	13 01	20 52	23 27	19 51	11 01	00 36	11 55	20 41	23 27	24
25	18 51	08 55	02 00	13 20	21 03	23 26	19 38	10 40	01 00	12 16	20 53	23 25	25
26	18 36	08 32	02 23	13 39	21 13	23 24	19 25	10 20	01 23	12 37	21 04	23 23	26
27	18 21	08 10	02 47	13 59	21 23	23 21	19 11	09 58	01 47	12 57	21 15	23 21	27
28	18 05	07 47	03 10	14 18	21 33	23 19	18 57	09 37	02 10	13 17	21 26	23 18	28
29	17 49		03 33	14 36	21 42	23 15	18 43	09 16	02 34	13 37	21 36	23 14	29
30	17 32		03 57	14 55	21 51	23 12	18 29	08 54	02 57	13 57	21 46	23 10	30
31	17 15		04 20		22 08		18 14	08 33		14 17		23 06	31

Second after Leap-Year. Sun's Declination, 1754, 1758, 1762, 1766, 1770.

Days	*Janua.* South	*Febru.* South	*March* South	*April* North	*May* North	*June* North	*July* North	*August* North	*Septem.* North	*Octob.* South	*Nov.* South	*Decem.* South	Days
1	23 01	17 03	07 30	04 37	15 08	22 06	23 08	18 02	08 15	03 14	14 31	21 54	1
2	22 55	16 45	07 07	05 00	15 26	22 14	23 04	17 47	07 54	03 38	14 50	22 03	2
3	22 50	16 27	06 44	05 23	15 44	22 22	22 59	17 31	07 32	04 02	15 09	22 12	3
4	22 43	16 10	06 21	05 46	16 01	22 29	22 54	17 15	07 10	04 25	15 28	22 20	4
5	22 37	15 52	05 58	06 09	16 18	22 36	22 49	16 59	06 47	04 48	15 47	22 27	5
6	22 30	15 33	05 35	06 32	16 35	22 43	22 43	16 43	06 25	05 11	16 05	22 34	6
7	22 23	15 14	05 12	06 54	16 52	22 49	22 37	16 27	06 02	05 34	16 23	22 41	7
8	22 15	14 55	04 48	07 17	17 09	22 54	22 30	16 10	05 39	05 57	16 41	22 48	8
9	22 06	14 36	04 25	07 39	17 25	22 59	22 23	15 53	05 17	06 20	16 58	22 54	9
10	21 57	14 17	04 02	08 01	17 41	23 04	22 16	15 35	04 54	06 43	17 15	23 00	10
11	21 48	13 57	03 38	08 23	17 56	23 08	22 08	15 17	04 32	07 05	17 32	23 05	11
12	21 38	13 37	03 14	08 45	18 11	23 12	22 00	14 59	04 09	07 28	17 48	23 10	12
13	21 28	13 17	02 51	09 07	18 26	23 15	21 52	14 41	03 46	07 50	18 04	23 14	13
14	21 17	12 57	02 26	09 29	18 41	23 18	21 43	14 23	03 23	08 13	18 20	23 17	14
15	21 06	12 36	02 03	09 50	18 56	23 21	21 34	14 04	03 00	08 35	18 35	23 20	15
16	20 55	12 15	01 40	10 11	19 10	23 23	21 24	13 45	02 37	08 57	18 50	23 23	16
17	20 43	11 54	01 16	10 32	19 24	23 25	21 14	13 26	02 14	09 19	19 05	23 25	17
18	20 31	11 33	00 52	10 53	19 37	23 27	21 04	13 07	01 50	09 41	19 19	23 27	18
19	20 18	11 11	00 29	11 14	19 50	23 28	20 53	12 47	01 27	10 03	19 33	23 28	19
20	20 05	10 50	00 05	11 35	20 02	23 29	20 42	12 27	01 03	10 25	19 47	23 29	20
21	19 52	10 28	Nor. 19	11 55	20 14	23 29	20 30	12 07	00 39	10 47	20 00	23 29	21
22	19 38	10 07	00 43	12 15	20 26	23 29	20 18	11 47	00 16	11 08	20 13	23 29	22
22	19 24	09 45	01 06	12 35	20 38	23 28	20 06	11 27	Sou. 08	11 29	20 26	23 28	23
24	18 10	09 23	01 30	12 55	20 50	23 27	19 54	11 06	00 31	11 50	20 38	23 27	24
25	18 55	09 00	01 53	13 15	21 01	23 26	19 41	10 45	00 54	12 11	20 50	23 26	25
26	18 40	08 38	02 17	13 34	21 11	23 24	19 29	10 24	01 17	12 32	21 02	23 24	26
27	18 25	08 15	02 40	13 53	21 21	23 21	19 15	10 03	01 40	12 52	21 13	23 21	27
28	18 09	07 53	03 04	14 12	21 31	23 18	19 01	09 42	02 04	13 12	21 24	23 18	28
29	17 53		03 27	14 31	21 40	23 15	18 47	09 20	02 28	13 32	21 34	23 15	29
30	17 37		03 15	14 50	21 49	23 12	18 33	08 58	02 52	13 52	21 44	23 11	30
31	17 20		04 14		21 58		18 18	08 36		14 12		23 07	31

TABLES *of the* SUN'S DECLINATION. *Adapted to the* New Stile.

Third after Leap-Year. Sun's Declination 1755, 1759, 1763, 1767, 1771.

Days	*Janu.* South	*Feb* South	*March* South	*April* North	*May* North	*June* North	*July* North	*August* North	*Septem.* North	*Oct.* South	*Nov.* South	*Decem.* South	Days
1	23 02	17 06	07 36	04 31	15 04	22 04	23 09	18 06	08 22	03 08	14 26	21 52	1
2	22 57	16 49	07 13	04 54	15 22	22 12	23 05	17 51	08 00	03 32	14 46	22 01	2
3	22 51	16 31	06 50	05 17	15 40	22 20	23 01	17 35	07 38	03 55	15 05	22 09	3
4	22 45	16 14	06 27	05 40	15 58	22 27	22 56	17 19	07 16	04 18	15 24	22 17	4
5	22 39	15 56	06 03	06 03	16 15	22 34	22 50	17 04	06 54	04 41	15 42	22 25	5
6	22 32	15 38	05 40	06 26	16 32	22 40	22 44	16 48	06 32	05 04	16 00	22 32	6
7	22 24	15 19	05 17	06 48	16 49	22 46	22 38	16 31	06 09	05 27	16 18	22 39	7
8	22 16	15 00	04 54	07 10	17 05	22 52	22 32	16 14	05 46	05 50	16 36	22 45	8
9	22 08	14 41	04 30	07 33	17 22	22 57	22 25	15 57	05 23	06 13	16 54	22 51	9
10	21 59	14 22	04 07	07 56	17 37	23 02	22 18	15 39	05 00	06 36	17 11	22 57	10
11	21 50	14 02	03 43	08 18	17 52	23 07	22 10	15 21	04 38	06 59	17 28	23 02	11
12	21 40	13 42	03 20	08 40	18 07	23 11	22 02	15 03	04 15	07 22	17 44	23 07	12
13	21 30	13 22	02 56	09 02	18 22	23 14	21 54	14 45	03 52	07 44	18 00	23 11	13
14	21 20	13 02	02 32	09 23	18 37	23 18	21 45	14 27	03 29	08 07	18 16	23 15	14
15	21 09	12 42	02 08	09 45	18 52	23 21	21 35	14 09	03 06	08 30	18 32	23 18	15
16	20 58	12 21	01 45	10 07	19 06	23 23	21 26	13 50	02 43	08 52	18 47	23 21	16
17	20 46	12 00	01 21	10 28	19 20	23 25	21 16	13 31	02 19	09 14	19 02	23 24	17
18	20 34	11 39	00 57	10 49	19 34	23 27	21 05	13 11	01 56	09 36	19 16	23 26	18
19	20 21	11 18	00 34	11 10	19 47	23 28	20 54	12 51	01 33	09 58	19 30	23 27	19
20	20 19	10 56	00 10	11 30	20 00	23 29	20 43	12 32	01 09	10 20	19 44	23 28	20
21	19 56	10 34	Nor. 14	11 50	20 12	23 29	20 33	12 12	00 45	10 42	19 57	23 29	21
22	19 42	10 12	00 37	12 10	20 24	23 29	20 23	11 52	00 22	11 04	20 10	23 29	22
23	19 28	09 50	01 01	12 30	20 36	23 28	20 11	11 32	Sou. 01	11 25	20 23	23 29	23
24	19 14	09 38	01 24	12 50	20 47	23 28	19 58	11 12	00 25	11 45	20 35	23 28	24
25	18 59	09 06	01 48	13 10	20 58	23 27	19 45	10 51	00 48	12 05	20 47	23 26	25
26	18 44	08 44	02 12	13 29	21 09	23 25	19 31	10 30	01 12	12 26	20 59	23 24	26
27	18 29	08 21	02 35	13 48	21 19	23 23	19 18	10 09	01 35	12 47	21 10	23 22	27
28	18 13	07 58	02 58	14 07	21 29	23 20	19 04	09 48	01 59	13 07	21 21	23 19	28
29	17 56		03 22	14 26	21 38	23 17	18 50	09 27	02 22	13 27	21 31	23 16	29
30	17 40		03 45	14 45	21 47	23 14	18 36	09 06	02 45	13 47	21 42	23 13	30
31	17 23		04 08		21 56		18 21	08 44		14 07		23 09	31

Leap-Year. Sun's Declination, 1756, 1760, 1764, 1768, 1772.

Days	*Janu.* South	*Febru.* South	*Mar.* South	*April* North	*May* North	*June* North	*July* North	*August* North	*Septem.* North	*Oct.* South	*Nov.* South	*Decem.* South	Days
1	23 03	17 11	07 18	04 49	15 17	22 10	23 07	17 55	08 05	03 27	14 41	21 58	1
2	22 58	16 54	06 55	05 12	15 34	22 18	23 03	17 40	07 43	03 50	15 00	22 07	2
3	22 52	16 36	06 32	05 35	15 52	22 25	22 58	17 24	07 21	04 13	15 19	22 15	3
4	22 46	16 18	06 09	05 57	16 10	22 32	22 52	17 08	06 59	04 36	15 37	22 23	4
5	22 40	16 00	05 46	06 20	16 27	22 39	22 47	16 52	06 37	04 59	15 55	22 30	5
6	22 33	15 42	05 23	06 43	16 44	22 45	22 41	16 35	06 14	05 22	16 13	22 37	6
7	22 26	15 24	04 59	07 05	17 00	22 51	22 34	16 18	05 52	05 45	16 31	22 44	7
8	22 18	15 05	04 36	07 27	17 17	22 56	22 27	16 01	05 29	06 08	16 49	22 50	8
9	22 10	14 46	04 13	07 50	17 33	23 01	22 20	15 44	05 06	06 31	17 06	22 56	9
10	22 01	14 27	03 49	08 12	17 48	23 06	22 13	15 26	04 44	06 54	17 23	23 01	10
11	21 52	14 07	03 26	08 34	18 04	23 10	22 05	15 08	04 21	07 17	17 39	23 06	11
12	21 42	13 47	03 02	08 56	18 19	23 14	21 56	14 51	03 57	07 39	17 55	23 11	12
13	21 33	13 27	02 38	09 18	18 34	23 17	21 48	14 33	03 34	08 02	18 11	23 15	13
14	21 23	13 07	02 15	09 39	18 48	23 20	21 38	14 14	03 11	08 24	18 27	23 18	14
15	21 12	12 46	01 51	10 00	19 02	23 22	21 29	13 55	02 48	08 47	18 42	23 21	15
16	21 01	12 26	01 28	10 21	19 16	23 25	21 19	13 36	02 25	09 09	18 57	23 23	16
17	20 49	12 05	01 04	10 42	19 29	23 26	21 09	13 16	02 02	09 31	19 12	23 25	17
18	20 37	11 43	00 40	11 03	19 42	23 28	20 58	12 57	01 38	09 53	19 26	23 27	18
19	20 25	11 22	00 16	11 24	19 55	23 28	20 47	12 37	01 15	10 14	19 40	23 28	19
20	20 12	11 01	Nor. 07	11 45	20 08	23 29	20 36	12 17	00 52	10 36	19 54	23 29	20
21	19 59	10 40	00 31	12 05	20 20	23 29	20 25	11 57	00 29	10 57	20 07	23 29	21
22	19 45	10 18	00 55	12 26	20 32	23 29	20 13	11 37	00 05	11 19	20 20	23 29	22
23	19 32	09 56	01 18	12 45	20 43	23 28	20 00	11 17	Sou. 19	11 40	20 32	23 28	23
24	19 17	09 34	01 41	13 05	20 54	23 27	19 48	10 56	00 42	12 01	20 44	23 27	24
25	19 03	09 12	02 05	13 25	21 05	23 25	19 35	10 35	01 06	12 21	20 56	23 25	25
26	18 48	08 49	02 29	13 44	21 15	23 23	19 21	10 14	01 29	12 42	21 07	23 22	26
27	18 32	08 27	02 52	14 03	21 25	23 20	19 08	09 53	01 53	13 02	21 18	23 19	27
28	18 17	08 04	03 15	14 22	21 35	23 18	18 54	09 32	02 16	13 22	21 29	23 16	28
29	18 01	07 41	03 39	14 41	21 44	23 15	18 40	09 11	02 39	13 42	21 39	23 13	29
30	17 45		04 02	14 59	21 53	23 11	18 25	08 49	03 03	14 02	21 48	23 09	30
31	17 28		04 25		22 02		18 10	08 27		14 22		23 05	31

A Large and very Useful

TABLE OF DIFFERENCE OF LATITUDE and DEPARTURE

IN

MINUTES and TENTH PARTS

TO

Every DEGREE and QUARTER-POINT

OF THE

COMPASS,

For the exact Working of a

TRAVERSE.

Dist.	1 Deg. Lat.	1 Deg. Dep.	2 Deg. Lat.	2 Deg. Dep.	¼ Point. Lat.	¼ Point. Dep.	3 Deg. Lat.	3 Deg. Dep.	4 Deg. Lat.	4 Deg. Dep.	5 Deg. Lat.	5 Deg. Dep.	½ Point. Lat.	½ Point. Dep.	Dist.
1	01.0	00.0	01.0	00.0	01.0	00.0	01.0	00.1	01.0	00.1	01.0	00.1	01.0	00.1	1
2	02.0	00.0	02.0	00.1	02.0	00.1	02.0	00.1	02.0	00.1	02.0	00.2	02.0	00.2	2
3	03.0	00.1	03.0	00.1	03.0	00.1	03.0	00.2	03.0	00.2	03.0	00.3	03.0	00.3	3
4	04.0	00.1	04.0	00.1	04.0	00.2	04.0	00.2	04.0	00.3	04.0	00.3	04.0	00.4	4
5	05.0	00.1	05.0	00.2	05.0	00.2	05.0	00.3	05.0	00.3	05.0	00.4	05.0	00.5	5
6	16.0	00.1	06.0	00.2	06.0	00.3	06.0	00.3	06.0	00.4	06.0	00.5	06.0	00.6	6
7	07.0	00.1	07.0	00.2	07.0	00.3	07.0	00.4	07.0	00.5	07.0	00.6	07.0	00.7	7
8	08.0	00.1	08.0	00.3	08.0	00.4	08.0	00.4	08.0	00.6	08.0	00.7	08.0	00.8	8
9	09.0	00.2	09.0	00.3	09.0	00.4	09.0	00.5	09.0	00.6	09.0	00.8	09.0	00.9	9
10	10.0	00.2	10.0	00.4	10.0	00.5	10.0	00.5	10.0	00.7	10.0	00.9	10.0	01.0	10
11	11.0	00.2	11.0	00.4	11.0	00.5	11.0	00.6	11.0	00.8	11.0	01.0	10.9	01.1	11
12	12.0	00.2	12.0	00.4	12.0	00.6	12.0	00.6	12.0	00.8	12.0	01.0	11.9	01.2	12
13	13.0	00.2	13.0	00.5	13.0	00.6	13.0	00.7	13.0	00.9	12.9	01.1	12.9	01.3	13
14	14.0	00.2	14.0	00.5	14.0	00.7	14.0	00.7	14.0	01.0	13.9	01.2	13.9	01.4	14
15	15.0	00.3	15.0	00.5	15.0	00.7	15.0	00.8	15.0	01.0	14.9	01.3	14.9	01.5	15
16	16.0	00.3	16.0	00.6	16.0	00.8	16.0	00.8	16.0	01.1	15.9	01.4	15.9	01.6	16
17	17.0	00.3	17.0	00.6	17.0	00.8	17.0	00.9	17.0	01.2	16.9	01.5	16.9	01.7	17
18	18.0	00.3	18.0	00.6	18.0	00.9	18.0	00.9	18.0	01.3	17.9	01.6	17.9	01.8	18
19	19.0	00.3	19.0	00.7	19.0	00.9	19.0	01.0	19.0	01.3	18.9	01.7	18.9	01.9	19
20	20.0	00.4	20.0	00.7	20.0	01.0	20.0	01.0	20.0	01.4	19.9	01.7	19.9	02.0	20
21	21.0	00.4	21.0	00.7	21.0	01.0	21.0	01.1	20.9	01.5	20.9	01.8	20.9	02.1	21
22	22.0	00.4	22.0	00.8	22.0	01.1	22.0	01.1	21.9	01.5	21.9	01.9	21.9	02.2	22
23	23.0	00.4	23.0	00.8	23.0	01.1	23.0	01.2	22.9	01.6	22.9	02.0	22.9	02.3	23
24	24.0	00.4	24.0	00.8	24.0	01.2	24.0	01.3	23.9	01.7	23.9	02.1	23.9	02.4	24
25	25.0	00.4	25.0	00.9	25.0	01.2	25.0	01.3	24.9	01.7	24.9	02.2	24.9	02.4	25
26	26.0	00.5	26.0	00.9	26.0	01.3	26.0	01.4	25.9	01.8	25.9	02.3	25.9	02.5	26
27	27.0	00.5	27.0	00.9	27.0	01.3	27.0	01.4	26.9	01.9	26.9	02.4	26.9	02.6	27
28	28.0	00.5	28.0	01.0	28.0	01.4	28.0	01.5	27.9	02.0	27.9	02.4	27.9	02.7	28
29	29.0	00.5	29.0	01.0	29.0	01.4	29.0	01.5	28.9	02.0	28.9	02.5	28.9	02.8	29
30	30.0	00.5	30.0	01.1	30.0	01.5	30.0	01.6	29.9	02.1	29.9	02.6	29.9	02.9	30
31	31.0	00.5	31.0	01.1	31.0	01.5	31.0	01.6	30.9	02.2	30.9	02.7	30.8	03.0	31
32	32.0	00.6	32.0	01.1	32.0	01.6	32.0	01.7	31.9	02.2	31.9	02.8	31.8	03.1	32
33	33.0	00.6	33.0	01.2	33.0	01.6	33.0	01.7	32.9	02.3	32.9	02.9	32.8	03.2	33
34	34.0	00.6	34.0	01.2	34.0	01.7	34.0	01.8	33.9	02.4	33.9	03.0	33.8	03.3	34
35	35.0	00.6	35.0	01.2	35.0	01.7	35.0	01.8	34.9	02.4	34.9	03.1	34.8	03.4	35
36	36.0	00.6	36.0	01.3	36.0	01.8	35.9	01.9	35.9	02.5	35.9	03.1	35.8	03.5	36
37	37.0	00.6	37.0	01.3	37.0	01.8	36.9	01.9	36.9	02.6	36.9	03.2	36.8	03.6	37
38	38.0	00.7	38.0	01.3	38.0	01.9	37.9	02.0	37.9	02.7	37.9	03.3	37.8	03.7	38
39	39.0	00.7	39.0	01.4	39.0	01.9	38.9	02.0	38.9	02.7	38.9	03.4	38.8	03.8	39
40	40.0	00.7	40.0	01.4	40.0	02.0	49.9	02.1	39.9	02.8	39.8	03.5	39.8	03.9	40
41	41.0	00.7	41.0	01.4	41.0	02.0	40.9	02.1	40.9	02.9	40.8	03.6	40.8	04.0	41
42	42.0	00.7	42.0	01.5	41.9	02.1	41.9	02.2	41.9	02.9	41.8	03.7	41.8	04.1	42
43	43.0	00.8	43.0	01.5	42.9	02.1	42.9	02.2	42.9	03.0	42.8	03.8	42.8	04.2	43
44	44.0	00.8	44.0	01.5	43.9	02.2	43.9	02.3	43.9	03.1	43.8	03.8	43.8	04.3	44
45	45.0	00.8	45.0	01.6	44.9	02.2	44.9	02.4	44.9	03.1	44.8	03.9	44.8	04.4	45
46	46.0	00.8	46.0	01.6	45.9	02.3	45.9	02.4	45.9	03.2	45.8	04.0	45.8	04.5	46
47	47.0	00.8	47.0	01.6	46.9	02.3	46.9	02.5	46.9	03.3	46.8	04.1	46.8	04.6	47
48	48.0	00.8	48.0	01.7	47.9	02.4	47.9	02.5	47.9	03.4	47.8	04.2	47.8	04.7	48
49	49.0	00.9	49.0	01.7	48.9	02.4	48.9	02.6	48.9	03.4	48.8	04.3	48.8	04.8	49
50	50.0	00.9	50.0	01.7	49.9	02.5	49.9	02.6	49.9	03.5	49.8	04.4	49.8	04.9	50
Dist.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dist.
	89 Deg.		88 Deg.		7¾ Point.		87 Deg.		86 Deg.		85 Deg.		7½ Point.		

Dist.	1 Deg.		2 Deg.		¼ Point.		3 Deg.		4 Deg.		5 Deg.		½ Point.		Dist.
	Lat.	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	
51	51.0	00.9	51.0	01.8	50.9	02.5	50.9	02.7	50.9	03.6	50.8	04.4	50.8	05.0	51
52	52.0	00.9	52.0	01.8	51.9	02.6	51.9	02.7	51.9	03.6	51.8	04.5	51.7	05.1	52
53	53.0	00.9	53.0	01.8	52.9	02.6	52.9	02.8	52.9	03.7	52.8	04.6	52.7	05.2	53
54	54.0	00.9	54.0	01.9	53.9	02.7	53.9	02.8	53.9	03.8	53.8	04.7	53.7	05.3	54
55	55.0	01.0	55.0	01.9	54.9	02.7	54.9	02.8	54.9	03.8	54.8	04.8	54.7	05.4	55
56	56.0	01.0	56.0	02.0	55.9	02.7	55.9	02.9	55.9	03.9	55.8	04.9	55.7	05.5	56
57	57.0	01.0	57.0	02.0	56.9	02.8	56.9	03.0	56.9	04.0	56.8	05.0	56.7	05.6	57
58	58.0	01.0	58.0	02.0	57.9	02.8	57.9	03.0	57.9	04.0	57.8	05.1	57.7	05.7	58
59	59.0	01.0	59.0	02.1	58.9	02.9	58.9	03.1	58.9	04.1	58.8	05.1	58.7	05.8	59
60	60.0	01.1	60.0	02.1	59.9	02.9	59.9	03.1	59.9	04.2	59.8	05.2	59.7	05.9	60
61	61.0	01.1	61.0	02.1	60.9	03.0	60.9	03.2	60.9	04.3	60.8	05.3	60.7	06.0	61
62	62.0	01.1	62.0	02.2	61.9	03.0	61.9	03.2	61.9	04.3	61.8	05.4	61.7	06.1	62
63	63.0	01.1	63.0	02.2	62.9	03.1	62.9	03.3	62.8	04.4	62.8	05.5	62.7	06.2	63
64	64.0	01.1	64.0	02.3	63.9	03.1	63.9	03.3	63.8	04.5	63.8	05.6	63.7	06.3	64
65	65.0	01.1	65.0	02.3	64.9	03.2	64.9	03.4	64.8	04.5	64.8	05.7	64.7	06.4	65
66	66.0	01.2	66.0	02.3	65.9	03.2	65.9	03.5	65.8	04.6	65.7	05.8	65.7	06.5	66
67	67.0	01.2	67.0	02.3	66.9	03.3	66.9	03.5	66.8	04.7	66.7	05.8	66.7	06.6	67
68	68.0	01.2	68.0	02.4	67.9	03.3	67.9	03.6	67.8	04.7	67.7	05.9	67.7	06.7	68
69	69.0	01.2	69.0	02.4	68.9	03.4	68.9	03.6	68.8	04.8	68.7	06.0	68.7	06.8	69
70	70.0	01.2	70.0	02.4	69.9	03.4	69.9	03.7	69.8	04.9	69.7	06.1	69.7	06.9	70
71	71.0	01.2	71.0	02.5	70.9	03.5	70.9	03.7	70.8	05.0	70.7	06.2	70.7	07.0	71
72	72.0	01.3	72.0	02.5	71.9	03.5	71.9	03.8	71.8	05.0	71.7	06.3	71.7	07.1	72
73	73.0	01.3	73.0	02.5	72.9	03.6	72.9	03.8	72.8	05.1	72.7	06.4	72.6	07.2	73
74	74.0	01.3	74.0	02.6	73.9	03.6	73.9	03.9	73.8	05.2	73.7	06.5	73.6	07.3	74
75	75.0	01.3	75.0	02.6	74.9	03.7	74.9	03.9	74.8	05.2	74.7	06.5	74.6	07.3	75
76	76.0	01.3	76.0	02.7	75.9	03.7	75.9	04.0	75.8	05.3	75.7	06.6	75.6	07.4	76
77	77.0	01.3	77.0	02.7	76.9	03.8	76.9	04.0	76.8	05.4	76.7	06.7	76.6	07.5	77
78	78.0	01.4	78.0	02.7	77.9	03.8	77.9	04.1	77.8	05.4	77.7	06.8	77.6	07.6	78
79	79.0	01.4	79.0	02.8	78.9	03.9	78.9	04.1	78.8	05.5	78.7	06.9	78.6	07.7	79
80	80.0	01.4	80.0	02.8	79.9	03.9	79.9	04.2	79.8	05.6	79.7	07.0	79.6	07.8	80
81	81.0	01.4	81.0	02.8	80.9	04.0	80.9	04.2	80.8	05.7	80.7	07.1	80.6	07.9	81
82	82.0	01.4	81.9	02.9	81.9	04.0	81.9	04.3	81.8	05.7	81.7	07.2	81.6	08.0	82
83	83.0	01.5	82.9	02.9	82.9	04.1	82.9	04.3	82.8	05.8	82.7	07.2	82.6	08.1	83
84	84.0	01.5	83.9	02.9	83.9	04.1	83.9	04.4	83.8	05.9	83.7	07.3	83.6	08.2	84
85	85.0	01.5	84.9	03.0	84.9	04.2	84.9	04.4	84.8	05.9	84.7	07.4	84.6	08.3	85
86	86.0	01.5	85.9	03.0	85.9	04.2	85.9	04.5	85.8	06.0	85.7	07.5	85.6	08.4	86
87	87.0	01.5	86.9	03.0	86.9	04.3	86.9	04.6	86.8	06.1	86.7	07.6	86.6	08.5	87
88	88.0	01.5	87.9	03.1	87.9	04.3	87.9	04.6	87.8	06.1	87.7	07.7	87.6	08.6	88
89	89.0	01.6	88.9	03.1	88.9	04.4	88.9	04.7	88.8	06.2	88.7	07.8	88.6	08.7	89
90	90.0	01.6	89.9	03.1	89.9	04.4	89.9	04.7	89.8	06.3	89.7	07.8	89.6	08.8	90
91	91.0	01.6	90.9	03.2	90.9	04.5	90.9	04.8	90.8	06.4	90.7	07.9	90.6	08.9	91
92	92.0	01.6	91.9	03.2	91.9	04.5	91.9	04.8	91.8	06.4	91.6	08.0	91.6	09.0	92
93	93.0	01.6	92.9	03.2	92.9	04.6	92.9	04.9	92.8	06.5	92.6	08.1	92.6	09.1	93
94	94.0	01.6	93.9	03.3	93.9	04.6	93.9	04.9	93.8	06.6	93.6	08.2	93.5	09.2	94
95	95.0	01.7	94.9	03.3	94.9	04.7	94.9	05.0	94.8	06.6	94.6	08.3	94.5	09.3	95
96	96.0	01.7	95.9	03.4	95.9	04.7	95.9	05.0	95.8	06.7	95.6	08.4	95.5	09.4	96
97	97.0	01.7	96.9	03.4	96.9	04.8	96.9	05.1	96.8	06.8	96.6	08.5	96.5	09.5	97
98	98.0	01.7	97.9	03.4	97.9	04.8	97.9	05.1	97.8	06.8	97.6	08.5	97.5	09.6	98
99	99.0	01.7	98.9	03.5	98.9	04.9	98.9	05.2	98.8	06.9	98.6	08.6	98.5	09.7	99
100	100.0	01.7	99.9	03.5	99.9	04.9	99.9	05.2	99.8	07.0	99.6	08.7	99.5	09.8	100
	Dep.	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	
Dist.	89 Deg.		88 Deg.		7¾ Point.		87 Deg.		86 Deg.		85 Deg.		7½ Point.		Dist.

Dist.	6 Deg. Lat.	6 Deg. Dep.	7 Deg. Lat.	7 Deg. Dep.	8 Deg. Lat.	8 Deg. Dep.	¾ Point. Lat.	¾ Point. Dep.	9 Deg. Lat.	9 Deg. Dep.	10 Deg. Lat.	10 Deg. Dep.	11 Deg. Lat.	11 Deg. Dep.	Dist.
1	01.0	00.1	01.0	00.1	01.0	00.1	01.0	00.1	01.0	00.2	01.0	00.2	01.0	00.2	1
2	02.0	00.2	02.0	00.2	02.0	00.3	02.0	00.3	02.0	00.3	02.0	00.3	02.0	00.4	2
3	03.0	00.3	03.0	00.4	03.0	00.4	03.0	00.4	03.0	00.5	03.0	00.5	02.9	00.6	3
4	04.0	00.4	04.0	00.5	04.0	00.6	04.0	00.6	03.9	00.6	03.9	00.7	03.9	00.8	4
5	05.0	00.5	05.0	00.6	05.0	00.7	04.9	00.7	04.9	00.8	04.9	00.9	04.9	01.0	5
6	06.0	00.6	06.0	00.7	05.9	00.8	05.9	00.9	05.9	00.9	05.9	01.0	05.9	01.1	6
7	07.0	00.7	06.9	00.9	06.9	01.0	06.9	01.0	06.9	01.1	06.9	01.2	06.9	01.3	7
8	08.0	00.8	07.9	01.0	07.9	01.1	07.9	01.2	07.9	01.3	07.9	01.4	07.9	01.5	8
9	08.9	00.9	08.9	01.1	08.9	01.2	08.9	01.3	08.9	01.4	08.9	01.6	08.8	01.7	9
10	09.9	01.0	09.9	01.2	09.9	01.4	09.9	01.5	09.9	01.6	09.8	01.7	09.8	01.9	10
11	10.9	01.1	10.9	01.3	10.9	01.5	10.9	01.6	10.9	01.7	10.8	01.9	10.8	02.1	11
12	11.9	01.3	11.9	01.5	11.9	01.7	11.9	01.8	11.9	01.9	11.8	02.1	11.8	02.3	12
13	12.9	01.4	12.9	01.6	12.9	01.8	12.9	01.9	12.8	02.0	12.8	02.3	12.8	02.5	13
14	13.9	01.5	13.9	01.7	13.9	01.9	13.8	02.1	13.8	02.2	13.8	02.4	13.7	02.7	14
15	14.9	01.6	14.9	01.8	14.9	02.1	14.8	02.2	14.8	02.3	14.8	02.6	14.7	02.9	15
16	15.9	01.7	15.9	01.9	15.8	02.2	15.8	02.3	15.8	02.5	15.8	02.8	15.7	03.1	16
17	16.9	01.8	16.9	02.1	16.8	02.4	16.8	02.5	16.8	02.7	16.7	03.0	16.7	03.2	17
18	17.9	01.9	17.9	02.2	17.8	02.5	17.8	02.6	17.8	02.8	17.7	03.1	17.7	03.4	18
19	18.9	02.0	18.9	02.3	18.8	02.6	18.8	02.8	18.8	03.0	18.7	03.3	18.6	03.6	19
20	19.9	02.1	19.8	02.4	19.8	02.8	19.8	02.9	19.8	03.1	19.7	03.5	19.6	03.8	20
21	20.9	02.2	20.8	02.6	20.8	02.9	20.8	03.1	20.7	03.3	20.7	03.6	20.6	04.0	21
22	21.9	02.3	21.8	02.7	21.8	03.1	21.8	03.2	21.7	03.4	21.7	03.8	21.6	04.2	22
23	22.9	02.4	22.8	02.8	22.8	03.2	22.8	03.4	22.7	03.6	22.6	04.0	22.6	04.4	23
24	23.9	02.5	23.8	02.9	23.8	03.3	23.7	03.5	23.7	03.8	23.6	04.2	23.6	04.6	24
25	24.9	02.6	24.8	03.0	24.8	03.5	24.7	03.7	24.7	03.9	24.6	04.3	24.5	04.8	25
26	25.9	02.7	25.8	03.2	25.7	03.6	25.7	03.8	25.7	04.1	25.6	04.5	25.5	05.0	26
27	26.9	02.8	26.8	03.3	26.7	03.8	26.7	04.0	26.7	04.2	26.6	04.7	26.5	05.2	27
28	27.8	02.9	27.8	03.4	27.7	03.9	27.7	04.1	27.7	04.4	27.6	04.9	27.5	05.3	28
29	28.8	03.0	28.8	03.5	28.7	04.0	28.7	04.3	28.6	04.5	28.6	05.0	28.5	05.5	29
30	29.8	03.1	29.8	03.7	29.7	04.2	29.7	04.4	29.6	04.7	29.5	05.2	29.4	05.7	30
31	30.8	03.2	30.8	03.8	30.7	04.3	30.7	04.5	30.6	04.9	30.5	05.4	30.4	05.9	31
32	31.8	03.3	31.8	03.9	31.7	04.5	31.7	04.7	31.6	05.1	31.5	05.6	31.4	06.1	32
33	32.8	03.4	32.8	04.0	32.7	04.6	32.6	04.8	32.6	05.2	32.5	05.7	32.4	06.3	33
34	33.8	03.6	33.7	04.1	33.7	04.7	33.6	05.0	33.6	05.4	33.5	05.9	33.4	06.5	34
35	34.8	03.7	34.7	04.3	34.7	04.9	34.6	05.1	34.6	05.5	34.5	06.1	34.4	06.7	35
36	35.8	03.8	35.7	04.4	35.6	05.0	35.6	05.3	35.6	05.6	35.4	06.2	35.3	06.9	36
37	36.8	03.9	36.7	04.5	36.6	05.1	36.6	05.4	36.5	05.8	36.4	06.4	36.3	07.1	37
38	37.8	04.0	37.7	04.6	37.6	05.3	37.6	05.6	37.5	05.9	37.4	06.6	37.3	07.3	38
39	38.8	04.1	38.7	04.8	38.6	05.4	38.6	05.7	38.5	06.1	38.4	06.8	38.3	07.4	39
40	39.8	04.2	39.7	04.9	39.6	05.6	39.6	05.9	39.5	06.3	39.4	06.9	39.3	07.6	40
41	40.8	04.3	40.7	05.0	40.6	05.7	40.6	06.0	40.5	06.4	40.4	07.1	40.2	07.8	41
42	41.8	04.4	41.7	05.1	41.6	05.8	41.5	06.2	41.5	06.6	41.4	07.3	41.2	08.0	42
43	42.8	04.5	42.7	05.2	42.6	06.0	42.5	06.3	42.5	06.7	42.3	07.5	42.2	08.2	43
44	43.8	04.6	43.7	05.4	43.6	06.1	43.5	06.5	43.5	06.9	43.3	07.6	43.2	08.4	44
45	44.8	04.7	44.7	05.5	44.6	06.3	44.5	06.6	44.4	07.0	44.3	07.8	44.2	08.6	45
46	45.7	04.8	45.7	05.6	45.6	06.4	45.5	06.7	45.4	07.2	45.3	08.0	45.2	08.8	46
47	46.7	04.9	46.6	05.7	46.5	06.6	46.5	06.9	46.4	07.3	46.3	08.2	46.1	09.0	47
48	47.7	05.0	47.6	05.9	47.5	06.7	47.5	07.0	47.4	07.5	47.3	08.3	47.1	09.2	48
49	48.7	05.1	48.6	06.0	48.5	06.8	48.5	07.2	48.4	07.7	48.3	08.5	48.1	09.3	49
50	49.7	05.2	49.6	06.1	49.5	07.0	49.5	07.3	49.4	07.8	49.2	08.7	49.1	09.5	50
Dist.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dist.
	84 Deg.		83 Deg.		82 Deg.		7¼ Point.		81 Deg.		80 Deg.		79 Deg.		

Dist.	6 Deg.		7 Deg.		8 Deg.		¾ Point.		9 Deg.		10 Deg.		11 Deg.		Dist.
	Lat.	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	
51	50.7	05.3	50.6	06.2	50.5	07.1	50.4	07.5	50.4	08.0	50.2	08.9	50.1	09.7	51
52	51.7	05.4	51.6	06.3	51.5	07.2	51.4	07.6	51.4	08.1	51.2	09.0	51.0	09.9	52
53	52.7	05.5	52.6	06.5	52.5	07.4	52.4	07.8	52.3	08.3	52.2	09.2	52.0	10.1	53
54	53.7	05.6	53.6	06.6	53.5	07.5	53.4	07.9	53.3	08.4	53.2	09.4	53.0	10.3	54
55	54.7	05.7	54.6	06.7	54.5	07.7	54.4	08.1	54.3	08.6	54.2	09.5	54.0	10.5	55
56	55.7	05.9	55.6	06.8	55.5	07.8	55.4	08.2	55.3	08.8	55.1	09.7	55.0	10.7	56
57	56.7	06.0	56.6	06.9	56.4	07.9	56.4	08.4	56.3	08.9	56.1	09.9	56.0	10.9	57
58	57.7	06.1	57.6	07.1	57.4	08.1	57.4	08.5	57.3	09.1	57.1	10.1	56.9	11.1	58
59	58.7	06.2	58.6	07.2	58.4	08.2	58.4	08.7	58.3	09.2	58.1	10.2	57.9	11.3	59
60	59.7	06.3	59.5	07.3	59.4	08.4	59.4	08.8	59.3	09.4	59.1	10.4	58.9	11.4	60
61	60.7	06.4	60.5	07.4	60.4	08.5	60.3	08.9	60.2	09.5	60.1	10.6	59.9	11.6	61
62	61.7	06.5	61.5	07.6	61.4	08.6	61.3	09.1	61.2	09.7	61.1	10.8	60.9	11.8	62
63	62.7	06.6	62.5	07.7	62.4	08.8	62.3	09.2	62.2	09.9	62.0	10.9	61.8	12.0	63
64	63.6	06.7	63.5	07.8	63.4	08.9	63.3	09.4	63.2	10.0	63.0	11.1	62.8	12.2	64
65	64.6	06.8	64.5	07.9	64.4	09.0	64.3	09.5	64.2	10.2	64.0	11.3	63.8	12.4	65
66	65.6	06.9	65.5	08.0	65.4	09.2	65.3	09.7	65.2	10.3	65.0	11.5	64.8	12.6	66
67	66.6	07.0	66.5	08.2	66.3	09.3	66.3	09.8	66.2	10.5	66.0	11.6	65.8	12.8	67
68	67.6	07.1	67.5	08.3	67.3	09.5	67.3	10.0	67.2	10.6	67.0	11.8	66.7	13.0	68
69	68.6	07.2	68.5	08.4	68.3	09.6	68.3	10.1	68.2	10.8	68.0	12.0	67.7	13.2	69
70	69.6	07.3	69.5	08.5	69.3	09.7	69.2	10.3	69.1	10.9	68.9	12.2	68.7	13.4	70
71	70.6	07.4	70.5	08.7	70.3	09.9	70.2	10.4	70.1	11.1	69.9	12.3	69.7	13.5	71
72	71.6	07.5	71.5	08.8	71.3	10.0	71.2	10.6	71.1	11.3	70.9	12.5	70.7	13.7	72
73	72.6	07.6	72.5	08.9	72.3	10.2	72.2	10.7	72.1	11.4	71.9	12.7	71.7	13.9	73
74	73.6	07.7	73.4	09.0	73.3	10.3	73.2	10.9	73.1	11.6	72.9	12.8	72.6	14.1	74
75	74.6	07.8	74.4	09.1	74.3	10.4	74.2	11.0	74.1	11.7	73.9	13.0	73.6	14.3	75
76	75.6	07.9	75.4	09.3	75.3	10.6	75.2	11.1	75.1	11.9	74.8	13.2	74.6	14.5	76
77	76.6	08.0	76.4	09.4	76.3	10.7	76.2	11.3	76.1	12.0	75.8	13.4	75.6	14.7	77
78	77.6	08.1	77.4	09.5	77.2	10.9	77.2	11.4	77.0	12.2	76.8	13.5	76.6	14.9	78
79	78.6	08.3	78.4	09.6	78.2	11.0	78.1	11.6	78.0	12.4	77.8	13.7	77.5	15.1	79
80	79.6	08.4	79.4	09.8	79.2	11.1	79.1	11.7	79.0	12.5	78.8	13.9	78.5	15.3	80
81	80.6	08.5	80.4	09.9	80.2	11.3	80.1	11.9	80.0	12.7	79.8	14.1	79.5	15.5	81
82	81.5	08.6	81.4	10.0	81.2	11.4	81.1	12.0	81.0	12.8	80.8	14.2	80.5	15.6	82
83	82.5	08.7	82.4	10.1	82.2	11.6	82.1	12.2	82.0	13.0	81.7	14.4	81.5	15.8	83
84	83.5	08.8	83.4	10.2	83.2	11.7	83.1	12.3	83.0	13.1	82.7	14.6	82.5	16.0	84
85	84.5	08.9	84.4	10.4	84.2	11.8	84.1	12.5	84.0	13.3	83.7	14.8	83.4	16.2	85
86	85.5	09.0	85.4	10.5	85.2	12.0	85.1	12.6	84.9	13.4	84.7	14.9	84.4	16.4	86
87	86.5	09.1	86.3	10.6	86.2	12.1	86.0	12.8	85.9	13.6	85.7	15.1	85.4	16.6	87
88	87.5	09.2	87.3	10.7	87.1	12.2	87.0	12.9	86.9	13.8	86.7	15.3	86.4	16.8	88
89	88.5	09.3	88.3	10.8	88.1	12.4	88.0	13.1	87.9	13.9	87.6	15.4	87.4	17.0	89
90	89.5	09.4	89.3	11.0	89.1	12.5	89.0	13.2	88.9	14.1	88.6	15.6	88.3	17.2	90
91	90.5	09.5	90.3	11.1	90.1	12.7	90.0	13.3	89.9	14.2	89.6	15.8	89.3	17.4	91
92	91.5	09.6	91.3	11.2	91.1	12.8	91.0	13.5	90.9	14.4	90.6	16.0	90.3	17.6	92
93	92.5	09.7	92.3	11.3	92.1	12.9	92.0	13.6	91.9	14.5	91.6	16.1	91.3	17.7	93
94	93.5	09.8	93.3	11.5	93.1	13.1	93.0	13.8	92.8	14.7	92.6	16.3	92.3	17.9	94
95	94.5	09.9	94.3	11.6	94.1	13.2	94.0	13.9	93.8	14.9	93.6	16.5	93.3	18.1	95
96	95.5	10.0	95.3	11.7	95.1	13.4	95.0	14.1	94.8	15.0	94.5	16.7	94.2	18.3	96
97	96.5	10.1	96.3	11.8	96.1	13.5	96.0	14.2	95.8	15.2	95.5	16.8	95.2	18.5	97
98	97.5	10.2	97.3	11.9	97.0	13.6	96.9	14.4	96.8	15.3	96.5	17.0	96.2	18.7	98
99	98.5	10.3	98.3	12.1	98.0	13.8	97.9	14.5	07.8	15.5	97.5	17.2	97.2	18.9	99
100	99.4	10.4	99.2	12.2	99.0	13.9	98.9	14.7	98.8	15.6	98.5	17.4	98.2	19.1	100
Dist.	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	Dist.
	84 Deg.		83 Deg.		82 Deg.		7¼ Point.		81 Deg.		80 Deg.		79 Deg.		

Dist.	1 Point.		12 Deg.		13 Deg.		14 Deg.		$1\frac{1}{4}$ Point.		15 Deg.		16 Deg.		Dist.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	01.0	00.2	01.0	00.2	01.0	00.2	01.0	00.2	01.0	00.2	01.0	00.3	01.0	00.3	1
2	02.0	00.4	02.0	00.4	01.9	00.4	01.9	00.5	01.9	00.5	01.9	00.5	01.9	00.6	2
3	02.9	00.6	02.9	00.6	02.9	00.7	02.9	00.7	02.9	00.7	02.9	00.8	02.9	00.8	3
4	03.9	00.8	03.9	00.8	03.9	00.9	03.9	01.0	03.9	01.0	03.9	01.0	03.8	01.1	4
5	04.9	01.0	04.9	01.0	04.9	01.1	04.9	01.2	04.8	01.2	04.8	01.3	04.8	01.4	5
6	05.9	01.2	05.9	01.2	05.8	01.3	05.8	01.4	05.8	01.5	05.8	01.5	05.8	01.7	6
7	06.9	01.4	06.8	01.5	06.8	01.6	06.8	01.7	06.8	01.7	06.8	01.8	06.7	01.9	7
8	07.8	01.6	07.8	01.7	07.8	01.8	07.8	01.9	07.8	01.9	07.7	02.1	07.7	02.2	8
9	08.8	01.8	08.8	01.9	08.8	02.0	08.7	02.2	08.7	02.2	08.7	02.3	08.7	02.5	9
10	09.8	02.0	09.8	02.1	09.7	02.2	09.7	02.4	09.7	02.4	09.7	02.6	09.6	02.8	10
11	10.8	02.1	10.8	02.3	10.7	02.5	10.7	02.7	10.7	02.7	10.6	02.8	10.6	03.0	11
12	11.8	02.3	11.7	02.5	11.7	02.7	11.6	02.9	11.6	02.9	11.6	03.1	11.5	03.3	12
13	12.7	02.5	12.7	02.7	12.7	02.9	12.6	03.1	12.6	03.2	12.6	03.4	12.5	03.6	13
14	13.7	02.7	13.7	02.9	13.6	03.1	13.6	03.4	13.6	03.4	13.5	03.6	13.5	03.9	14
15	14.7	02.9	14.7	03.1	14.6	03.4	14.6	03.6	14.5	03.6	14.5	03.9	14.4	04.1	15
16	15.7	03.1	15.6	03.3	15.6	03.6	15.5	03.9	15.5	03.9	15.5	04.1	15.4	04.4	16
17	16.7	03.3	16.6	03.5	16.6	03.8	16.5	04.1	16.5	04.1	16.4	04.4	16.3	04.7	17
18	17.7	03.5	17.6	03.7	17.5	04.0	17.5	04.4	17.5	04.4	17.4	04.7	17.3	05.0	18
19	18.6	03.7	18.6	03.9	18.5	04.3	18.4	04.6	18.4	04.6	18.4	04.9	18.3	05.2	19
20	19.6	03.9	19.6	04.2	19.5	04.5	19.4	04.8	19.4	04.9	19.3	05.2	19.2	05.5	20
21	20.6	04.1	20.5	04.4	20.5	04.7	20.4	05.1	20.4	05.1	20.3	05.4	20.2	05.8	21
22	21.6	04.3	21.5	04.6	21.4	04.9	21.3	05.3	21.3	05.3	21.2	05.7	21.1	06.1	22
23	22.6	04.5	22.5	04.8	22.4	05.2	22.3	05.6	22.3	05.6	22.2	06.0	22.1	06.3	23
24	23.5	04.7	23.5	05.0	23.4	05.4	23.3	05.8	23.3	05.8	23.2	06.2	23.1	06.6	24
25	24.5	04.9	24.5	05.2	24.4	05.6	24.3	06.0	24.2	06.1	24.1	06.5	24.0	06.9	25
26	25.5	05.1	25.4	05.4	25.3	05.8	25.2	06.3	25.2	06.3	25.1	06.7	25.0	07.2	26
27	26.5	05.3	26.4	05.6	26.3	06.1	26.2	06.5	26.2	06.6	26.1	07.0	26.0	07.4	27
28	27.5	05.5	27.4	05.8	27.3	06.3	27.2	06.8	27.2	06.8	27.0	07.2	26.9	07.7	28
29	28.4	05.7	28.4	06.0	28.3	06.5	28.1	07.0	28.1	07.0	28.0	07.5	27.9	08.0	29
30	29.4	05.9	29.3	06.2	29.2	06.7	29.1	07.3	29.1	07.3	29.0	07.8	28.8	08.3	30
31	30.4	06.0	30.3	06.4	30.2	07.0	30.1	07.5	30.1	07.5	29.9	08.0	29.8	08.5	31
32	31.4	06.2	31.3	06.7	31.2	07.2	31.0	07.7	31.0	07.8	30.9	08.3	30.8	08.8	32
33	32.4	06.4	32.3	06.9	32.2	07.4	32.0	08.0	32.0	08.0	31.9	08.5	31.7	09.1	33
34	33.3	06.6	33.3	07.1	33.1	07.6	33.0	08.2	33.0	08.3	32.8	08.8	32.7	09.4	34
35	34.3	06.8	34.2	07.3	34.1	07.9	34.0	08.5	33.9	08.5	33.8	09.1	33.6	09.6	35
36	35.3	07.0	35.2	07.5	35.1	08.1	34.9	08.7	34.9	08.7	34.8	09.3	34.6	09.9	36
37	36.3	07.2	36.2	07.7	36.1	08.3	35.9	09.0	35.9	09.0	35.7	09.6	35.6	10.2	37
38	37.3	07.4	37.2	07.9	37.0	08.5	36.9	09.2	36.9	09.2	36.7	09.8	36.6	10.5	38
39	38.3	07.6	38.1	08.1	38.0	08.8	37.8	09.4	37.8	09.5	37.7	10.1	37.5	10.7	39
40	39.2	07.8	39.1	08.3	39.0	09.0	38.8	09.7	38.8	09.7	38.6	10.4	38.5	11.0	40
41	40.2	08.0	40.1	08.5	39.9	09.2	39.8	09.9	39.8	10.0	39.6	10.6	39.4	11.3	41
42	41.2	08.2	41.1	08.7	40.9	09.4	40.8	10.2	40.7	10.2	40.6	10.9	40.4	11.6	42
43	42.2	08.4	42.1	08.9	41.9	09.7	41.7	10.4	41.7	10.4	41.5	11.1	41.3	11.8	43
44	43.2	08.6	43.0	09.1	42.9	09.9	42.7	10.6	42.7	10.7	42.5	11.4	42.3	12.1	44
45	44.1	08.8	44.0	09.4	43.8	10.1	43.7	10.9	43.6	10.9	43.5	11.6	43.3	12.4	45
46	45.1	09.0	45.0	09.6	44.8	10.3	44.6	11.1	44.6	11.2	44.4	11.9	44.2	12.7	46
47	46.1	09.2	46.0	09.8	45.8	10.6	45.6	11.4	45.6	11.4	45.4	12.2	45.2	13.0	47
48	47.1	09.4	47.0	10.0	46.8	10.8	46.6	11.6	46.6	11.7	46.4	12.4	46.1	13.2	48
49	48.1	09.6	47.9	10.2	47.7	11.0	47.5	11.9	47.5	11.9	47.3	12.7	47.1	13.5	49
50	49.0	09.8	48.9	10.4	48.7	11.2	48.5	12.1	48.5	12.1	48.3	12.9	48.1	13.8	50
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
Dist.	7 Point.		78 Deg.		77 Deg.		76 Deg.		$6\frac{3}{4}$ Point.		75 Deg.		74 Deg.		Dist.

Dist.	1 Point		12 Deg.		13 Deg.		14 Deg.		1¼ Point.		15 Deg.		16 Deg.		Dist.
	Lat.	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	
51	50.0	10.0	49.9	10.6	49.7	11.5	49.5	12.3	49.5	12.4	49.3	13.2	49.0	14.1	51
52	51.0	10.1	50.9	10.8	50.7	11.7	50.5	12.6	50.4	12.6	50.2	13.5	50.0	14.3	52
53	52.0	10.3	51.8	11.0	51.6	11.9	51.4	12.8	51.4	12.9	51.2	13.7	50.9	14.6	53
54	53.0	10.5	52.8	11.2	52.6	12.1	52.4	13.1	52.4	13.1	52.2	14.0	51.9	14.9	54
55	53.9	10.7	53.8	11.4	53.6	12.4	53.4	13.3	53.3	13.4	53.1	14.2	52.9	15.2	55
56	54.9	10.9	54.8	11.6	54.6	12.6	54.3	13.5	54.3	13.6	54.1	14.5	53.8	15.4	56
57	55.9	11.1	55.8	11.8	55.5	12.8	55.3	13.8	55.3	13.9	55.1	14.8	54.8	15.7	57
58	56.9	11.3	56.7	12.1	56.5	13.0	56.3	14.0	56.3	14.1	56.0	15.0	55.8	16.0	58
59	57.9	11.5	57.7	12.3	57.5	13.3	57.2	14.3	57.2	14.3	57.0	15.3	56.7	16.3	59
60	58.8	11.7	58.7	12.5	58.5	13.5	58.2	14.5	58.2	14.6	58.0	15.5	57.7	16.5	60
61	59.8	11.9	59.7	12.7	59.4	13.7	59.2	14.8	59.2	14.8	58.9	15.8	58.6	16.8	61
62	60.8	12.1	60.6	12.9	60.4	13.9	60.2	15.0	60.1	15.1	59.9	16.0	59.6	17.1	62
63	61.8	12.3	61.6	13.1	61.4	14.2	61.1	15.2	61.1	15.3	60.9	16.3	60.6	17.4	63
64	62.8	12.5	62.6	13.3	62.4	14.4	62.1	15.5	62.1	15.6	61.8	16.6	61.5	17.6	64
65	63.8	12.7	63.6	13.5	63.3	14.6	63.1	15.7	63.0	15.8	62.8	16.8	62.5	17.9	65
66	64.7	12.9	64.6	13.7	64.3	14.8	64.0	16.0	64.0	16.0	63.7	17.1	63.4	18.2	66
67	65.7	13.1	65.5	13.9	65.3	15.1	65.0	16.2	65.0	16.3	64.7	17.3	64.4	18.5	67
68	66.7	13.3	66.5	14.1	66.3	15.3	66.0	16.4	66.0	16.5	65.7	17.6	65.4	18.7	68
69	67.7	13.5	67.5	14.3	67.2	15.5	66.9	16.7	66.9	16.8	66.6	17.9	66.3	19.0	69
70	68.7	13.7	68.5	14.6	68.2	15.7	67.9	16.9	67.9	17.0	67.6	18.1	67.3	19.3	70
71	69.6	13.9	69.4	14.8	69.2	16.0	68.9	17.2	68.9	17.3	68.6	18.4	68.2	19.6	71
72	70.6	14.0	70.4	15.0	70.2	16.2	69.9	17.4	69.8	17.5	69.5	18.6	69.2	19.8	72
73	71.6	14.2	71.4	15.2	71.1	16.4	70.8	17.7	70.8	17.7	70.5	18.9	70.2	20.1	73
74	72.6	14.4	72.4	15.4	72.1	16.6	71.8	17.9	71.8	18.0	71.5	19.2	71.1	20.4	74
75	73.6	14.6	73.4	15.6	73.1	16.9	72.8	18.1	72.7	18.2	72.4	19.4	72.1	20.7	75
76	74.5	14.8	74.3	15.8	74.1	17.1	73.7	18.4	73.7	18.5	73.4	19.7	73.1	20.9	76
77	75.5	15.0	75.3	16.0	75.0	17.3	74.7	18.6	74.7	18.7	74.4	19.9	74.0	21.2	77
78	76.5	15.2	76.3	16.2	76.0	17.5	75.7	18.9	75.7	18.9	75.3	20.2	75.0	21.5	78
79	77.5	15.4	77.3	16.4	77.0	17.8	76.7	19.1	76.6	19.2	76.3	20.4	75.9	21.8	79
80	78.5	15.6	78.2	16.6	78.0	18.0	77.6	19.4	77.6	19.4	77.3	20.7	76.9	22.0	80
81	79.4	15.8	79.2	16.8	78.9	18.2	78.6	19.6	78.6	19.7	78.2	21.0	77.9	22.3	81
82	80.4	16.0	80.2	17.0	79.9	18.4	79.6	19.8	79.5	19.9	79.2	21.2	78.8	22.6	82
83	81.4	16.2	81.2	17.3	80.9	18.7	80.5	20.1	80.5	20.2	80.2	21.5	79.8	22.9	83
84	82.4	16.4	82.2	17.5	81.8	18.9	81.5	20.3	81.5	20.4	81.1	21.7	80.7	23.1	84
85	83.4	16.6	83.1	17.7	82.8	19.1	82.5	20.6	82.4	20.7	82.1	22.0	81.7	23.4	85
86	84.3	16.8	84.1	17.9	83.8	19.3	83.4	20.8	83.4	20.9	83.1	22.3	82.7	23.7	86
87	85.3	17.0	85.1	18.1	84.8	19.6	84.4	21.0	84.4	21.1	84.0	22.5	83.6	24.0	87
88	86.3	17.2	86.1	18.3	85.7	19.8	85.4	21.3	85.4	21.4	85.0	22.8	84.6	24.3	88
89	87.3	17.4	87.1	18.5	86.7	20.0	86.4	21.5	86.3	21.6	86.0	23.0	85.6	24.5	89
90	88.3	17.6	88.0	18.7	87.7	20.2	87.3	21.8	87.3	21.9	86.9	23.3	86.5	24.8	90
91	89.3	17.8	89.0	18.9	88.7	20.5	88.3	22.0	88.3	22.1	87.9	23.5	87.5	25.1	91
92	90.2	17.9	90.0	19.1	89.6	20.7	89.3	22.3	89.2	22.4	88.9	23.8	88.4	25.4	92
93	91.2	18.1	91.0	19.3	90.6	20.9	90.2	22.5	90.2	22.6	89.8	24.1	89.4	25.6	93
94	92.2	18.3	91.9	19.5	91.6	21.1	91.2	22.7	91.2	22.8	90.8	24.3	90.4	25.9	94
95	93.2	18.5	92.9	19.7	92.6	21.4	92.2	23.0	92.1	23.1	91.8	24.6	91.3	26.2	95
96	94.2	18.7	93.9	20.0	93.5	21.6	93.1	23.2	93.1	23.3	92.7	24.8	92.3	26.5	96
97	95.1	18.9	94.9	20.2	94.5	21.8	94.1	23.5	94.1	23.6	93.7	25.1	93.2	26.7	97
98	96.1	19.1	95.9	20.4	95.5	22.0	95.1	23.7	95.1	23.8	94.7	25.4	94.2	27.0	98
99	97.1	19.3	96.8	20.6	96.5	22.3	96.1	23.9	96.0	24.1	95.6	25.6	95.2	27.3	99
100	98.1	19.5	97.8	20.8	97.4	22.5	97.0	24.2	97.0	24.3	96.6	25.9	96.1	27.6	100
Dist.	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	Dist.
	7 Point.		78 Deg.		77 Deg.		76 Deg.		6¾ Point.		75 Deg.		74 Deg.		

Dist.	1½ Point.		17 Deg.		18 Deg.		19 Deg.		1¾ Point.		20 Deg.		21 Deg.		Dist.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	01.0	00.3	01.0	00.3	01.0	00.3	00.9	00.3	00.9	00.3	00.9	00.3	00.9	00.4	1
2	01.9	00.6	01.9	00.6	01.9	00.6	01.9	00.7	01.9	00.7	01.9	00.7	01.9	00.7	2
3	02.9	00.9	02.9	00.9	02.9	00.9	02.8	01.0	02.8	01.0	02.8	01.0	02.8	01.1	3
4	03.8	01.2	03.8	01.2	03.8	01.2	03.8	01.3	03.8	01.3	03.8	01.4	03.7	01.4	4
5	04.8	01.5	04.8	01.5	04.8	01.5	04.7	01.6	04.7	01.7	04.7	01.7	04.7	01.8	5
6	05.7	01.7	05.7	01.8	05.7	01.9	05.7	02.0	05.6	02.0	05.6	02.1	05.6	02.1	6
7	06.7	02.0	06.7	02.0	06.7	02.2	06.6	02.3	06.6	02.4	06.6	02.4	06.5	02.5	7
8	07.7	02.3	07.6	02.3	07.6	02.5	07.6	02.6	07.5	02.7	07.5	02.7	07.5	02.9	8
9	08.6	02.6	08.6	02.6	08.6	02.8	08.5	02.9	08.5	03.0	08.5	03.1	08.4	03.2	9
10	09.6	02.9	09.6	02.9	09.5	03.1	09.5	03.3	09.4	03.4	09.4	03.4	09.3	03.6	10
11	10.5	03.2	10.5	03.2	10.5	03.4	10.4	03.6	10.4	03.7	10.3	03.8	10.3	03.9	11
12	11.5	03.5	11.5	03.5	11.4	03.7	11.3	03.9	11.3	04.0	11.3	04.1	11.2	04.3	12
13	12.4	03.8	12.4	03.8	12.4	04.0	12.3	04.2	12.2	04.4	12.2	04.4	12.1	04.7	13
14	13.4	04.1	13.4	04.1	13.3	04.3	13.2	04.6	13.2	04.7	13.2	04.8	13.1	05.0	14
15	14.4	04.4	14.3	04.4	14.3	04.6	14.2	04.9	14.1	05.1	14.1	05.1	14.0	05.4	15
16	15.3	04.6	15.3	04.7	15.2	04.9	15.1	05.2	15.1	05.4	15.0	05.5	14.9	05.7	16
17	16.3	04.9	16.3	05.0	16.2	05.3	16.1	05.5	16.0	05.7	16.0	05.8	15.9	06.1	17
18	17.2	05.2	17.2	05.3	17.1	05.6	17.0	05.9	16.9	06.1	16.9	06.2	16.8	06.4	18
19	18.2	05.5	18.2	05.6	18.1	05.9	18.0	06.2	17.9	06.4	17.9	06.5	17.7	06.8	19
20	19.1	05.8	19.1	05.8	19.0	06.2	18.9	06.5	18.8	06.7	18.8	06.8	18.7	07.2	20
21	20.1	06.1	20.1	06.1	20.0	06.5	19.9	06.8	19.8	07.1	19.7	07.2	19.6	07.5	21
22	21.1	06.4	21.0	06.4	20.9	06.8	20.8	07.2	20.7	07.4	20.7	07.5	20.5	07.9	22
23	22.0	06.7	22.0	06.7	21.9	07.1	21.7	07.5	21.7	07.7	21.6	07.9	21.5	08.2	23
24	23.0	07.0	22.9	07.0	22.8	07.4	22.7	07.8	22.6	08.1	22.6	08.2	22.4	08.6	24
25	23.9	07.3	23.9	07.3	23.8	07.7	23.6	08.1	23.5	08.4	23.5	08.5	23.3	09.0	25
26	24.9	07.5	24.9	07.6	24.7	08.0	24.6	08.5	24.5	08.8	24.4	08.9	24.3	09.3	26
27	25.8	07.8	25.8	07.9	25.7	08.3	25.5	08.8	25.4	09.1	25.4	09.2	25.2	09.7	27
28	26.8	08.1	26.8	08.2	26.6	08.7	26.5	09.1	26.4	09.4	26.3	09.6	26.1	10.0	28
29	27.8	08.4	27.7	08.5	27.6	09.0	27.4	09.4	27.3	09.8	27.3	09.9	27.1	10.4	29
30	28.7	08.7	28.7	08.8	28.5	09.3	28.4	09.8	28.2	10.1	28.2	10.3	28.0	10.8	30
31	29.7	09.0	29.6	09.1	29.5	09.6	29.3	10.1	29.2	10.4	29.1	10.6	28.9	11.1	31
32	30.6	09.3	30.6	09.4	30.4	10.0	30.3	10.4	30.1	10.8	30.1	10.9	29.9	11.5	32
33	31.6	09.6	31.6	09.6	31.4	10.2	31.2	10.7	31.1	11.1	31.0	11.3	30.8	11.8	33
34	32.5	09.9	32.5	09.9	32.3	10.5	32.1	11.1	32.0	11.5	31.9	11.6	31.7	12.2	34
35	33.5	10.2	33.5	10.2	33.3	10.8	33.1	11.4	33.0	11.8	32.9	12.0	32.7	12.5	35
36	34.4	10.4	34.4	10.5	34.2	11.1	34.0	11.7	33.9	12.1	33.8	12.3	33.6	12.9	36
37	35.4	10.7	35.4	10.8	35.2	11.2	35.0	12.0	34.8	12.5	34.8	12.7	34.5	13.3	37
38	36.4	11.0	36.3	11.1	36.1	11.7	35.9	12.4	35.8	12.8	35.7	13.0	35.5	13.6	38
39	37.3	11.3	37.3	11.4	37.1	12.0	36.9	12.7	36.7	13.1	36.6	13.3	36.4	14.0	39
40	38.3	11.6	38.3	11.7	38.0	12.4	37.8	13.0	37.7	13.5	37.6	13.7	37.3	14.3	40
41	39.2	11.9	39.2	12.0	39.0	12.7	38.8	13.3	38.6	13.8	38.5	14.0	38.3	14.7	41
42	40.2	12.2	40.2	12.3	39.9	13.0	39.7	13.7	39.5	14.1	39.5	14.4	39.2	15.1	42
43	41.1	12.5	41.1	12.6	40.9	13.3	40.7	14.0	40.5	14.5	40.4	14.7	40.1	15.4	43
44	42.1	12.8	42.1	12.9	41.8	13.6	41.6	14.3	41.4	14.8	41.3	15.0	41.1	15.8	44
45	43.1	13.1	43.0	13.1	42.8	13.9	42.5	14.7	42.4	15.2	42.3	15.4	42.0	16.1	45
46	44.0	13.4	44.0	13.4	43.7	14.2	43.5	15.0	43.3	15.5	43.2	15.7	42.9	16.5	46
47	45.0	13.6	44.9	13.7	44.7	14.5	44.4	15.3	44.2	15.8	44.2	16.1	43.9	16.8	47
48	45.9	13.9	45.9	14.0	45.7	14.8	45.4	15.6	45.2	16.2	45.1	16.4	44.8	17.2	48
49	46.9	14.2	46.9	14.3	46.6	15.1	46.3	16.0	46.1	16.5	46.0	16.8	45.7	17.6	49
50	47.8	14.5	47.8	14.6	47.6	15.4	47.3	16.3	47.1	16.8	47.0	17.1	46.7	17.9	50
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
Dist.	6½ Point.		73 Deg.		72 Deg.		71 Deg.		6¼ Point.		70 Deg.		69 Deg.		Dist.

Dist.	1½ Point. Lat.	1½ Point. Dep.	17 Deg. Lat.	17 Deg. Dep.	18 Deg. Lat.	18 Deg. Dep.	19 Deg. Lat.	19 Deg. Dep.	1¾ Point. Lat.	1¾ Point. Dep.	20 Deg. Lat.	20 Deg. Dep.	21 Deg. Lat.	21 Deg. Dep.	Dist.
51	48.8	14.8	48.8	14.9	48.5	15.8	48.2	16.6	48.0	17.2	47.9	17.4	47.6	18.3	51
52	49.7	15.1	49.7	15.2	49.4	16.1	49.2	16.9	49.0	17.5	48.9	17.8	48.5	18.6	52
53	50.7	15.3	50.7	15.5	50.4	16.4	50.1	17.3	49.9	17.9	49.8	18.1	49.5	19.0	53
54	51.7	15.7	51.6	15.8	51.3	16.7	51.0	17.6	50.8	18.2	50.7	18.5	50.4	19.4	54
55	52.6	16.0	52.6	16.1	52.3	17.0	52.0	17.9	51.8	18.5	51.7	18.8	51.3	19.7	55
56	53.6	16.2	53.5	16.4	53.3	17.3	52.9	18.2	52.7	18.9	52.6	19.2	52.3	20.1	56
57	54.5	16.5	54.5	16.7	54.2	17.6	53.9	18.6	53.7	19.2	53.6	19.5	53.2	20.4	57
58	55.5	16.8	55.5	17.0	55.2	17.9	54.8	18.9	54.6	19.5	54.5	19.8	54.1	20.8	58
59	56.5	17.1	56.4	17.3	56.1	18.2	55.8	19.2	55.5	19.9	55.4	20.2	55.1	21.1	59
60	57.4	17.4	57.4	17.5	57.1	18.5	56.7	19.5	56.5	20.2	56.4	20.5	56.0	21.5	60
61	58.4	17.7	58.3	17.8	58.0	18.8	57.7	19.9	57.4	20.6	57.3	20.9	56.9	21.9	61
62	59.3	18.0	59.3	18.:	59.0	19.2	58.6	20.2	58.4	20.9	58.3	21.2	57.9	22.2	62
63	60.3	18.3	60.2	18.4	59.9	19.5	59.6	20.5	59.3	21.2	59.2	21.5	58.8	22.6	63
64	61.2	18.6	61.2	18.7	60.9	19.8	60.5	20.8	60.3	21.6	60.1	21.9	59.7	22.9	64
65	62.2	18.9	62.2	19.0	61.8	20.1	61.5	21.2	61.2	21.9	61.1	22.2	60.7	23.3	65
66	63.2	19.2	63.1	19.3	62.8	20.4	62.4	21.5	62.1	22.2	62.0	22.6	61.6	23.7	66
67	64.1	19.4	64.1	19.6	63.7	20.7	63.3	21.8	63.1	22.6	63.0	22.9	62.6	24.0	67
68	65.1	19.7	65.0	19.9	64.7	21.0	64.3	22.1	64.0	22.9	63.9	23.3	63.5	24.4	68
69	66.0	20.0	66.0	20.2	65.6	21.3	65.2	22.5	65.0	23.2	64.8	23.6	64.4	24.7	69
70	67.0	20.3	66.9	20.5	66.6	21.6	66.2	22.8	65.9	23.6	65.8	23.9	65.4	25.1	70
71	67.9	20.6	67.9	20.8	67.5	21.9	67.1	23.1	66.8	23.9	66.7	24.3	66.3	25.4	71
72	68.9	20.9	68.8	21.1	68.5	22.2	68.1	23.4	67.8	24.3	67.7	24.6	67.2	25.8	72
73	69.9	21.2	69.8	21.3	69.4	22.6	69.0	23.8	68.7	24.6	68.6	25.0	68.2	26.2	73
74	70.8	21.5	70.8	21.6	70.4	22.9	70.0	24.1	69.7	24.9	69.5	25.3	69.1	26.5	74
75	71.8	21.8	71.7	21.9	71.3	23.2	70.9	24.4	70.6	25.3	70.5	25.6	70.0	26.9	75
76	72.7	22.1	72.7	22.2	72.3	23.5	71.9	24.7	71.6	25.6	71.4	26.0	71.0	27.2	76
77	73.7	22.4	73.6	22.5	73.2	23.8	72.8	25.1	72.5	25.9	72.4	26.3	71.9	27.6	77
78	74.6	22.6	74.6	22.8	74.2	24.1	73.7	25.4	73.4	26.3	73.3	26.7	72.8	28.0	78
79	75.6	22.9	75.5	23.1	75.1	24.4	74.7	25.7	74.4	26.6	74.2	27.0	73.8	28.3	79
80	76.6	23.2	76.5	23.4	76.1	24.7	75.6	26.0	75.3	27.0	75.2	27.4	74.7	28.7	80
81	77.5	23.5	77.5	23.7	77.0	25.0	76.6	26.4	76.3	27.3	76.1	27.7	75.6	29.0	81
82	78.5	23.8	78.4	24.0	78.0	25.3	77.5	26.7	77.2	27.6	77.1	28.0	76.6	29.4	82
83	79.4	24.1	79.4	24.3	78.9	25.6	78.5	27.0	78.1	28.0	78.0	28.4	77.5	29.7	83
84	80.4	24.4	80.3	24.5	79.9	26.0	79.4	27.3	79.1	28.3	78.9	28.7	78.4	30.1	84
85	81.3	24.7	81.3	24.8	80.8	26.3	80.4	27.7	80.0	28.6	79.9	29.1	79.4	30.5	85
86	82.3	25.0	82.2	25.1	81.8	26.6	81.3	28.0	81.0	29.0	80.8	29.4	80.3	30.8	86
87	83.3	25.3	83.2	25.4	82.7	26.9	82.3	28.3	81.9	29.3	81.8	29.8	81.2	31.2	87
88	84.2	25.5	84.2	25.7	83.7	27.2	83.2	28.7	82.9	29.6	82.7	30.1	82.2	31.5	88
89	85.2	25.8	85.1	26.0	84.6	27.5	84.1	29.0	83.8	30.0	83.6	30.4	83.1	31.9	89
90	86.1	26.1	86.1	26.3	85.6	27.8	85.1	29.3	84.7	30.3	84.6	30.8	84.0	32.3	90
91	87.1	26.4	87.0	26.6	86.5	28.1	86.0	29.6	85.7	30.7	85.5	31.1	85.0	32.6	91
92	88.0	26.7	88.0	26.9	87.5	28.4	87.0	30.0	86.6	31.0	86.5	31.5	85.9	33.0	92
93	89.0	27.0	88.9	27.2	88.4	28.7	87.9	30.3	87.6	31.3	87.4	31.8	86.8	33.3	93
94	90.0	27.3	89.9	27.5	89.4	29.0	88.9	30.6	88.5	31.7	88.3	32.1	87.8	33.7	94
95	90.9	27.6	90.8	27.8	90.4	29.4	89.8	30.9	89.4	32.0	89.3	32.5	88.7	34.0	95
96	91.9	27.9	91.8	28.1	91.3	29.7	90.8	31.3	90.4	32.3	90.2	32.8	89.6	34.4	96
97	92.8	28.2	92.8	28.4	92.3	30.0	91.7	31.6	91.3	32.7	91.1	33.2	90.6	34.8	97
98	93.8	28.4	93.7	28.7	93.2	30.3	92.7	31.9	92.3	33.0	92.1	33.5	91.5	35.1	98
99	94.7	28.7	94.7	28.9	94.2	30.6	93.6	32.2	93.2	33.4	93.0	33.9	92.4	35.5	99
100	95.7	29.0	95.6	29.2	95.1	30.9	94.5	32.6	94.2	33.7	94.0	34.2	93.4	35.8	100
Dist.	6½ Point. Dep.	6½ Point. Lat.	73 Deg. Dep.	73 Deg. Lat.	72 Deg. Dep.	72 Deg. Lat.	71 Deg. Dep.	71 Deg. Lat.	6¼ Point. Dep.	6¼ Point. Lat.	70 Deg. Dep.	70 Deg. Lat.	69 Deg. Dep.	69 Deg. Lat.	Dist.

A Table *of* Difference

Dist.	22 Deg.		2 Points		23 Deg.		24 Deg.		25 Deg.		$2\frac{1}{4}$ Point.		26 Deg.		Dist.
	Lat.	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	
1	00.9	00.4	00.9	00.4	00.9	00.4	00.9	00.4	00.9	00.4	00.9	00.4	00.9	00.4	1
2	01.9	00.7	01.8	00.8	01.8	00.8	01.8	00.8	01.8	00.8	01.8	00.9	01.8	00.9	2
3	02.8	01.1	02.8	01.1	02.8	01.2	02.7	01.2	02.7	01.3	02.7	01.3	02.7	01.3	3
4	03.7	01.5	03.7	01.5	03.7	01.6	03.6	01.6	03.6	01.7	03.6	01.7	03.6	01.8	4
5	04.6	01.9	04.6	01.9	04.6	02.0	04.6	02.0	04.5	02.1	04.5	02.1	04.5	02.2	5
6	05.6	02.2	05.5	02.3	05.5	02.3	05.5	02.4	05.4	02.5	05.4	02.6	05.4	02.6	6
7	06.5	02.6	06.5	02.7	06.4	02.7	06.4	02.8	06.3	03.0	06.3	03.0	06.3	03.1	7
8	07.4	03.0	07.4	03.1	07.4	03.1	07.3	03.2	07.2	03.4	07.2	03.4	07.2	03.5	8
9	08.3	03.4	08.3	03.4	08.3	03.5	08.2	03.7	08.2	03.8	08.1	03.8	08.1	03.9	9
10	09.3	03.7	09.2	03.8	09.2	03.9	09.1	04.1	09.1	04.2	09.0	04.3	09.0	04.4	10
11	10.2	04.1	10.2	04.2	10.1	04.3	10.0	04.5	10.0	04.6	09.9	04.7	09.9	04.8	11
12	11.1	04.5	11.1	04.6	11.0	04.7	11.0	04.9	10.9	05.1	10.8	05.1	10.8	05.3	12
13	12.1	04.9	12.0	05.0	12.0	05.1	11.9	05.3	11.8	05.5	11.7	05.6	11.7	05.7	13
14	13.0	05.2	12.9	05.4	12.9	05.5	12.8	05.7	12.7	05.9	12.7	06.0	12.6	06.1	14
15	13.9	05.6	13.9	05.7	13.8	05.9	13.7	06.1	13.6	06.3	13.6	06.4	13.5	06.6	15
16	14.8	06.0	14.8	06.1	14.7	06.2	14.6	06.5	14.5	06.8	14.5	06.8	14.4	06.9	16
17	15.8	06.4	15.7	06.5	15.6	06.6	15.5	06.9	15.4	07.2	15.4	07.3	15.3	07.5	17
18	16.7	06.7	16.6	06.9	16.6	07.0	16.4	07.3	16.3	07.6	16.3	07.7	16.2	07.9	18
19	17.6	07.1	17.6	07.3	17.5	07.4	17.4	07.7	17.2	08.0	17.2	08.1	17.1	08.3	19
20	18.5	07.5	18.5	07.7	18.4	07.8	18.3	08.1	18.1	08.5	18.1	08.6	18.0	08.8	20
21	19.5	07.9	19.4	08.0	19.3	08.2	19.2	08.5	19.0	08.9	19.0	09.0	18.9	09.2	21
22	20.4	08.2	20.3	08.4	20.3	08.6	20.1	08.9	19.9	09.3	19.9	09.4	19.8	09.6	22
23	21.3	08.6	21.2	08.8	21.2	09.0	21.0	09.4	20.8	09.7	20.8	09.8	20.7	10.1	23
24	22.3	09.0	22.2	09.2	22.1	09.4	21.9	09.8	21.8	10.1	21.7	10.3	21.6	10.5	24
25	23.2	09.4	23.1	09.6	23.0	09.8	22.8	10.2	22.7	10.6	22.6	10.7	22.5	11.0	25
26	24.1	09.7	24.0	09.9	23.9	10.2	23.8	10.6	23.6	11.0	23.5	11.1	23.4	11.4	26
27	25.0	10.1	24.9	10.2	24.9	10.5	24.7	11.0	24.5	11.4	24.4	11.5	24.3	11.8	27
28	26.0	10.5	25.9	10.7	25.8	10.9	25.6	11.4	25.4	11.8	25.3	12.0	25.2	12.3	28
29	26.9	10.9	26.8	11.1	26.7	11.3	26.5	11.8	26.3	12.3	26.2	12.4	26.1	12.7	29
30	27.8	11.2	27.7	11.5	27.6	11.7	27.4	12.2	27.2	12.7	27.1	12.8	27.0	13.2	30
31	28.7	11.6	28.6	11.9	28.5	12.1	28.3	12.6	28.1	13.1	28.0	13.3	27.9	13.6	31
32	29.7	12.0	29.6	12.3	29.5	12.5	29.2	13.0	29.0	13.5	28.9	13.7	28.8	14.0	32
33	30.6	12.4	30.5	12.6	30.4	12.9	30.1	13.4	29.9	13.9	29.8	14.1	29.7	14.5	33
34	31.5	12.7	31.4	13.0	31.3	13.3	31.1	13.8	30.8	14.4	30.7	14.5	30.6	14.9	34
35	32.5	13.1	32.3	13.4	32.2	13.7	32.0	14.2	31.7	14.8	31.6	15.1	31.5	15.3	35
36	33.4	13.5	33.3	13.8	33.1	14.1	32.9	14.6	32.6	15.2	32.5	15.4	32.4	15.8	36
37	34.3	13.9	34.2	14.2	34.1	14.4	33.8	15.0	33.5	15.6	33.4	15.8	33.3	16.2	37
38	35.2	14.2	35.1	14.5	35.0	14.8	34.7	15.5	34.4	16.1	34.3	16.2	34.2	16.7	38
39	36.2	14.6	36.0	14.9	35.9	15.2	35.6	15.9	35.3	16.5	35.3	16.7	35.1	17.1	39
40	37.1	15.0	37.0	15.3	36.8	15.6	36.5	16.3	36.3	16.9	36.2	17.1	36.0	17.5	40
41	38.0	15.4	37.9	15.7	37.7	16.0	37.5	16.7	37.2	17.3	37.1	17.5	36.8	18.0	41
42	38.9	15.7	38.8	16.1	38.7	16.4	38.4	17.1	38.1	17.7	38.0	18.0	37.7	18.4	42
43	39.9	16.1	39.7	16.5	39.6	16.8	39.3	17.5	39.0	18.2	38.9	18.4	38.6	18.9	43
44	40.8	16.5	40.7	16.8	40.5	17.2	40.2	17.9	39.9	18.6	39.8	18.8	39.5	19.3	44
45	41.7	16.9	41.6	17.2	41.4	17.6	41.1	18.3	40.8	19.0	40.7	19.2	40.4	19.7	45
46	42.7	17.2	42.5	17.6	42.3	18.0	42.0	18.7	41.7	19.4	41.6	19.7	41.3	20.2	46
47	43.6	17.6	43.4	18.0	43.3	18.4	42.9	19.1	42.6	19.9	42.5	20.1	42.2	20.6	47
48	44.5	18.0	44.3	18.4	44.2	18.8	43.8	19.5	43.5	20.3	43.4	20.5	43.1	21.0	48
49	45.4	18.4	45.3	18.8	45.1	19.2	44.8	19.9	44.4	20.7	44.3	20.9	44.0	21.5	49
50	46.4	18.7	46.2	19.1	46.0	19.5	45.7	20.3	45.3	21.1	45.2	21.4	44.9	21.9	50
Dist.	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	Dist.
	68 Deg.		6 Points		67 Deg.		66 Deg.		65 Deg.		$5\frac{3}{4}$ Point.		64 Deg.		

Dist.	22 Deg.		2 Points		23 Deg.		24 Deg.		25 Deg.		$2\frac{1}{4}$ Point.		26 Deg.		Dist.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep	Lat.	Dep.	Lat.	Dep.	
51	47.3	19.1	47.1	19.5	46.9	19.9	46.6	20.7	46.2	21.6	46.1	21.8	45.8	22.4	51
52	48.2	19.5	48.0	19.9	47.9	20.3	47.5	21.1	47.1	22.0	47.0	22.2	46.7	22.8	52
53	49.1	19.9	49.0	20.3	48.8	20.7	48.4	21.6	48.0	22.4	47.9	22.7	47.6	23.2	53
54	50.1	20.2	49.9	20.7	49.7	21.1	49.3	22.0	48.9	22.8	48.8	23.1	48.5	23.7	54
55	51.0	20.6	50.8	21.0	50.6	21.5	50.2	22.4	49.8	23.2	49.7	23.5	49.4	24.1	55
56	51.9	21.0	51.7	21.4	51.5	21.9	51.2	22.8	50.8	23.7	50.6	23.9	50.3	24.5	56
57	52.8	21.4	52.7	21.8	52.5	22.3	52.1	23.2	51.7	24.1	51.5	24.4	51.2	25.0	57
58	53.8	21.7	53.6	22.2	53.4	22.7	53.0	23.6	52.6	24.5	52.4	24.8	52.1	25.4	58
59	54.7	22.1	54.5	22.6	54.3	23.1	53.9	24.0	53.5	24.9	53.3	25.2	53.0	25.9	59
60	55.6	22.5	55.4	23.0	55.2	23.4	54.8	24.4	54.4	25.4	54.2	25.7	53.9	26.3	60
61	56.5	22.8	56.4	23.3	56.1	23.8	55.7	24.8	55.3	25.8	55.1	26.1	54.8	26.7	61
62	57.5	23.2	57.3	23.7	57.1	24.2	56.6	25.2	56.2	26.2	56.0	26.5	55.7	27.2	62
63	58.4	23.6	58.2	24.1	58.0	24.6	57.5	25.6	57.1	26.6	57.0	26.9	56.6	27.6	63
64	59.3	24.0	59.1	24.5	58.9	25.0	58.5	26.0	58.0	27.0	57.9	27.4	57.5	28.0	64
65	60.3	24.3	60.1	24.9	59.8	25.4	59.4	26.4	58.9	27.5	58.8	27.8	58.4	28.5	65
66	61.2	24.7	61.0	25.3	60.8	25.8	60.3	26.8	59.8	27.9	59.7	28.2	59.3	28.9	66
67	62.1	25.1	61.9	25.6	61.7	26.2	61.2	27.2	60.7	28.3	60.6	28.6	60.2	29.4	67
68	63.0	25.5	62.8	26.0	62.6	26.6	62.1	27.7	61.6	28.7	61.5	29.1	61.1	29.8	68
69	64.0	25.8	63.7	26.4	63.5	27.0	63.0	28.1	62.5	29.2	62.4	29.5	62.0	30.2	69
70	64.9	26.2	64.7	26.8	64.4	27.3	63.9	28.5	63.4	29.6	63.3	29.9	62.9	30.7	70
71	65.8	26.6	65.6	27.2	65.4	27.7	64.9	28.9	64.3	30.0	64.2	30.4	63.8	31.1	71
72	66.8	27.0	66.5	27.6	66.3	28.1	65.8	29.3	65.2	30.4	65.1	30.8	64.7	31.6	72
73	67.7	27.3	67.4	27.9	67.2	28.5	66.7	29.7	66.2	30.8	66.0	31.2	65.6	32.0	73
74	68.6	27.7	68.4	28.3	68.1	28.9	67.6	30.1	67.1	31.3	66.9	31.6	66.5	32.4	74
75	69.5	28.1	69.3	28.7	69.0	29.3	68.5	30.5	68.0	31.7	67.8	32.1	67.4	32.9	75
76	70.5	28.5	70.2	29.1	70.0	29.7	69.4	30.9	68.9	32.1	68.7	32.5	68.3	33.3	76
77	71.4	28.8	71.1	29.5	70.9	30.1	70.3	31.3	69.8	32.5	69.6	32.9	69.2	33.8	77
78	72.3	29.2	72.1	29.8	71.8	30.5	71.3	31.7	70.7	33.0	70.5	33.3	70.1	34.2	78
79	73.2	29.6	73.0	30.2	72.7	30.9	72.2	32.1	71.6	33.4	71.4	33.8	71.0	34.6	79
80	74.2	30.0	73.9	30.6	73.6	31.3	73.1	32.5	72.5	33.8	72.3	34.2	71.9	35.1	80
81	75.1	30.3	74.8	31.0	74.6	31.6	74.0	32.9	73.4	34.2	73.2	34.6	72.8	35.5	81
82	76.0	30.7	75.8	31.4	75.5	32.0	74.9	33.3	74.3	34.7	74.1	35.1	73.7	35.9	82
83	77.0	31.1	76.7	31.8	76.4	32.4	75.8	33.7	75.2	35.1	75.0	35.5	74.6	36.4	83
84	77.9	31.5	77.6	32.1	77.3	32.8	76.7	34.1	76.1	35.5	75.9	35.9	75.5	36.8	84
85	78.8	31.8	78.5	32.5	78.2	33.2	77.6	34.6	77.0	35.9	76.8	36.3	76.4	37.3	85
86	79.7	32.2	79.5	32.9	79.2	33.6	78.6	35.0	77.9	36.3	77.7	36.8	77.3	37.7	86
87	80.7	32.6	80.4	33.3	80.1	34.0	79.5	35.4	78.8	36.8	78.6	37.2	78.2	38.1	87
88	81.6	33.0	81.3	33.7	81.0	34.4	80.4	35.8	79.8	37.2	79.6	37.6	79.1	38.6	88
89	82.5	33.3	82.2	34.1	81.9	34.8	81.3	36.2	80.7	37.6	80.5	38.1	80.0	39.0	89
90	83.4	33.7	83.2	34.4	82.8	35.2	82.2	36.6	81.6	38.0	81.4	38.5	80.9	39.5	90
91	84.4	34.1	84.1	34.8	83.8	35.6	83.1	37.0	82.5	38.5	82.3	38.9	81.8	39.9	91
92	85.3	34.5	85.0	35.2	84.7	35.9	84.0	37.4	83.4	38.9	83.2	39.3	82.7	40.3	92
93	86.2	34.8	85.9	35.6	85.6	36.3	85.0	37.8	84.3	39.3	84.1	39.8	83.6	40.8	93
94	87.2	35.2	86.8	36.0	86.5	36.7	85.9	38.2	85.2	39.7	85.0	40.2	84.5	41.2	94
95	88.1	35.6	87.8	36.4	87.4	37.1	86.8	38.6	86.1	40.1	85.9	40.6	85.4	41.6	95
96	89.0	36.0	88.7	36.7	88.3	37.5	87.7	39.0	87.0	40.6	86.8	41.0	86.3	42.1	96
97	89.9	36.3	89.6	37.1	89.3	37.9	88.6	39.4	87.9	41.0	87.7	41.5	87.2	42.5	97
98	90.9	36.7	90.5	37.5	90.2	38.4	89.5	39.9	88.8	41.4	88.6	41.9	88.1	43.0	98
99	91.8	37.1	91.5	37.9	91.1	38.7	90.4	40.3	89.7	41.8	89.5	42.3	89.0	43.4	99
100	92.7	37.5	92.4	38.3	92.0	39.1	91.4	40.7	90.6	42.3	90.4	42.8	89.9	43.8	100
Dist.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dist.
	68 Deg.		6 Points		67 Deg.		66 Deg.		65 Deg.		$5\frac{3}{4}$ Point.		64 Deg.		

Dist.	27 Deg.		28 Deg.		2½ Point.		29 Deg.		30 Deg.		2¾ Point.		31 Deg.		Dist.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	00.9	00.5	00.9	00.5	00.9	00.5	00.9	00.5	00.9	00.5	00.9	00.5	00.9	00.5	1
2	01.8	00.9	01.8	00.9	01.8	00.9	01.7	01.0	01.7	01.0	01.7	01.0	01.7	01.0	2
3	02.7	01.4	02.6	01.4	02.6	01.4	02.6	01.5	02.6	01.5	02.6	01.5	02.6	01.5	3
4	03.6	01.8	03.5	01.9	03.5	01.9	03.5	01.9	03.5	02.0	03.4	02.1	03.4	02.1	4
5	04.5	02.3	04.4	02.3	04.4	02.4	04.4	02.4	04.3	02.5	04.3	02.6	04.3	02.6	5
6	05.3	02.7	05.3	02.8	05.3	02.8	05.2	02.9	05.2	03.0	05.1	03.1	05.1	03.1	6
7	06.2	03.2	06.2	03.3	06.2	03.3	06.1	03.4	06.1	03.5	06.0	03.6	06.0	03.6	7
8	07.1	03.6	07.1	03.8	07.1	03.8	07.0	03.9	06.9	04.0	06.9	04.1	06.9	04.1	8
9	08.0	04.1	07.9	04.2	07.9	04.2	07.9	04.4	07.8	04.5	07.7	04.6	07.7	04.6	9
10	08.9	04.5	08.8	04.7	08.8	04.7	08.7	04.8	08.7	05.0	08.6	05.1	08.6	05.1	10
11	09.8	05.0	09.7	05.2	09.7	05.2	09.6	05.3	09.5	05.5	09.4	05.7	09.4	05.7	11
12	10.7	05.4	10.6	05.6	10.6	05.7	10.5	05.8	10.4	06.0	10.3	06.2	10.3	06.2	12
13	11.6	05.9	11.5	06.1	11.5	06.1	11.4	06.3	11.3	06.5	11.1	06.7	11.1	06.7	13
14	12.5	06.4	12.4	06.6	12.3	06.6	12.2	06.8	12.1	07.0	12.0	07.2	12.0	07.2	14
15	13.4	06.8	13.2	07.0	13.2	07.1	13.1	07.3	13.0	07.5	12.9	07.7	12.9	07.7	15
16	14.3	07.3	14.1	07.5	14.1	07.5	14.0	07.8	13.9	08.0	13.7	08.2	13.7	08.2	16
17	15.1	07.7	15.0	08.0	15.0	08.0	14.9	08.2	14.7	08.5	14.6	08.7	14.6	08.8	17
18	16.0	08.2	15.9	08.5	15.9	08.5	15.7	08.7	15.6	09.0	15.4	09.3	15.4	09.3	18
19	16.9	08.6	16.8	08.9	16.8	09.0	16.6	09.2	16.5	09.5	16.3	09.8	16.3	09.8	19
20	17.8	09.1	17.7	09.4	17.6	09.4	17.5	09.7	17.3	10.0	17.2	10.3	17.1	10.3	20
21	18.7	09.5	18.5	09.9	18.5	09.9	18.4	10.2	18.2	10.5	18.0	10.8	18.0	10.8	21
22	19.6	10.0	19.4	10.3	19.4	10.4	19.2	10.7	19.1	11.0	18.9	11.3	18.9	11.3	22
23	20.5	10.4	20.3	10.8	20.3	10.8	20.1	11.1	19.9	11.5	19.7	11.8	19.7	11.8	23
24	21.4	10.9	21.2	11.3	21.2	11.3	21.0	11.6	20.8	12.0	20.6	12.3	20.6	12.3	24
25	22.3	11.3	22.1	11.7	22.0	11.8	21.9	12.1	21.6	12.5	21.4	12.9	21.4	12.9	25
26	23.2	11.8	23.0	12.2	22.9	12.3	22.7	12.6	22.5	13.0	22.3	13.4	22.3	13.4	26
27	24.1	12.3	23.8	12.7	23.8	12.7	23.6	13.1	23.4	13.5	23.2	13.9	23.1	13.9	27
28	24.9	12.7	24.7	13.1	24.7	13.2	24.5	13.6	24.2	14.0	24.0	14.4	24.0	14.4	28
29	25.8	13.2	25.6	13.6	25.6	13.7	25.4	14.1	25.1	14.5	24.9	14.9	24.9	14.9	29
30	26.7	13.6	26.5	14.1	26.5	14.1	26.2	14.5	26.0	15.0	25.7	15.4	25.7	15.4	30
31	27.6	14.1	27.4	14.6	27.3	14.6	27.1	15.0	26.8	15.5	26.6	15.9	26.6	16.0	31
32	28.5	14.5	28.3	15.0	28.2	15.1	28.0	15.5	27.7	16.0	27.4	16.4	27.4	16.5	32
33	29.4	15.0	29.1	15.5	29.1	15.6	28.9	16.0	28.6	16.5	28.3	17.0	28.3	17.0	33
34	30.3	15.4	30.0	16.0	30.0	16.0	29.7	16.5	29.4	17.0	29.2	17.5	29.1	17.5	34
35	31.2	15.9	30.9	16.4	30.9	16.5	30.6	17.0	30.3	17.5	30.0	18.0	30.0	18.0	35
36	32.1	16.3	31.8	16.9	31.7	17.0	31.5	17.5	31.2	18.0	30.9	18.5	30.9	18.5	36
37	33.0	16.8	32.7	17.4	32.6	17.4	32.4	17.9	32.0	18.5	31.7	19.0	31.7	19.1	37
38	33.9	17.3	33.5	17.8	33.5	17.9	33.2	18.4	32.9	19.0	32.6	19.5	32.6	19.6	38
39	34.7	17.7	34.4	18.3	34.4	18.4	34.1	18.9	33.8	19.5	33.4	20.0	33.4	20.1	39
40	35.6	18.2	35.3	18.8	35.3	18.9	35.0	19.4	34.6	20.0	34.3	20.6	34.3	20.6	40
41	36.5	18.6	36.2	19.2	36.2	19.3	35.9	19.9	35.5	20.5	35.2	21.1	35.1	21.1	41
42	37.4	19.1	37.1	19.7	37.0	19.8	36.7	20.4	36.4	21.0	36.0	21.6	36.0	21.6	42
43	38.3	19.5	38.0	20.2	37.9	20.3	37.6	20.8	37.2	21.5	36.9	22.1	36.9	22.1	43
44	39.2	20.0	38.8	20.7	38.8	20.7	38.5	21.3	38.1	22.0	37.7	22.6	37.7	22.7	44
45	40.1	20.4	39.7	21.1	39.7	21.2	39.4	21.8	39.0	22.5	38.6	23.1	38.6	23.2	45
46	41.0	20.9	40.6	21.6	40.6	21.7	40.2	22.3	39.8	23.0	39.5	23.6	39.4	23.7	46
47	41.9	21.3	41.5	22.1	41.4	22.2	41.1	22.8	40.7	23.5	40.3	24.2	40.3	24.2	47
48	42.8	21.8	42.4	22.5	42.3	22.6	42.0	23.3	41.6	24.0	41.2	24.7	41.1	24.7	48
49	43.7	22.2	43.3	23.0	43.2	23.1	42.9	23.8	42.4	24.5	42.0	25.2	42.0	25.2	49
50	44.5	22.7	44.1	23.5	44.1	23.6	43.7	24.2	43.3	25.0	42.9	25.7	42.9	25.7	50
	Dep.	Lat.	Dep	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
Dist.	63 Deg.		62 Deg.		5½ Point.		61 Deg.		60 Deg.		5¼ Point.		59 Deg.		Dist.

Dist.	27 Deg.		28 Deg.		2½ Point.		29 Deg.		30 Deg.		2¾ Point.		31 Deg.		Dist.
	Lat.	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	
51	45.4	23.2	45.0	23.9	45.0	24.0	44.6	24.7	44.2	25.5	43.7	26.2	43.7	26.3	51
52	46.3	23.6	45.9	24.4	45.9	24.5	45.5	25.2	45.0	26.0	44.6	26.7	44.6	26.8	52
53	47.2	24.1	46.8	24.9	46.7	25.0	46.4	25.7	45.9	26.5	45.5	27.2	45.4	27.3	53
54	48.1	24.5	47.7	25.4	47.6	25.5	47.2	26.2	46.8	27.0	46.3	27.8	46.3	27.8	54
55	49.0	25.0	48.6	25.8	48.5	25.9	48.1	26.7	47.6	27.5	47.2	28.3	47.1	28.3	55
56	49.9	25.4	49.4	26.3	49.4	26.4	49.0	27.1	48.5	28.0	48.0	28.8	48.0	28.8	56
57	50.8	25.9	50.3	26.8	50.3	26.9	49.9	27.6	49.4	28.5	48.9	29.3	48.9	29.4	57
58	51.7	26.3	51.2	27.2	51.2	27.3	50.7	28.1	50.2	29.0	49.7	29.8	49.7	29.9	58
59	52.6	26.8	52.1	27.7	52.0	27.8	51.6	28.6	51.1	29.5	50.6	30.3	50.6	30.4	59
60	53.5	27.2	53.0	28.2	52.9	28.3	52.5	29.1	52.0	30.0	51.5	30.8	51.4	30.9	60
61	54.4	27.7	53.9	28.6	53.8	28.8	53.3	29.6	52.8	30.5	52.3	31.4	52.3	31.4	61
62	55.2	28.1	54.7	29.1	54.7	29.2	54.2	30.1	53.7	31.0	53.2	31.9	53.1	31.9	62
63	56.1	28.6	55.6	29.6	55.6	29.7	55.1	30.5	54.6	31.5	54.0	32.4	54.0	32.4	63
64	57.0	29.1	56.5	30.0	56.4	30.2	56.0	31.0	55.4	32.0	54.9	32.9	54.9	33.0	64
65	57.9	29.5	57.4	30.5	57.3	30.6	56.8	31.5	56.3	32.5	55.7	33.4	55.7	33.5	65
66	58.8	30.0	58.3	31.0	58.2	31.1	57.7	32.0	57.2	33.0	56.6	33.9	56.6	34.0	66
67	59.7	30.4	59.2	31.5	59.1	31.6	58.6	32.5	58.0	33.5	57.5	34.4	57.4	34.5	67
68	60.6	30.9	60.0	31.9	60.0	32.1	59.5	33.0	58.9	34.0	58.3	35.0	58.3	35.0	68
69	61.5	31.3	60.9	32.4	60.9	32.5	60.3	33.5	59.8	34.5	59.2	35.5	59.1	35.5	69
70	62.4	31.8	61.8	32.9	61.7	33.0	61.2	33.9	60.6	35.0	60.0	36.0	60.0	36.0	70
71	63.3	32.2	62.7	33.3	62.6	33.5	62.1	34.4	61.5	35.5	60.9	36.5	60.9	36.6	71
72	64.2	32.7	63.6	33.8	63.5	33.9	63.0	34.9	62.4	36.0	61.8	37.0	61.7	37.2	72
73	65.0	33.1	64.5	34.3	64.4	34.4	63.8	35.4	63.2	36.5	62.6	37.5	62.6	37.6	73
74	65.9	33.6	65.3	34.7	65.3	34.9	64.7	35.9	64.1	37.0	63.5	38.0	63.4	38.1	74
75	66.8	34.1	66.2	35.2	66.1	35.4	65.6	36.4	64.9	37.5	64.3	38.6	64.3	38.6	75
76	67.7	34.5	67.1	35.7	67.0	35.8	66.5	36.8	65.8	38.0	65.2	39.1	65.1	39.1	76
77	68.6	35.0	68.0	36.2	67.9	36.3	67.3	37.3	66.7	38.5	66.0	39.6	66.0	39.7	77
78	69.5	35.4	68.9	36.6	68.8	36.8	68.2	37.8	67.5	39.0	66.9	40.1	66.9	40.2	78
79	70.4	35.9	69.7	37.1	69.7	37.2	69.1	38.3	68.4	39.5	67.8	40.6	67.7	40.7	79
80	71.3	36.3	70.6	37.6	70.6	37.7	70.0	38.8	69.3	40.0	68.6	41.1	68.6	41.2	80
81	72.2	36.8	71.5	38.0	71.4	38.2	70.8	39.3	70.1	40.5	69.5	41.6	69.4	41.7	81
82	73.1	37.2	72.4	38.5	72.3	38.7	71.7	39.8	71.0	41.0	70.3	42.2	70.3	42.2	82
83	74.0	37.7	73.3	39.0	73.2	39.1	72.6	40.2	71.9	41.5	71.2	42.7	71.1	42.7	83
84	74.8	38.1	74.2	39.4	74.1	39.6	73.5	40.7	72.7	42.0	72.0	43.2	72.0	43.3	84
85	75.7	38.6	75.0	39.9	75.0	40.1	74.3	41.2	73.6	42.5	72.9	43.7	72.9	43.8	85
86	76.6	39.0	75.9	40.4	75.8	40.5	75.2	41.7	74.5	43.0	73.8	44.2	73.7	44.3	86
87	77.5	39.5	76.8	40.8	76.7	41.0	76.1	42.2	75.3	43.5	74.6	44.7	74.6	44.8	87
88	78.4	40.0	77.7	41.3	77.6	41.5	77.0	42.7	76.2	44.0	75.5	45.2	75.4	45.3	88
89	79.3	40.4	78.6	41.8	78.5	42.0	77.8	43.1	77.1	44.5	76.3	45.8	76.3	45.8	89
90	80.2	40.9	79.5	42.3	79.4	42.4	78.7	43.6	77.9	45.0	77.2	46.3	77.1	46.3	90
91	81.1	41.3	80.3	42.7	80.3	42.9	79.6	44.1	78.8	45.5	78.1	46.8	78.0	46.9	91
92	82.0	41.8	81.2	43.2	81.1	43.4	80.5	44.6	79.7	46.0	78.9	47.3	78.9	47.4	92
93	82.9	42.2	82.1	43.7	82.0	43.8	81.3	45.1	80.5	46.5	79.8	47.8	79.7	47.9	93
94	83.8	42.7	83.0	44.1	82.9	44.3	82.2	45.6	81.4	47.0	80.6	48.3	80.6	48.4	94
95	84.6	43.1	83.9	44.6	83.8	44.8	83.1	46.1	82.3	47.5	81.5	48.8	81.4	48.9	95
96	85.5	43.6	84.8	45.1	84.7	45.3	84.0	46.5	83.1	48.0	82.3	49.4	82.3	49.4	96
97	86.4	44.0	85.6	45.5	85.5	45.7	84.8	47.0	84.0	48.5	83.2	49.9	83.1	50.0	97
98	87.3	44.5	86.5	46.0	86.4	46.2	85.7	47.5	84.9	49.0	84.1	50.4	84.0	50.5	98
99	88.2	44.9	87.4	46.5	87.3	46.7	86.6	48.0	85.7	49.5	84.9	50.9	84.9	51.0	99
100	89.1	45.4	88.3	46.9	88.2	47.1	87.5	48.5	86.6	50.0	85.8	51.4	85.7	51.5	100
Dist.	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	Dist.
	63 Deg.		62 Deg.		5½ Point.		61 Deg.		60 Deg.		5¼ Point.		59 Deg.		

Dist.	32 Deg.		33 Deg.		3 Points		34 Deg.		35 Deg.		36 Deg.		$3\frac{1}{4}$ Point.		Dist.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	00.8	00.9	00.8	00.5	00.8	00.6	00.8	00.6	00.8	00.6	00.8	00.6	00.8	00.6	1
2	01.7	01.1	01.7	01.1	01.7	01.1	01.6	01.1	01.6	01.1	01.6	01.2	01.6	01.2	2
3	02.5	01.6	02.5	01.6	02.5	01.7	02.5	01.7	02.5	01.7	02.4	01.8	02.4	01.8	3
4	03.4	02.1	03.4	02.2	03.3	02.2	03.3	02.2	03.3	02.3	03.2	02.4	03.2	02.4	4
5	04.2	02.6	04.2	02.7	04.2	02.8	04.1	02.8	04.1	02.9	04.0	02.9	04.0	03.0	5
6	05.1	03.2	05.0	03.3	05.0	03.3	05.0	03.3	04.9	03.4	04.8	03.5	04.8	03.6	6
7	05.9	03.7	05.9	03.8	05.8	03.9	05.8	03.9	05.7	04.0	05.7	04.1	05.6	04.2	7
8	06.8	04.2	06.7	04.4	06.6	04.4	06.6	04.5	06.6	04.6	06.5	04.7	06.4	04.8	8
9	07.6	04.8	07.5	04.9	07.5	05.0	07.5	05.0	07.4	05.2	07.3	05.3	07.2	05.4	9
10	08.5	05.3	08.4	05.4	08.3	05.6	08.3	05.6	08.2	05.7	08.1	05.9	08.0	06.0	10
11	09.3	05.8	09.2	06.0	09.1	06.1	09.1	06.2	09.0	06.3	08.9	06.5	08.8	06.6	11
12	10.2	06.4	10.1	06.5	10.0	06.7	09.9	06.7	09.8	06.9	09.7	07.0	09.6	07.1	12
13	11.0	06.9	10.9	07.1	10.8	07.2	10.8	07.3	10.6	07.5	10.5	07.6	10.4	07.7	13
14	11.9	07.4	11.7	07.6	11.6	07.8	11.6	07.8	11.5	08.0	11.3	08.2	11.2	08.3	14
15	12.7	07.9	12.6	08.2	12.5	08.3	12.4	08.3	12.3	08.6	12.1	08.8	12.0	08.9	15
16	13.6	08.5	13.4	08.7	13.3	08.9	13.3	08.9	13.1	09.2	12.9	09.4	12.9	09.5	16
17	14.4	09.0	14.3	09.3	14.1	09.4	14.1	09.5	13.9	09.8	13.8	10.0	13.7	10.1	17
18	15.3	09.5	15.1	09.8	15.0	10.0	14.9	10.1	14.7	10.3	14.6	10.6	14.5	10.7	18
19	16.1	10.1	15.9	10.3	15.8	10.6	15.8	10.6	15.6	10.9	15.4	11.2	15.3	11.3	19
20	17.0	10.6	16.8	10.9	16.6	11.1	16.6	11.2	16.4	11.5	16.2	11.8	16.1	11.9	20
21	17.8	11.1	17.6	11.4	17.5	11.7	17.4	11.7	17.2	12.0	17.0	12.3	16.9	12.5	21
22	18.7	11.7	18.5	12.0	18.3	12.2	18.2	12.3	18.0	12.6	17.8	12.9	17.7	13.1	22
23	19.5	12.2	19.3	12.5	19.1	12.8	19.1	12.9	18.8	13.2	18.6	13.5	18.5	13.7	23
24	20.4	12.7	20.1	13.1	20.0	13.3	19.9	13.4	19.7	13.8	19.4	14.1	19.3	14.3	24
25	21.2	13.2	21.0	13.6	20.8	13.9	20.7	14.0	20.5	14.3	20.2	14.7	20.1	14.9	25
26	22.0	13.9	21.8	14.2	21.6	14.4	21.6	14.5	21.3	14.9	21.0	15.3	20.9	15.5	26
27	22.9	14.3	22.6	14.7	22.4	15.0	22.4	15.1	22.1	15.5	21.8	15.9	21.7	16.1	27
28	23.7	14.8	23.5	15.2	23.3	15.6	23.2	15.6	22.9	16.1	22.7	16.5	22.5	16.7	28
29	24.6	15.4	24.3	15.8	24.1	16.1	24.0	16.2	23.8	16.6	23.5	17.0	23.3	17.3	29
30	25.4	15.9	25.2	16.3	24.9	16.7	24.9	16.8	24.6	17.2	24.3	17.6	24.1	17.9	30
31	26.3	16.4	26.0	16.9	25.8	17.2	25.7	17.3	25.4	17.8	25.1	18.2	24.9	18.5	31
32	27.1	17.0	26.8	17.4	26.6	17.8	26.5	17.9	26.2	18.4	25.9	18.8	25.7	19.1	32
33	28.0	17.5	27.7	18.0	27.4	18.3	27.4	18.5	27.0	18.9	26.7	19.4	26.5	19.7	33
34	28.8	18.0	28.5	18.5	28.3	18.9	28.2	19.0	27.9	19.5	27.5	20.0	27.3	20.3	34
35	29.7	18.5	29.4	19.1	29.1	19.4	29.0	19.6	28.7	20.1	28.3	20.6	28.1	20.8	35
36	30.5	19.1	30.2	19.6	29.9	20.0	29.8	20.1	29.5	20.6	29.1	21.2	28.9	21.4	36
37	31.4	19.6	31.0	20.1	30.8	20.6	30.7	20.7	30.3	21.2	29.9	21.7	29.7	22.0	37
38	32.2	20.1	31.9	20.7	31.6	21.1	31.5	21.2	31.1	21.8	30.7	22.3	30.5	22.6	38
39	33.1	20.7	32.7	21.2	32.4	21.7	32.3	21.8	32.0	22.4	31.6	22.9	31.3	23.2	39
40	33.9	21.2	33.6	21.8	33.3	22.2	33.2	22.4	32.8	22.9	32.4	23.5	32.1	23.8	40
41	34.8	21.7	34.4	22.3	34.1	22.8	34.0	22.9	33.6	23.5	33.2	24.1	32.9	24.4	41
42	35.6	22.3	35.2	22.9	34.9	23.3	34.8	23.5	34.4	24.1	34.0	24.7	33.7	25.0	42
43	36.5	22.8	36.1	23.4	35.8	23.9	35.6	24.0	35.2	24.7	34.8	25.3	34.5	25.6	43
44	37.3	23.3	36.9	24.0	36.6	24.4	36.5	24.6	36.0	25.2	35.6	25.9	35.3	26.2	44
45	38.2	23.8	37.7	24.5	37.4	25.0	37.3	25.2	36.9	25.8	36.4	26.5	36.1	26.8	45
46	39.0	24.4	38.6	25.1	38.2	25.5	38.1	25.7	37.7	26.4	37.2	27.0	36.9	27.4	46
47	39.9	24.9	39.4	25.6	39.1	26.1	39.0	26.3	38.5	27.0	38.0	27.6	37.7	28.0	47
48	40.7	25.4	40.3	26.1	39.9	26.7	39.8	26.8	39.3	27.5	38.8	28.2	38.6	28.6	48
49	41.6	26.0	41.1	26.7	40.7	27.2	40.6	27.4	40.1	28.1	39.6	28.8	39.4	29.2	49
50	42.4	26.5	41.9	27.2	41.6	27.8	41.4	28.0	41.0	28.7	40.4	29.4	40.2	29.8	50
Dist.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dist.
	58 Deg.		57 Deg.		5 Points		56 Deg.		55 Deg.		54 Deg.		$4\frac{3}{4}$ Point.		

Dist.	32 Deg.		33 Deg.		3 Points		34 Deg.		35 Deg.		36 Deg.		3¼ Point.		Dist.
	Lat.	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	
51	43.2	27.0	42.8	27.8	42.4	28.3	42.3	28.5	41.8	29.3	41.3	30.0	41.0	30.4	51
52	44.1	27.6	43.6	28.3	43.2	28.9	43.1	29.1	42.6	29.8	42.1	30.6	41.8	31.0	52
53	44.9	28.1	44.5	28.9	44.1	29.4	43.9	29.6	43.4	30.4	42.9	31.2	42.6	31.6	53
54	45.8	28.6	45.3	29.4	44.9	30.0	44.8	30.2	44.2	31.0	43.7	31.7	43.4	32.2	54
55	46.6	29.1	46.1	30.0	45.7	30.6	45.6	30.8	45.1	31.5	44.5	32.3	44.2	32.8	55
56	47.5	29.7	47.0	30.5	46.6	31.1	46.4	31.3	45.9	32.1	45.3	32.9	45.0	33.4	56
57	48.3	30.2	47.8	31.0	47.4	31.7	47.3	31.9	46.7	32.7	46.1	33.5	45.8	34.0	57
58	49.2	30.7	48.6	31.6	48.2	32.2	48.1	32.4	47.5	33.3	46.9	34.1	46.6	34.5	58
59	50.0	31.3	49.5	32.1	49.1	32.8	48.9	33.0	48.3	33.8	47.7	34.7	47.4	35.1	59
60	50.9	31.8	50.3	32.7	49.9	33.3	49.7	33.6	49.2	34.4	48.5	35.3	48.2	35.7	60
61	51.7	32.3	51.2	33.2	50.7	33.9	50.6	34.1	50.0	35.0	49.3	35.9	49.0	36.3	61
62	52.6	32.8	52.0	33.8	51.6	34.4	51.4	34.7	50.8	35.6	50.2	36.4	49.8	36.9	62
63	53.4	33.4	52.8	34.3	52.4	35.0	52.2	35.2	51.6	36.1	51.0	37.0	50.6	37.5	63
64	54.3	33.9	53.7	34.9	53.2	35.6	53.1	35.8	52.4	36.7	51.8	37.6	51.4	38.1	64
65	55.1	34.4	54.5	35.4	54.0	36.1	53.9	36.3	53.2	37.3	52.6	38.2	52.2	38.7	65
66	56.0	35.0	55.4	35.9	54.9	36.7	54.7	36.9	54.1	37.9	53.4	38.8	53.0	39.3	66
67	56.8	35.5	56.2	36.5	55.7	37.2	55.5	37.5	54.9	38.4	54.2	39.4	53.8	39.9	67
68	57.7	36.0	57.0	37.0	56.5	37.8	56.4	38.0	55.7	39.0	55.0	40.0	54.6	40.5	68
69	58.5	36.6	57.9	37.6	57.4	38.3	57.2	38.6	56.5	39.6	55.8	40.6	55.4	41.1	69
70	59.4	37.1	58.7	38.1	58.2	38.9	58.0	39.1	57.3	40.2	56.6	41.1	56.2	41.7	70
71	60.2	37.6	59.5	38.7	59.0	39.4	58.9	39.7	58.2	40.7	57.4	41.7	57.0	42.3	71
72	61.1	38.2	60.4	39.2	59.9	40.0	59.7	40.3	59.0	41.3	58.2	42.3	57.8	42.9	72
73	61.9	38.7	61.2	39.8	60.7	40.6	60.5	40.8	59.8	41.9	59.1	42.9	58.6	43.5	73
74	62.8	39.2	62.1	40.3	61.5	41.1	61.3	41.4	60.6	42.4	59.9	43.5	59.4	44.1	74
75	63.6	39.7	62.9	40.8	62.4	41.7	62.2	41.9	61.4	43.0	60.7	44.1	60.2	44.7	75
76	64.4	40.3	63.7	41.4	63.2	42.2	63.0	42.5	62.3	43.6	61.5	44.7	61.0	45.3	76
77	65.3	40.8	64.6	41.9	64.0	42.8	63.8	43.1	63.1	44.2	62.3	45.3	61.8	45.9	77
78	66.1	41.3	65.4	42.5	64.9	43.3	64.7	43.6	63.9	44.7	63.1	45.8	62.6	46.5	78
79	67.0	41.9	66.3	43.0	65.7	43.9	65.5	44.2	64.7	45.3	63.9	46.4	63.5	47.1	79
80	67.8	42.4	67.1	43.6	66.5	44.4	66.3	44.7	65.5	45.9	64.7	47.0	64.3	47.7	80
81	68.7	42.9	67.9	44.1	67.4	45.0	67.1	45.3	66.4	46.5	65.5	47.6	65.1	48.3	81
82	69.5	43.4	68.8	44.7	68.2	45.6	68.0	45.9	67.2	47.0	66.3	48.2	65.9	48.8	82
83	70.4	44.0	69.6	45.2	69.0	46.1	68.8	46.4	68.0	47.6	67.1	48.8	66.7	49.4	83
84	71.2	44.5	70.5	45.7	69.8	46.7	69.6	47.0	68.8	48.2	68.0	49.4	67.5	50.0	84
85	72.1	45.0	71.3	46.3	70.7	47.2	70.5	47.5	69.6	48.8	68.8	50.0	68.3	50.6	85
86	72.9	45.6	72.1	46.8	71.5	47.8	71.3	48.1	70.5	49.3	69.6	50.5	69.1	51.2	86
87	73.8	46.1	73.0	47.4	72.3	48.3	72.1	48.6	71.3	49.9	70.4	51.1	69.9	51.8	87
88	74.6	46.6	73.8	47.9	73.2	48.9	73.0	49.2	72.1	50.5	71.2	51.7	70.7	52.4	88
89	75.5	47.2	74.6	48.5	74.0	49.4	73.8	49.8	72.9	51.0	72.0	52.3	71.5	53.0	89
90	76.3	47.7	75.5	49.0	74.8	50.0	74.6	50.3	73.7	51.6	72.8	52.9	72.3	53.6	90
91	77.2	48.2	76.3	49.6	75.7	50.6	75.4	50.9	74.5	52.2	73.6	53.5	73.1	54.2	91
92	78.0	48.7	77.2	50.1	76.5	51.1	76.3	51.4	75.4	52.8	74.4	54.1	73.9	54.8	92
93	78.9	49.3	78.0	50.6	77.3	51.7	77.1	52.0	76.2	53.3	75.2	54.7	74.7	55.4	93
94	79.7	49.8	78.8	51.2	78.2	52.2	77.9	52.6	77.0	53.9	76.0	55.3	75.5	56.0	94
95	80.6	50.3	79.7	51.7	79.0	52.8	78.8	53.1	77.8	54.5	76.9	55.8	76.3	56.6	95
96	81.4	50.9	80.5	52.3	79.8	53.3	79.6	53.7	78.6	55.1	77.7	56.4	77.1	57.2	96
97	82.3	51.4	81.4	52.8	80.7	53.9	80.4	54.2	79.5	55.6	78.5	57.0	77.9	57.8	97
98	83.1	51.9	82.2	53.4	81.5	54.4	81.2	54.8	80.3	56.2	79.3	57.6	78.7	58.4	98
99	84.0	52.5	83.0	53.9	82.3	55.0	82.1	55.4	81.1	56.8	80.1	58.2	79.5	59.0	99
100	84.8	53.0	83.9	54.5	83.1	55.6	82.9	55.9	81.9	57.4	80.9	58.8	80.3	59.6	100
Dist.	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	*Dep.*	*Lat.*	Dist.
	58 Deg.		57 Deg.		5 Points		56 Deg.		55 Deg.		54 Deg.		4¾ Point.		

Dist.	37 Deg.		38 Deg.		39 Deg.		3½ Point.		40 Deg.		41 Deg.		42 Deg.		Dist.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	00.8	00.6	00.8	00.6	00.8	00.6	00.8	00.6	00.8	00.6	00.8	00.7	00.7	00.7	1
2	01.6	01.2	01.6	01.2	01.6	01.3	01.5	01.3	01.5	01.3	01.5	01.3	01.5	01.3	2
3	02.4	01.8	02.4	01.8	02.3	01.9	02.3	01.9	02.3	01.9	02.3	02.0	02.2	02.0	3
4	03.2	02.4	03.2	02.5	03.1	02.5	03.1	02.5	03.1	02.6	03.0	02.6	03.0	02.7	4
5	04.0	03.0	03.9	03.1	03.9	03.1	03.9	03.2	03.8	03.2	03.8	03.3	03.7	03.3	5
6	04.8	03.6	04.7	03.7	04.7	03.8	04.6	03.8	04.6	03.9	04.5	03.9	04.5	04.0	6
7	05.6	04.2	05.5	04.3	05.4	04.4	05.4	04.4	05.4	04.5	05.3	04.6	05.2	04.7	7
8	06.4	04.8	06.3	04.9	06.2	05.0	06.2	05.1	06.1	05.1	06.0	05.2	05.9	05.4	8
9	07.2	05.4	07.1	05.5	07.0	05.7	07.0	05.7	06.9	05.8	06.8	05.9	06.7	06.0	9
10	08.0	06.0	07.9	06.2	07.8	06.3	07.7	06.3	07.7	06.4	07.5	06.6	07.4	06.7	10
11	08.8	06.6	08.7	06.8	08.5	06.9	08.5	07.0	08.4	07.1	08.3	07.2	08.2	07.4	11
12	09.6	07.2	09.5	07.4	09.3	07.6	09.3	07.6	09.2	07.7	09.1	07.9	08.9	08.0	12
13	10.4	07.8	10.2	08.0	10.1	08.2	10.0	08.2	10.0	08.4	09.8	08.5	09.7	08.7	13
14	11.2	08.4	11.0	08.6	10.9	08.8	10.8	08.9	10.7	09.0	10.6	09.2	10.4	09.4	14
15	12.0	09.0	11.8	09.2	11.7	09.4	11.6	09.5	11.5	09.6	11.3	09.8	11.1	10.0	15
16	12.8	09.6	12.6	09.9	12.4	10.1	12.4	10.1	12.3	10.3	12.1	10.5	11.9	10.7	16
17	13.6	10.2	13.4	10.5	13.2	10.7	13.1	10.8	13.0	10.9	12.8	11.2	12.6	11.4	17
18	14.4	10.8	14.2	11.1	14.0	11.3	13.9	11.4	13.8	11.6	13.6	11.8	13.4	12.0	18
19	15.2	11.4	15.0	11.7	14.8	12.0	14.7	12.1	14.6	12.2	14.3	12.5	14.1	12.7	19
20	16.0	12.0	15.8	12.3	15.5	12.6	15.5	12.7	15.3	12.9	15.1	13.1	14.9	13.4	20
21	16.8	12.6	16.5	12.9	16.3	13.2	16.2	13.3	16.1	13.5	15.8	13.8	15.6	14.0	21
22	17.6	13.2	17.3	13.5	17.1	13.8	17.0	14.0	16.9	14.1	16.6	14.4	16.3	14.7	22
23	18.4	13.8	18.1	14.2	17.9	14.5	17.8	14.6	17.6	14.8	17.4	15.1	17.1	15.4	23
24	19.2	14.4	18.9	14.8	18.6	15.1	18.6	15.2	18.4	15.4	18.1	15.7	17.8	16.1	24
25	20.0	15.0	19.7	15.4	19.4	15.7	19.3	15.9	19.1	16.1	18.9	16.4	18.6	16.7	25
26	20.8	15.6	20.5	16.0	20.2	16.4	20.1	16.5	19.9	16.7	19.6	17.1	19.3	17.4	26
27	21.6	16.2	21.3	16.6	21.0	17.0	20.9	17.1	20.7	17.4	20.4	17.7	20.1	18.1	27
28	22.4	16.8	22.1	17.2	21.8	17.6	21.6	17.8	21.4	18.0	21.1	18.4	20.8	18.7	28
29	23.2	17.5	22.9	17.9	22.5	18.2	22.4	18.4	22.2	18.6	21.9	19.0	21.5	19.4	29
30	24.0	18.1	23.6	18.5	23.3	18.9	23.2	19.0	23.0	19.3	22.6	19.7	22.3	20.1	30
31	24.8	18.7	24.4	19.1	24.1	19.5	24.0	19.7	23.7	19.9	23.4	20.3	23.0	20.7	31
32	25.6	19.3	25.2	19.7	24.9	20.1	24.7	20.3	24.5	20.6	24.1	21.0	23.8	21.4	32
33	26.4	19.9	26.0	20.3	25.6	20.8	25.5	20.9	25.3	21.2	24.9	21.7	24.5	22.1	33
34	27.2	20.5	26.8	20.9	26.4	21.4	26.3	21.6	26.0	21.9	25.7	22.3	25.3	22.7	34
35	28.0	21.1	27.6	21.5	27.2	22.0	27.1	22.2	26.8	22.5	26.4	23.0	26.0	23.4	35
36	28.7	21.7	28.4	22.2	28.0	22.7	27.8	22.8	27.6	23.1	27.2	23.6	26.8	24.1	36
37	29.5	22.3	29.2	22.8	28.8	23.3	28.6	23.5	28.3	23.8	27.9	24.3	27.5	24.8	37
38	30.3	22.9	29.9	23.4	29.5	23.9	29.4	24.1	29.1	24.4	28.7	24.9	28.2	25.4	38
39	31.1	23.5	30.7	24.0	30.3	24.5	30.1	24.7	29.9	25.1	29.4	25.6	29.0	26.1	39
40	31.9	24.1	31.5	24.6	31.1	25.2	30.9	25.4	30.6	25.7	30.2	26.2	29.7	26.8	40
41	32.7	24.7	32.3	25.2	31.9	25.8	31.7	26.0	31.4	26.4	30.9	26.9	30.5	27.4	41
42	33.5	25.3	33.1	25.9	32.6	26.4	32.5	26.6	32.2	27.0	31.7	27.6	31.2	28.1	42
43	34.3	25.9	33.9	26.5	33.4	27.1	33.2	27.3	32.9	27.6	32.5	28.2	32.0	28.8	43
44	35.1	26.5	34.7	27.1	34.2	27.7	34.0	27.9	33.7	28.3	33.2	28.9	32.7	29.4	44
45	35.9	27.1	35.5	27.7	35.0	28.3	34.8	28.5	34.5	28.9	34.0	29.5	33.4	30.1	45
46	36.7	27.7	36.2	28.3	35.7	28.9	35.6	29.2	35.2	29.6	34.7	30.2	34.2	30.8	46
47	37.5	28.3	37.0	28.9	36.5	29.6	36.3	29.8	36.0	30.2	35.5	30.8	34.9	31.4	47
48	38.3	28.9	37.8	29.6	37.3	30.2	37.1	30.5	36.8	30.9	36.2	31.5	35.7	32.1	48
49	39.1	29.5	38.6	30.2	38.1	30.8	37.9	31.1	37.5	31.5	37.0	32.1	36.4	32.8	49
50	39.9	30.1	39.4	30.8	38.9	31.5	38.6	31.7	38.3	32.1	37.7	32.8	37.2	33.5	50
Dist.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dist.
	53 Deg.		52 Deg.		51 Deg.		4½ Point.		50 Deg.		49 Deg.		48 Deg.		

Dist.	37 Deg.		38 Deg.		39 Deg.		3½ Point.		40 Deg.		41 Deg.		42 Deg.		Dist.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
51	40.7	30.7	40.2	31.4	39.6	32.1	39.4	32.4	39.1	32.8	38.5	33.5	37.9	34.1	51
52	41.5	31.3	41.0	32.0	40.4	32.7	40.2	33.0	39.8	33.4	39.2	34.1	38.6	34.8	52
53	42.3	31.9	41.8	32.6	41.2	33.4	41.0	33.6	40.6	34.1	40.0	34.8	39.4	35.5	53
54	43.1	32.5	42.6	33.2	42.0	34.0	41.7	34.3	41.4	34.7	40.8	35.4	40.1	36.1	54
55	43.9	33.1	43.3	33.9	42.7	34.6	42.5	34.9	42.1	35.4	41.5	36.1	40.9	36.8	55
56	44.7	33.7	44.1	34.5	43.5	35.2	43.3	35.5	42.9	36.0	42.3	36.7	41.6	37.5	56
57	45.5	34.3	44.9	35.1	44.3	35.9	44.1	36.2	43.7	36.6	43.0	37.4	42.4	38.1	57
58	46.3	34.9	45.7	35.7	45.1	36.5	44.8	36.8	44.4	37.3	43.8	38.1	43.1	38.8	58
59	47.1	35.5	46.5	36.3	45.8	37.1	45.6	37.4	45.2	37.9	44.5	38.7	43.8	39.5	59
60	47.9	36.1	47.3	36.9	46.6	37.8	46.4	38.1	46.0	38.6	45.3	39.4	44.6	40.1	60
61	48.7	36.7	48.1	37.6	47.4	38.4	47.2	38.7	46.7	39.2	46.0	40.0	45.3	40.8	61
62	49.5	37.3	48.9	38.2	48.2	39.0	47.9	39.3	47.5	39.9	46.8	40.7	46.1	41.5	62
63	50.3	37.9	49.6	38.8	49.0	39.6	48.7	40.0	48.3	40.5	47.5	41.3	46.8	42.2	63
64	51.1	38.5	50.4	39.4	49.7	40.3	49.5	40.6	49.0	41.1	48.3	42.0	47.6	42.8	64
65	51.9	39.1	51.2	40.0	50.5	40.9	50.2	41.2	49.8	41.8	49.1	42.6	48.3	43.5	65
66	52.7	39.7	52.0	40.6	51.3	41.5	51.0	41.9	50.6	42.4	49.8	43.3	49.0	44.2	66
67	53.5	40.3	52.8	41.3	52.1	42.2	51.8	42.5	51.3	43.1	50.6	44.0	49.8	44.8	67
68	54.3	40.9	53.6	41.9	52.8	42.8	52.6	43.1	52.1	43.7	51.3	44.6	50.5	45.5	68
69	55.1	41.5	54.4	42.5	53.6	43.4	53.3	43.8	52.9	44.4	52.1	45.3	51.3	46.2	69
70	55.9	42.1	55.2	43.1	54.4	44.1	54.1	44.4	53.6	45.0	52.8	45.9	52.0	46.8	70
71	56.7	42.7	55.9	43.7	55.2	44.7	54.9	45.0	54.4	45.6	53.6	46.6	52.8	47.5	71
72	57.5	43.3	56.7	44.3	56.0	45.3	55.7	45.7	55.2	46.3	54.3	47.2	53.5	48.2	72
73	58.3	43.9	57.5	44.9	56.7	45.9	56.4	46.3	55.9	46.9	55.1	47.9	54.2	48.8	73
74	59.1	44.5	58.3	45.6	57.5	46.6	57.2	46.9	56.7	47.6	55.8	48.6	55.0	49.5	74
75	59.9	45.1	59.1	46.2	58.3	47.2	58.0	47.6	57.5	48.2	56.6	49.2	55.7	50.2	75
76	60.7	45.7	59.9	46.8	59.1	47.8	58.7	48.2	58.2	48.9	57.4	49.9	56.5	50.9	76
77	61.5	46.3	60.7	47.4	59.8	48.5	59.5	48.8	59.0	49.5	58.1	50.5	57.2	51.5	77
78	62.3	46.9	61.5	48.0	60.6	49.1	60.3	49.5	59.7	50.1	58.9	51.2	58.0	52.2	78
79	63.1	47.5	62.3	48.6	61.4	49.7	61.1	50.1	60.5	50.8	59.6	51.8	58.7	52.9	79
80	63.9	48.1	63.0	49.3	62.2	50.3	61.8	50.8	61.3	51.4	60.4	52.5	59.4	53.5	80
81	64.7	48.7	63.8	49.9	62.9	51.0	62.6	51.4	62.0	52.1	61.1	53.1	60.2	54.2	81
82	65.5	49.3	64.6	50.5	63.7	51.6	63.4	52.0	62.8	52.7	61.9	53.8	60.9	54.9	82
83	66.3	49.9	65.4	51.1	64.5	52.2	64.2	52.7	63.6	53.4	62.6	54.5	61.7	55.5	83
84	67.1	50.6	66.2	51.7	65.3	52.9	64.9	53.3	64.3	54.0	63.4	55.1	62.4	56.2	84
85	67.9	51.2	67.0	52.3	66.1	53.5	65.7	53.9	65.1	54.6	64.1	55.8	63.2	56.9	85
86	68.7	51.8	67.8	52.9	66.8	54.1	66.5	54.6	65.9	55.3	64.9	56.4	63.9	57.5	86
87	69.5	52.4	68.6	53.6	67.6	54.7	67.3	55.2	66.6	55.9	65.7	57.1	64.6	58.2	87
88	70.3	53.0	69.3	54.2	68.4	55.4	68.0	55.8	67.4	56.6	66.4	57.7	65.4	58.9	88
89	71.1	53.6	70.1	54.8	69.2	56.0	68.8	56.5	68.2	57.2	67.2	58.4	66.1	59.6	89
90	71.9	54.2	70.9	55.4	69.9	56.6	69.6	57.1	68.9	57.9	67.9	59.0	66.9	60.2	90
91	72.7	54.8	71.7	56.0	70.7	57.3	70.3	57.7	69.7	58.5	68.7	59.7	67.6	60.9	91
92	73.5	55.4	72.5	56.6	71.5	57.9	71.1	58.4	70.5	59.1	69.4	60.4	68.4	61.6	92
93	74.3	56.0	73.3	57.3	72.3	58.5	71.9	59.0	71.2	59.8	70.2	61.0	69.1	62.2	93
94	75.1	56.6	74.1	57.9	73.0	59.2	72.7	59.6	72.0	60.4	70.9	61.7	69.9	62.9	94
95	75.9	57.2	74.9	58.5	73.8	59.8	73.4	60.3	72.8	61.1	71.7	62.3	70.6	63.6	95
96	76.7	57.8	75.6	59.1	74.6	60.4	74.2	60.9	73.5	61.7	72.5	63.0	71.3	64.2	96
97	77.5	58.4	76.4	59.7	75.4	61.0	75.0	61.5	74.3	62.4	73.2	63.6	72.1	64.9	97
98	78.3	59.0	77.2	60.3	76.2	61.7	75.8	62.2	75.1	63.0	74.0	64.3	72.8	65.6	98
99	79.1	59.6	78.0	61.0	76.9	62.3	76.5	62.8	75.8	63.6	74.7	65.0	73.6	66.2	99
100	79.9	60.2	78.8	61.6	77.7	62.9	77.3	63.4	76.6	64.3	75.5	65.6	74.3	66.9	100
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
Dist.	53 Deg.		52 Deg.		51 Deg.		4½ Point.		50 Deg.		49 Deg.		48 Deg.		Dist.

Dist.	3¾ Point.		43 Deg.		44 Deg.		4 Points		Dist.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	00.7	00.7	00.7	00.7	00.7	00.7	00.7	00.7	1
2	01.5	01.3	01.5	01.4	01.4	01.4	01.4	01.4	2
3	02.2	02.0	02.2	02.0	02.2	02.1	02.1	02.1	3
4	03.0	02.7	03.0	02.7	02.9	02.8	02.8	02.8	4
5	03.7	03.4	03.7	03.4	03.6	03.5	03.5	03.5	5
6	04.4	04.0	04.4	04.1	04.3	04.2	04.2	04.2	6
7	05.2	04.7	05.1	04.8	05.0	04.9	04.9	04.9	7
8	05.9	05.4	05.9	05.5	05.8	05.6	05.7	05.7	8
9	06.7	06.0	06.6	06.1	06.5	06.3	06.4	06.4	9
10	07.4	06.7	07.3	06.8	07.2	06.9	07.1	07.1	10
11	08.2	07.4	08.0	07.5	07.9	07.6	07.8	07.8	11
12	08.9	08.1	08.8	08.2	08.6	08.3	08.5	08.5	12
13	09.6	08.7	09.5	08.9	09.3	09.0	09.2	09.2	13
14	10.4	09.4	10.2	09.5	10.1	09.7	09.9	09.9	14
15	11.1	10.1	11.0	10.2	10.8	10.4	10.6	10.6	15
16	11.9	10.7	11.7	10.9	11.5	11.1	11.3	11.3	16
17	12.6	11.4	12.4	11.6	12.2	11.8	12.0	12.0	17
18	13.3	12.1	13.2	12.3	12.9	12.5	12.7	12.7	18
19	14.1	12.8	13.9	13.0	13.7	13.2	13.4	13.4	19
20	14.8	13.4	14.6	13.6	14.4	13.9	14.1	14.1	20
21	15.6	14.1	15.4	14.3	15.1	14.6	14.8	14.8	21
22	16.3	14.8	16.1	15.0	15.8	15.3	15.6	15.6	22
23	17.0	15.4	16.8	15.7	16.5	16.0	16.3	16.3	23
24	17.8	16.1	17.6	16.4	17.3	16.7	17.0	17.0	24
25	18.5	16.8	18.3	17.1	18.0	17.4	17.7	17.7	25
26	19.3	17.5	19.0	17.7	18.7	18.1	18.4	18.4	26
27	20.0	18.1	19.7	18.4	19.4	18.8	19.1	19.1	27
28	20.7	18.8	20.5	19.1	20.1	19.5	19.8	19.8	28
29	21.5	19.5	21.2	19.8	20.9	20.1	20.5	20.5	29
30	22.2	20.1	21.9	20.5	21.6	20.8	21.2	21.2	30
31	23.0	20.8	22.7	21.1	22.3	21.5	21.9	21.9	31
32	23.7	21.5	23.4	21.8	23.0	22.2	22.6	22.6	32
33	24.5	22.2	24.1	22.5	23.7	22.9	23.3	23.3	33
34	25.2	22.8	24.9	23.2	24.5	23.6	24.0	24.0	34
35	25.9	23.5	25.6	23.9	25.2	24.3	24.7	24.7	35
36	26.7	24.2	26.3	24.6	25.9	25.0	25.5	25.5	36
37	27.4	24.8	27.0	25.2	26.6	25.7	26.2	26.2	37
38	28.2	25.5	27.8	25.9	27.3	26.4	26.9	26.9	38
39	28.9	26.2	28.5	26.6	28.1	27.1	27.6	27.6	39
40	29.6	26.9	29.3	27.3	28.8	27.8	28.3	28.3	40
41	30.4	27.5	30.0	28.0	29.5	28.5	29.0	29.0	41
42	31.1	28.2	30.7	28.6	30.2	29.2	29.7	29.7	42
43	31.8	28.9	31.4	29.3	30.9	29.9	30.4	30.4	43
44	32.6	29.5	32.2	30.0	31.6	30.6	31.1	31.1	44
45	33.3	30.2	32.9	30.7	32.4	31.3	31.8	31.8	45
46	34.1	30.9	33.6	31.4	33.1	32.0	32.5	32.5	46
47	34.8	31.6	34.4	32.1	33.8	32.6	33.2	33.2	47
48	35.6	32.2	35.1	32.7	34.5	33.3	33.9	33.9	48
49	36.3	32.9	35.8	33.4	35.2	34.0	34.6	34.6	49
50	37.0	33.6	36.6	34.1	36.0	34.7	35.4	35.4	50
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
Dist.	4¼ Point.		47 Deg.		46 Deg.		4 Points		Dist.

Dist.	3¾ Point.		43 Deg.		44 Deg.		4 Points		Dist.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
51	37.8	34.3	37.3	34.8	36.7	35.4	36.1	36.1	51
52	38.5	34.9	38.0	35.5	37.4	36.1	36.8	36.8	52
53	39.3	35.6	38.8	36.1	38.1	36.8	37.5	37.5	53
54	40.0	36.3	39.5	36.8	38.8	37.5	38.2	38.2	54
55	40.8	36.9	40.2	37.5	39.6	38.2	38.9	38.9	55
56	41.5	37.6	41.0	38.2	40.3	38.9	39.6	39.6	56
57	42.2	38.3	41.7	38.9	41.0	39.6	40.3	40.3	57
58	43.0	39.0	42.4	39.6	41.7	40.3	41.0	41.0	58
59	43.7	39.6	43.2	40.2	42.4	41.0	41.7	41.7	59
60	44.5	40.3	43.9	40.9	43.2	41.7	42.4	42.4	60
61	45.2	41.0	44.6	41.6	43.9	42.4	43.1	43.1	61
62	45.9	41.6	45.3	42.3	44.6	43.1	43.8	43.8	62
63	46.7	42.3	46.1	43.0	45.3	43.8	44.5	44.5	63
64	47.4	43.0	46.8	43.6	46.0	44.5	45.3	45.3	64
65	48.2	43.7	47.5	44.3	46.8	45.2	46.0	46.0	65
66	48.9	44.3	48.3	45.0	47.5	45.8	46.7	46.7	66
67	49.6	45.0	49.0	45.7	48.2	46.5	47.4	47.4	67
68	50.4	45.7	49.7	46.4	48.9	47.2	48.1	48.1	68
69	51.1	46.3	50.5	47.1	49.6	47.9	48.8	48.8	69
70	51.9	47.0	51.2	47.7	50.4	48.6	49.5	49.5	70
71	52.6	47.7	51.9	48.4	51.1	49.3	50.2	50.2	71
72	53.4	48.4	52.7	49.1	51.8	50.0	50.9	50.9	72
73	54.1	49.0	53.4	49.8	52.5	50.7	51.6	51.6	73
74	54.8	49.7	54.1	50.5	53.2	51.4	52.3	52.3	74
75	55.6	50.4	54.9	51.1	53.9	52.1	53.0	53.0	75
76	56.3	51.0	55.6	51.8	54.7	52.8	53.7	53.7	76
77	57.1	51.7	56.3	52.5	55.4	53.5	54.4	54.4	77
78	57.8	52.4	57.0	53.2	56.1	54.2	55.2	55.2	78
79	58.5	53.1	57.8	53.9	56.8	54.9	55.9	55.9	79
80	59.3	53.7	58.5	54.6	57.5	55.6	56.6	56.6	80
81	60.0	54.4	59.2	55.2	58.3	56.3	57.3	57.3	81
82	60.8	55.1	60.0	55.9	59.0	57.0	58.0	58.0	82
83	61.5	55.7	60.7	56.6	59.7	57.7	58.7	58.7	83
84	62.2	56.4	61.4	57.3	60.4	58.4	59.4	59.4	84
85	63.0	57.1	62.2	58.0	61.1	59.0	60.1	60.1	85
86	63.7	57.8	62.9	58.7	61.9	59.7	60.8	60.8	86
87	64.5	58.4	63.6	59.3	62.6	60.4	61.5	61.5	87
88	65.2	59.1	64.4	60.0	63.3	61.1	62.2	62.2	88
89	65.9	59.8	65.1	60.7	64.0	61.8	62.9	62.9	89
90	66.7	60.4	65.8	61.4	64.7	62.5	63.6	63.6	90
91	67.4	61.1	66.6	62.1	65.5	63.2	64.3	64.3	91
92	68.2	61.8	67.3	62.7	66.2	63.9	65.1	65.1	92
93	68.9	62.5	68.0	63.4	66.9	64.6	65.8	65.8	93
94	69.7	63.1	68.8	64.1	67.6	65.3	66.5	66.5	94
95	70.4	63.8	69.5	64.8	68.3	66.0	67.2	67.2	95
96	71.1	64.5	70.2	65.5	69.1	66.7	67.9	67.9	96
97	71.9	65.1	70.9	66.2	69.8	67.4	68.6	68.6	97
98	72.6	65.8	71.7	66.8	70.5	68.1	69.3	69.3	98
99	73.4	66.5	72.4	67.5	71.2	68.8	70.0	70.0	99
100	74.1	67.2	73.1	68.2	71.9	69.5	70.7	70.7	100
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
Dist.	4¼ Point.		47 Deg.		46 Deg.		4 Points		Dist.

A Table of Meridional Parts.

L.	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	L.
M	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	M
0	0	60.0	120.0	180.1	240.2	300.4	360.7	421.1	481.6	542.2	603.1	664.1	725.3	786.8	848.5	910.5	0
1	1.0	61.0	121.0	181.1	241.2	301.4	361.7	422.1	482.6	543.3	604.1	665.1	726.4	787.9	849.5	911.5	1
2	2.0	62.0	122.0	182.1	242.2	302.4	362.7	423.1	483.6	544.3	605.1	666.1	727.4	788.9	850.6	912.6	2
3	3.0	63.0	123.0	183.1	243.2	303.4	363.7	424.1	484.6	545.3	606.1	667.1	728.4	789.9	851.6	913.6	3
4	4.0	64.0	124.0	184.1	244.2	304.4	364.7	425.1	485.6	546.3	607.1	668.1	729.4	790.9	852.6	914.6	4
5	5.0	65.0	125.0	185.1	245.2	305.4	365.7	426.1	486.6	547.3	608.2	669.2	730.5	392.0	853.7	915.7	5
6	6.0	66.0	126.0	186.1	246.2	306.4	366.7	427.1	487.6	548.3	609.2	670.2	731.5	793.0	854.7	916.7	6
7	7.0	67.0	127.0	187.1	247.2	307.4	367.7	428.1	488.6	549.3	610.2	671.2	732.5	794.0	855.7	917.7	7
8	8.0	68.0	128.0	188.1	248.2	308.4	368.7	429.1	489.6	550.3	611.2	672.2	733.5	795.0	856.8	918.8	8
9	9.0	69.0	129.0	189.1	249.2	309.4	369.7	430.1	490.7	551.4	612.2	673.2	734.6	796.1	857.8	919.8	9
10	10.0	70.0	130.0	190.1	250.2	310.4	370.7	431.1	491.7	552.4	613.2	674.3	735.6	797.1	858.9	920.8	10
11	11.0	71.0	131.0	191.1	251.2	311.4	371.7	432.1	492.7	553.4	614.2	675.3	736.6	798.1	859.9	921.9	11
12	12.0	72.0	132.0	192.1	252.2	312.4	372.7	433.1	493.7	554.4	615.3	676.3	737.6	799.1	861.0	922.9	12
13	13.0	73.0	133.0	193.1	253.2	313.4	373.7	434.2	494.7	555.4	616.3	677.3	738.7	800.2	862.0	923.9	13
14	14.0	74.0	134.0	194.1	254.2	314.4	374.7	435.2	495.7	556.4	617.3	678.3	739.7	801.2	863.0	925.0	14
15	15.0	75.0	135.0	195.1	255.2	315.4	375.8	436.2	496.7	557.4	618.3	679.4	740.7	802.2	864.1	926.0	15
16	16.0	76.0	136.0	196.1	256.2	316.5	376.8	437.2	497.7	558.4	619.3	680.4	741.7	803.2	865.1	927.0	16
17	17.0	77.0	137.0	197.1	257.2	317.5	377.8	438.2	498.7	559.4	620.3	681.4	742.8	804.3	866.1	928.1	17
18	18.0	78.0	138.0	198.1	258.2	318.5	378.8	439.2	499.8	560.5	621.3	682.4	743.8	805.3	867.2	929.1	18
19	19.0	79.0	139.0	199.1	259.3	319.5	379.8	440.2	500.8	561.5	622.4	683.4	744.8	806.3	868.2	930.1	19
20	20.0	80.0	140.0	200.1	260.3	320.5	380.8	441.2	501.8	562.5	623.4	684.5	745.8	807.3	869.2	931.2	20
21	21.0	81.0	141.0	201.1	261.3	321.5	381.8	442.2	502.8	563.5	624.4	685.5	746.9	808.4	870.3	932.2	21
22	22.0	82.0	142.0	202.1	262.3	322.5	382.8	443.2	503.8	564.5	625.4	686.5	747.9	809.4	871.3	933.2	22
23	23.0	83.0	143.0	203.1	263.3	323.5	383.8	444.2	504.8	565.5	626.4	687.5	748.9	810.4	872.3	934.3	23
24	24.0	84.0	144.0	204.1	264.3	324.5	384.8	445.2	505.8	566.6	627.4	688.5	749.9	811.4	873.4	935.3	24
25	25.0	85.0	145.0	205.1	265.3	325.5	385.8	446.3	506.8	567.6	628.5	689.6	751.0	812.5	874.4	936.3	25
26	26.0	86.0	146.0	206.1	266.3	326.5	386.8	447.3	507.8	568.6	629.5	690.6	752.0	813.5	875.4	937.4	26
27	27.0	87.0	147.0	207.1	267.3	327.5	387.8	448.3	508.9	569.6	630.5	691.6	753.0	814.5	876.5	938.4	27
28	28.0	88.0	148.1	208.1	268.3	328.5	388.8	449.3	509.9	570.6	631.5	692.6	754.0	815.5	877.5	939.4	28
29	29.0	89.0	149.1	209.1	269.3	329.5	389.8	450.3	510.9	571.6	632.5	693.6	755.1	816.6	878.5	940.5	29
30	30.0	90.0	150.1	210.1	270.3	330.5	390.8	451.3	511.9	572.6	633.5	694.7	756.1	817.6	879.6	941.5	30
31	31.0	91.0	151.1	211.1	271.3	331.5	391.9	452.3	512.9	573.7	634.6	695.7	757.1	818.6	880.6	942.5	31
32	32.0	92.0	152.1	212.1	272.3	332.5	392.9	453.3	513.9	574.7	635.6	696.7	758.1	819.6	881.6	943.6	32
33	33.0	93.0	153.1	213.1	273.3	333.5	393.9	454.3	514.9	575.7	636.6	697.7	759.2	820.7	882.7	944.6	33
34	34.0	94.0	154.1	214.1	274.3	334.5	394.9	455.3	515.9	576.7	637.6	698.7	760.2	821.7	883.7	945.6	34
35	35.0	95.0	155.1	215.1	275.3	335.5	395.9	456.3	516.9	577.7	638.6	699.8	761.2	822.7	884.7	946.7	35
36	36.0	96.0	156.1	216.1	276.3	336.5	396.9	457.3	518.0	578.7	639.6	700.8	762.2	823.7	885.8	947.7	36
37	37.0	97.0	157.1	217.1	277.3	337.5	397.9	458.4	519.0	579.7	640.6	701.8	763.3	824.8	886.8	948.7	37
38	38.0	98.0	158.1	218.2	278.3	338.6	398.9	459.4	520.0	580.8	641.7	702.8	764.3	825.8	887.8	949.8	38
39	39.0	99.0	159.1	219.2	279.3	339.6	399.9	460.4	521.0	581.8	642.7	703.8	765.3	826.8	888.9	950.8	39
40	40.0	100.0	160.1	220.2	280.3	340.6	400.9	461.4	522.0	582.8	643.7	704.9	766.3	827.9	889.9	951.9	40
41	41.0	101.0	161.1	221.2	281.3	341.6	401.9	462.4	523.0	583.8	644.7	705.9	767.4	828.9	890.9	952.9	41
42	42.0	102.0	162.1	222.2	282.3	342.6	402.9	463.4	524.0	584.8	645.7	706.9	768.4	829.9	892.0	953.9	42
43	43.0	103.0	163.1	223.2	283.3	343.6	403.9	464.4	525.0	585.8	646.7	707.9	769.4	831.0	893.0	955.0	43
44	44.0	104.0	164.1	224.2	284.3	344.6	404.9	465.4	526.0	586.8	647.7	708.9	770.4	832.0	894.0	956.0	44
45	45.0	105.0	165.1	225.2	285.3	345.6	405.9	466.4	527.1	587.9	648.8	710.0	771.5	833.0	895.1	957.1	45
46	46.0	106.0	166.1	226.2	286.3	346.6	407.0	467.4	528.1	588.9	649.8	711.0	772.5	834.1	896.1	958.1	46
47	47.0	107.0	167.1	227.2	287.3	347.6	408.0	468.4	529.1	589.9	650.8	712.0	773.5	835.1	897.1	959.2	47
48	48.0	108.0	168.1	228.2	288.3	348.6	409.0	469.5	530.1	590.9	651.8	713.0	774.5	836.1	898.2	960.2	48
49	49.0	109.0	169.1	229.2	289.3	349.6	410.0	470.5	531.1	591.9	652.8	714.1	775.6	837.2	899.2	961.3	49
50	50.0	110.0	170.1	230.2	290.3	350.6	411.0	471.5	532.1	592.9	653.9	715.1	776.6	838.2	900.2	962.3	50
51	51.0	111.0	171.1	231.2	291.4	351.6	412.0	472.5	533.1	593.9	654.9	716.1	777.6	839.2	901.2	963.4	51
52	52.0	112.0	172.1	232.2	292.4	352.6	413.0	473.5	534.1	595.0	655.9	717.1	778.6	840.3	902.3	964.4	52
53	53.0	113.0	173.1	233.2	293.4	353.6	414.0	474.5	535.1	596.0	656.9	718.2	779.7	841.3	903.3	965.5	53
54	54.0	114.0	174.1	234.2	294.4	354.6	415.0	475.5	536.2	597.0	657.9	719.2	780.7	842.3	904.3	966.5	54
55	55.0	115.0	175.1	235.2	295.4	355.6	416.0	476.5	537.2	598.0	659.0	720.2	781.7	843.4	905.4	967.6	55
56	56.0	116.0	176.1	236.2	296.4	356.6	417.0	477.5	538.2	599.0	660.0	721.2	782.7	844.4	906.4	968.6	56
57	57.0	117.0	177.1	237.2	297.4	357.6	418.0	478.5	539.2	600.0	661.0	722.3	783.8	845.4	907.4	969.6	57
58	58.0	118.0	178.1	238.2	298.4	358.7	419.0	479.6	540.2	601.0	662.0	723.3	784.8	846.5	908.4	970.7	58
59	59.0	119.0	179.1	239.2	299.4	359.7	420.0	480.6	541.2	602.1	663.0	724.3	785.8	847.5	909.5	971.7	59
M	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	M
L.	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	L.

A Table of Meridional Parts.

L.	16	17	18	19	20	21	22	23	24	25	20	27	28	29	L.
M	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	M
0	972.8	1035.3	1098.2	1161.5	1225.1	1289.2	1353.7	1418.7	1484.1	1550.0	1616.5	1683.6	1751.2	1819.5	0
1	973.8	1036.3	1099.3	1162.5	1226.2	1290.2	1354.8	1419.7	1485.2	1551.1	1617.6	1684.7	1752.3	1820.6	1
2	974.8	1037.4	1100.0	1163.6	1227.3	1291.3	1355.8	1420.8	1486.3	1552.2	1618.7	1685.8	1753.4	1821.7	2
3	975.9	1038.4	1101.4	1164.7	1228.3	1292.4	1356.9	1421.9	1487.3	1553.3	1619.8	1686.9	1754.6	1822.9	3
4	976.9	1039.5	1102.4	1165.7	1229.4	1293.5	1358.0	1423.0	1488.4	1554.4	1620.9	1688 0	1755.7	1824.0	4
5	978.0	1040.5	1103.5	1166.8	1230.4	1294.5	1359.0	1424.1	1489.5	1555.5	1622.0	1689.1	1756.8	1825.2	5
6	979.0	1041.6	1104.5	1167.8	1231.5	1295.6	1360.1	1425.1	1490.6	1536.6	1623.2	1690.3	1758.0	1826.3	6
7	980.0	1042.6	1105.6	1168.9	1232.6	1296.7	1361.2	1426.2	1491.7	1557.7	1624.3	1691.4	1759.1	1827.5	7
8	981.1	1043.7	1106.6	1170.0	1233.6	1297.8	1362.3	1427.3	1492.8	1558.8	1625.4	1692.5	1760.2	1828.6	8
9	982.1	1044.7	1107.7	1171.0	1234.7	1298.8	1363.3	1428.4	1493.9	1559.9	1626.5	1693.6	1761.4	1829.7	9
10	983.2	1045.8	1108.7	1172.1	1235.8	1299.9	1364.4	1429.5	1495.0	1561.0	1627.6	1694.8	1762.5	1830.9	10
11	984.2	1046.8	1109.8	1173.1	1236.8	1301.0	1365.5	1430.6	1496.1	1562.1	1628.7	1695.9	1763.6	1832.0	11
12	985.2	1047.9	1110.8	1174.2	1237.9	1302.0	1366.6	1431.7	1497.2	1563.2	1629.8	1697.0	1764.8	1833.2	12
13	986.3	1048.9	1111.9	1175.2	1239.0	1303.1	1367.6	1432.8	1498.3	1564.3	1631.0	1698.1	1765.9	1834.3	13
14	987.3	1049.9	1112.9	1176.3	1240.0	1304.2	1368.7	1433.9	1499.4	1565.4	1632.0	1699.3	1767.0	1835.5	14
15	988.4	1051.0	1114.0	1177.4	1241.1	1305.3	1369.8	1434.9	1500.5	1566.5	1633.2	1700.4	1768.2	1836.6	15
16	989 4	1052.0	1115.0	1178.4	1242.2	1306.3	1370.9	1436.0	1501.6	1567.6	1634.3	1701.5	1769.3	1837.8	16
17	990.4	1053.1	1116.1	1179.5	1243.2	1307.4	1372.0	1437.1	1502.7	1568.7	1635.4	1702.6	1770.5	1838.9	17
18	991.5	1054.1	1117.1	1180.5	1244.3	1308.5	1373.1	1438.2	1503.8	1569.8	1636.5	1703.8	1771.6	1840.1	18
19	992.5	1055.2	1118.2	1181,6	1245.4	1309.6	1374.2	1439.3	1504.9	1571.0	1637.7	1704.9	1772.7	1841.2	19
20	993.6	1056.2	1119.2	1182.7	1246 4	1310.6	1375.3	1440.4	1506.0	1572.1	1638.8	1706.0	1773.9	1842.4	20
21	994.6	1057.3	1120.3	1183.7	1247.5	1311.7	1376.4	1441.5	1507.1	1573.2	1639.9	1707.1	1775.0	1843.5	21
22	995.6	1058.3	1121.3	1184.8	1248.6	1312.8	1377.4	1442.6	1508.2	1574.3	1641.0	1708.3	1776.1	1844.6	22
23	996.7	1059.3	1122.4	1185.8	1249.6	1313.8	1378.5	1443.7	1509.3	1575.4	1642.1	1709.4	1777.2	1845.8	23
24	997.7	1060.4	1123.4	1186.9	1250.7	1314.9	1379.6	1444.8	1510.4	1576.5	1643.2	1710.5	1778.4	1846.9	24
25	998.8	1061.4	1124.5	1188.0	1251.8	1316.0	1380.7	1445.8	1511.5	1577.6	1644.3	1711.6	1779.5	1848.1	25
26	999.8	1062.5	1125.5	1189.0	1252.8	1317.1	1381.8	1446.9	1512.6	1578.7	1645.5	1712.8	1780.6	1849.2	26
27	1000.8	1063.5	1126.6	1190.1	1253.9	1318.1	1382.8	1448.0	1513.7	1579.8	1646.6	1713.9	1781.8	1850.4	27
28	1001.9	1064.6	1127.6	1191.1	1455.0	1319.2	1383.9	1449.1	1514.1	1580.9	1647.7	1715.0	1783.0	1851.5	28
29	1002.9	1065.6	1128.7	1192.2	1256.0	1320.3	1385.0	1450.2	1515.9	1582.0	1648.8	1716.1	1784.1	1852.7	29
30	1004.0	1066.7	1129.7	1193.2	1257.1	1321.4	1386.1	1451.3	1517.0	1583.2	1649.9	1717.3	1785.2	1853.8	30
31	1005.0	1067.7	1130.8	1194.3	1258.2	1322.4	1387.2	1452.4	1518.1	1584.3	1651.0	1718.4	1786.4	1855.0	31
32	1006.1	1068.8	1131.8	1195.4	1259.2	1323.5	1388.3	1453.5	1519.2	1585.4	1652.2	1719.5	1787.5	1856.1	32
33	1007.1	1069.8	1132.9	1196.4	1260.3	1324.6	1389.4	1454.6	1520.3	1586.5	1653.3	1720.7	1788.6	1857.2	33
34	1008.1	1070.9	1134.0	1197.5	1261.4	1325.7	1390.4	1455.7	1521.4	1587.6	1654.4	1721.8	1789.8	1858.4	34
35	1009.2	1072.0	1135.1	1198.5	1262.4	1326.7	1391.5	1456.8	1522.5	1588.7	1655.5	1722.9	1790.9	1859.6	35
36	1010.2	1073.0	1136.1	1199.6	1263.5	1327.8	1392.6	1457.9	1523.5	1589.8	1656.6	1724.0	1792.1	1860.7	36
37	1011.3	1074.1	1137.2	1200.7	1264.6	1328.9	1393.7	1458.9	1524 7	1590.9	1657.8	1725.2	1793.2	1861.9	37
38	1012.3	1075.1	1138.2	1201.7	1265.6	1330.0	1394.8	1460.0	1525.8	1592.0	1658.9	1726.3	1794.3	1863.0	38
39	1013.4	1076.2	1139.3	1202.8	1266.7	1331.0	1395.8	1461.1	1526.9	1593.2	1660.0	1727.4	1795.5	1864.2	39
40	1014.4	1077.2	1140.3	1203.9	1267.8	1332.1	1396.9	1462.2	1528.0	1594.3	1661.1	1728.6	1796.6	1865.3	40
41	1015.4	1078.3	1141.4	1204.9	1268.8	1333.2	1398.0	1463.3	1529.1	1595.4	1662.2	1729.7	1797.8	1866.5	41
42	1016.5	1079.3	1142.4	1206.0	1269.9	1334.2	1399.1	1464.4	1530.2	1596.5	1663.4	1730.8	1798.9	1867.6	42
43	1017.5	1080.4	1143.5	1207.1	1271.0	1335.2	1400.2	1465.5	1531.3	1597.6	1664.5	1731.9	1800.0	1868.8	43
44	1018.6	1081.4	1144.6	1208.1	1272.1	1336.4	1401.3	1466.6	1532.4	1598.7	1665.6	1733.1	1801.2	1869.9	44
45	1019.6	1082.5	1145.6	1209 2	1273.1	1337.5	1402.4	1467.7	1533.5	1599.8	1666.7	1734.2	1802.3	1871.1	45
46	1020.6	1083.5	1146.7	1210.2	1274,2	1338.6	1403.4	1468.8	1534.6	1600.9	1667.8	1735.3	1803.5	1872.2	46
47	1021.7	1084.6	1147.7	1211.3	1275.3	1339.7	1404.5	1469.8	1535.7	1602.0	1669.0	1736.6	1804.6	1873.4	47
48	1022.7	1085.6	1148.8	1212.4	1276.3	1340.7	1405.6	1470.9	1536.8	1603.1	1670.1	1737.6	1805.7	1874.5	48
49	1023.8	1086.7	1149.8	1213.4	1277.4	1341.8	1406.7	1472.0	1537.9	1604.3	1671.2	1738.7	1806.9	1875.7	49
50	1024.8	1087.7	1150.9	1214.5	1278.5	1342.9	1407.8	1473.1	1539.0	1605.4	1672.3	1739.9	1808.0	1876.8	50
51	1025.9	1088.8	1152.0	1215.5	1279.5	1344.0	1408.8	1474.2	1540.1	1606.5	1673.4	1741.0	1809.2	1878.0	51
52	1026.9	1089.8	1153.0	1216.6	1280.6	1345.0	1409.9	1475.3	1541.2	1607.6	1674.6	1742.1	1810.3	1879.2	52
53	1028.0	1090.9	1154.1	1217.7	1281.7	1346.1	1411.0	1476.4	1542.3	1608.7	1675.7	1743.2	1811.4	1880.3	53
54	1029.0	1091.9	1155.1	1218.7	1282.7	1347.2	1412.1	1477.5	1543.4	1609.8	1676.9	1744.4	1812.6	1881.5	54
55	1030 1	1093.0	1156.2	1119.8	1283.8	1348.3	1413.2	1478.6	1544.5	1610.9	1678.0	1745.5	1813.7	1882.6	55
56	1031.1	1094.0	1157.2	1220.9	1284.9	1349.4	1414.3	1479.7	1545.6	1612.0	1679.1	1746.6	1814.9	1883.8	56
57	1032.2	1095.1	1158.3	1221.9	1286.0	1350.4	1415.4	1480.8	1546.7	1613.1	1680.2	1747.8	1816.0	1884.9	57
58	1033 2	1096.1	1159.4	1223.0	1287.0	1351.5	1416.5	1481.9	1547.8	1614.3	1681.3	1748.9	1817.2	1886.1	58
59	1034.3	1007.2	1160.4	1224.1	1288.1	1351.6	1417.7	1483.0	1548.9	1615.4	1682.4	1750.0	1818.3	1887.2	59
M	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	M
L.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	L.

L. / M	30 Min.	31 Min.	32 Min.	33 Min.	34 Min.	35 Min.	36 Min.	37 Min.	38 Min.	39 Min.	40 Min.	41 Min.	42 Min.	L. / M
0	1888.4	1958.1	2028.4	2099.6	2171.5	2244.3	2318.0	2392.7	2468.3	2545.0	2622.7	2701.6	2781.7	0
1	1889.5	1959.2	2029.6	2100.7	2172.7	2245.5	2319.3	2393.9	2469.6	2546.2	2624.0	2702.9	2783.1	1
2	1890.7	1960.4	2030.7	2101.9	2173.9	2246.8	2320.5	2395.2	2470.8	2547.5	2625.3	2704.3	2784.4	2
3	1891.9	1961.5	2031.9	2103.1	2175.1	2248.0	2321.7	2396.4	2472.1	2548.8	2626.6	2705.6	2785.8	3
4	1893.0	1962.7	2033.1	2104.3	2176.3	2249.2	2323.0	2397.7	2473.4	2550.1	2627.9	2706.9	2787.1	4
5	1894.2	1963.9	2034.3	2105.5	2177.5	2250.4	2324.2	2398.9	2474.6	2551.4	2629.2	2708.3	2788.5	5
6	1895.3	1965.0	2035.5	2106.7	2178.7	2251.6	2325.4	2400.2	2475.9	2552.7	2630.5	2709.6	2789.8	6
7	1896.5	1966.2	2036.7	2107.9	2180.0	2252.9	2326.7	2401.4	2477.1	2554.0	2631.9	2710.9	2791.2	7
8	1897.6	1967.4	2037.8	2109.1	2181.2	2254.1	2327.9	2402.7	2478.5	2555.3	2633.2	2712.2	2792.5	8
9	1898.8	1968.5	2039.0	2110.3	2182.4	2255.3	2329.2	2403.9	2479.7	2556.6	2634.5	2713.6	2793.8	9
10	1899.9	1969.7	2040.2	2111.5	2183.6	2256.5	2330.4	2405.2	2481.0	2557.8	2635.8	2714.9	2795.1	10
11	1901.1	1970.9	2041.4	2112.7	2184.8	2257.8	2331.6	2406.4	2482.3	2559.1	2637.1	2716.2	2796.5	11
12	1902.3	1972.0	2042.6	2113.9	2186.0	2259.0	2332.9	2407.7	2483.5	2560.4	2638.4	2717.5	2797.9	12
13	1903.4	1973.2	2043.8	2115.1	2187.2	2260.2	2334.1	2409.0	2484.8	2561.7	2639.7	2718.9	2799.3	13
14	1904.6	1974.4	2044.9	2116.3	2188.4	2261.4	2335.3	2410.2	2486.1	2563.0	2641.0	2720.2	2800.6	14
15	1905.7	1975.6	2046.1	2117.5	2189.6	2262.7	2336.6	2411.5	2487.4	2564.3	2642.3	2721.5	2802.0	15
16	1906.9	1976.8	2047.3	2118.7	2150.8	2263.9	2337.8	2412.7	2488.6	2565.6	2643.6	2722.9	2803.3	16
17	1908.1	1977.9	2048.5	2119.8	2192.0	2265.1	2339.0	2414.0	2489.9	2566.9	2644.9	2724.2	2804.7	17
18	1909.2	1979.1	2049.7	2121.0	2193.3	2266.3	2340.3	2415.2	2491.2	2568.2	2646.3	2725.5	2806.0	18
19	1910.4	1980.3	2050.8	2122.2	2194.5	2267.6	2341.5	2416.5	2492.5	2569.5	2647.6	2726.9	2807.4	19
20	1911.5	1981.4	2052.0	2123.4	2195.7	2268.8	2342.8	2417.8	2493.7	2570.7	2648.9	2728.2	2808.7	20
21	1912.7	1982.6	2053.2	2124.6	2196.9	2270.0	2344.0	2419.0	2495.0	2572.0	2650.2	2729.5	2810.1	21
22	1913.8	1983.7	2054.4	2125.8	2198.1	2271.2	2345.3	2420.3	2496.3	2573.3	2651.5	2730.8	2811.4	22
23	1915.0	1984.9	2055.6	2127.0	2199.3	2272.5	2346.5	2421.5	2497.6	2574.6	2652.8	2732.2	2812.8	23
24	1916.2	1986.1	2056.8	2128.2	2200.5	2273.7	2347.8	2422.8	2498.8	2575.9	2654.1	2733.5	2814.1	24
25	1917.3	1987.3	2058.0	2129.4	2201.7	2274.9	2349.0	2424.0	2500.1	2577.2	2655.4	2734.8	2815.5	25
26	1918.5	1988.4	2059.1	2130.6	2203.0	2276.1	2350.2	2425.3	2501.4	2578.5	2656.8	2736.2	2816.8	26
27	1919.6	1989.6	2060.3	2131.8	2204.2	2277.4	2351.5	2426.5	2502.7	2579.8	2658.1	2737.5	2818.2	27
28	1920.8	1990.8	2061.5	2133.0	2205.4	2278.6	2352.7	2427.8	2503.4	2581.1	2659.4	2738.8	2819.5	28
29	1921.9	1992.0	2062.7	2134.2	2206.6	2279.8	2354.0	2429.1	2505.2	2582.4	2660.7	2740.2	2820.9	29
30	1923.1	1993.1	2063.9	2135.4	2207.8	2281.0	2355.2	2430.3	2506.5	2583.7	2662.0	2741.5	2822.3	30
31	1924.3	1994.3	2065.1	2136.6	2209.0	2282.3	2356.5	2431.6	2507.8	2585.0	2663.3	2742.9	2823.6	31
32	1925.4	1995.5	2066.2	2137.8	2210.2	2283.5	2357.7	2432.9	2509.0	2586.3	2664.6	2744.2	2825.0	32
33	1926.6	1996.6	2067.4	2139.0	2211.4	2284.7	2358.9	2434.1	2510.3	2587.6	2666.0	2745.5	2826.3	33
34	1927.8	1997.8	2068.6	2140.2	2212.7	2286.0	2360.2	2435.4	2511.6	2588.9	2667.3	2746.9	2827.7	34
35	1928.9	1999.0	2069.8	2141.4	2213.9	2287.2	2361.4	2436.7	2512.9	2590.2	2668.6	2748.2	2829.0	35
36	1930.1	2000.2	2071.0	2142.6	2215.1	2288.4	2362.7	2437.9	2514.2	2591.5	2669.9	2749.5	2830.4	36
37	1931.3	2001.3	2072.2	2143.8	2216.3	2289.7	2363.9	2439.2	2515.4	2592.8	2671.2	2750.9	2831.8	37
38	1932.4	2002.5	2073.4	2145.0	2217.5	2290.9	2365.2	2440.4	2516.7	2594.1	2672.5	2752.2	2833.1	38
39	1933.6	2003.7	2074.6	2146.2	2218.7	2292.1	2366.4	2441.7	2518.0	2595.4	2673.9	2753.5	2834.5	39
40	1934.7	2004.9	2075.7	2147.4	2219.9	2293.3	2367.7	2443.0	2519.3	2596.7	2675.1	2754.4	2835.8	40
41	1935.9	2006.0	2076.9	2148.6	2221.2	2294.6	2368.9	2444.2	2520.6	2598.0	2676.5	2756.2	2837.2	41
42	1937.1	2007.2	2078.1	2149.8	2222.4	2295.8	2370.2	2445.5	2521.8	2599.3	2677.8	2757.6	2838.6	42
43	1938.2	2008.4	2079.3	2151.0	2223.6	2297.0	2371.4	2446.8	2523.1	2600.6	2679.1	2758.9	2839.9	43
44	1939.4	2009.6	2080.5	2152.2	2224.8	2298.3	2372.7	2448.0	2524.4	2601.9	2680.5	2760.2	2841.3	44
45	1940.5	2010.7	2081.7	2153.4	2226.0	2299.5	2373.9	2449.3	2525.7	2603.2	2681.8	2761.5	2841.6	45
46	1941.7	2011.9	2082.9	2154.6	2227.2	2300.7	2375.2	2450.6	2527.0	2604.5	2683.1	2762.9	2844.0	46
47	1942.9	2013.1	2084.1	2155.8	2228.5	2302.0	2376.4	2451.8	2528.3	2605.8	2684.4	2764.3	2845.4	47
48	1944.0	2014.3	2085.3	2157.0	2229.7	2303.2	2377.7	2453.1	2529.5	2607.1	2685.7	2765.6	2846.7	48
49	1945.2	2015.4	2086.5	2158.2	2230.9	2304.4	2378.9	2454.3	2530.8	2608.4	2687.1	2766.9	2848.1	49
50	1946.4	2016.6	2087.7	2159.4	2232.1	2305.7	2380.1	2455.6	2532.1	2609.7	2688.4	2768.3	2849.5	50
51	1947.5	2017.8	2088.9	2160.7	2233.3	2306.9	2381.4	2456.9	2533.4	2611.0	2689.7	2769.6	2850.8	51
52	1948.7	2019.0	2090.1	2161.9	2234.6	2308.1	2382.6	2458.1	2534.7	2612.3	2691.0	2771.0	2852.2	52
53	1949.9	2020.2	2091.3	2163.1	2235.8	2309.4	2383.9	2459.4	2536.0	2613.6	2692.3	2772.3	2853.6	53
54	1951.0	2021.3	2092.5	2164.3	2237.0	2310.6	2385.1	2460.7	2537.2	2614.9	2693.7	2773.7	2854.9	54
55	1952.2	2022.5	2093.7	2165.5	2238.2	2311.8	2386.4	2461.9	2538.5	2616.2	2695.0	2775.0	2856.3	55
56	1953.4	2023.7	2094.9	2166.7	2239.4	2313.1	2387.6	2463.2	2539.8	2617.5	2696.3	2776.4	2857.7	56
57	1954.5	2024.9	2096.1	2167.9	2240.7	2314.3	2388.9	2464.5	2541.1	2618.8	2697.6	2777.7	2859.1	57
58	1955.7	2026.0	2097.3	2169.1	2241.9	2315.5	2390.2	2465.8	2542.4	2620.1	2699.0	2779.0	2860.5	58
59	1956.9	2027.2	2098.5	2170.3	2243.1	2316.7	2391.4	2467.0	2543.7	2621.4	2700.3	2780.4	2861.8	59
M / L.	Min. 30	Min. 31	Min. 32	Min. 33	Min. 34	Min. 35	Min. 36	Min. 37	Min. 38	Min. 39	Min. 40	Min. 41	Min. 42	M / L.

A Table of Meridional Parts.

L.	43	44	45	46	47	48	49	50	51	52	53	54	55	L.
M	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	M
0	2863.1	2945.7	3030.0	3115.6	3202.8	3291.6	3382.1	3474.5	3568.8	3665.2	3763.8	3864.7	3968.0	0
1	2864.5	2947.2	3031.4	3117.0	3204.2	3293.1	3383.6	3476.1	3570.4	3666.9	3765.5	3866.4	3969.7	1
2	2865.8	2948.6	3032.8	3118.5	3205.7	3294.6	3385.2	3477.6	3572.0	3668.5	3767.1	3868.1	3971.5	2
3	2867.2	2950.0	3034.2	3119.9	3207.2	3296.1	3386.7	3479.2	3573.6	3670.1	3768.8	3869.8	3973.2	3
4	2868.5	2951.4	3035.6	3121.4	3208.6	3297.5	3388.2	3480.7	3575.2	3671.7	3770.4	3871.5	3975.0	4
5	2870.0	2952.8	3037.0	3122.8	3210.1	3299.0	3389.7	3482.3	3576.8	3673.4	3772.1	3873.2	3976.7	5
6	2871.3	2954.2	3038.4	3124.2	3211.6	3300.5	3391.3	3483.9	3578.4	3675.0	3773.8	3874.9	3978.5	6
7	2872.7	2955.6	3039.8	3125.7	3213.0	3302.0	3392.8	3485.4	3580.0	3676.6	3775.4	3876.6	3980.2	7
8	2874.1	2957.0	3041.3	3127.1	3214.5	3303.5	3394.3	3487.0	3581.6	3678.2	3777.1	3878.3	3982.0	8
9	2875.4	2958.4	3042.7	3128.6	3216.0	3305.0	3395.9	3488.5	3583.2	3679.9	3778.8	3880.0	3983.7	9
10	2876.8	2959.8	3044.1	3130.0	3217.4	3306.5	3397.4	3490.1	3584.8	3681.5	3780.4	3881.7	3985.5	10
11	2878.2	2961.1	3045.5	3131.5	3218.9	3308.0	3398.9	3491.7	3586.4	3683.1	3782.1	3883.4	3987.2	11
12	2879.5	2962.5	3047.0	3132.9	3220.4	3309.5	3400.4	3493.2	3588.0	3684.8	3783.8	3885.1	3989.0	12
13	2880.9	2963.9	3048.4	3134.3	3221.9	3311.0	3402.0	3494.8	3589.5	3686.4	3785.5	3886.8	3990.7	13
14	2882.3	2965.3	3049.8	3135.8	3223.3	3312.5	3403.5	3496.3	3591.1	3688.0	3787.1	3888.6	3992.5	14
15	2883.7	2966.7	3051.2	3137.2	3224.8	3314.0	3405.0	3497.9	3592.7	3689.7	3788.8	3890.3	3994.2	15
16	2885.0	2968.1	3052.6	3138.7	3226.3	3315.5	3406.6	3499.5	3594.3	3691.3	3790.5	3892.0	3996.0	16
17	2886.4	2969.5	3054.1	3140.1	3227.7	3317.0	3408.1	3501.0	3595.9	3692.9	3792.1	3893.7	3997.7	17
18	2887.8	2970.9	3055.5	3141.6	3229.2	3318.5	3409.6	3502.6	3597.5	3694.6	3793.8	3895.8	3999.5	18
19	2889.2	2972.3	3056.9	3143.0	3230.7	3320.0	3411.2	3504.2	3599.1	3696.2	3795.5	3897.1	4001.3	19
20	2890.5	2973.7	3058.3	3144.5	3232.2	3321.5	3412.7	3505.7	3600.7	3697.8	3797.2	3898.8	4003.0	20
21	2891.9	2975.1	3059.7	3145.9	3233.6	3323.1	3414.2	3507.3	3602.3	3699.5	3798.8	3900.5	4004.8	21
22	2893.3	2976.5	3061.2	3147.4	3235.1	3324.6	3415.8	3508.9	3603.9	3701.1	3800.5	3902.3	4006.5	22
23	2894.7	2977.9	3062.6	3148.8	3236.6	3326.1	3417.3	3510.5	3605.5	3702.7	3802.2	3904.0	4008.3	23
24	2896.0	2979.3	3064.0	3150.3	3238.1	3327.6	3418.8	3512.0	3607.1	3704.4	3803.9	3905.7	4010.0	24
25	2897.4	2980.7	3065.4	3151.7	3239.5	3329.1	3420.4	3513.6	3608.7	3706.0	3805.5	3907.4	4011.8	25
26	2898.8	2982.1	3066.9	3153.2	3241.0	3330.6	3421.9	3515.1	3610.3	3707.7	3807.2	3909.1	4013.6	26
27	2900.2	2983.5	3068.3	3154.6	3242.5	3332.1	3423.5	3516.7	3611.9	3709.3	3808.9	3910.9	4015.3	27
28	2901.5	2984.9	3069.7	3156.1	3244.0	3333.6	3425.0	3518.3	3613.6	3710.9	3810.6	3912.6	4017.1	28
29	2902.9	2986.3	3071.1	3157.5	3245.5	3335.1	3426.5	3519.8	3615.2	3712.6	3812.3	3914.3	4018.9	29
30	2904.3	2987.7	3072.6	3159.0	3246.9	3336.6	3428.1	3521.4	3616.8	3714.2	3813.9	3916.0	4020.6	30
31	2905.7	2989.1	3074.0	3160.4	3248.4	3338.1	3429.6	3523.0	3618.4	3715.9	3815.6	3917.7	4022.4	31
32	2907.1	2990.5	3075.4	3161.9	3249.9	3339.6	3431.2	3524.6	3620.0	3717.5	3817.3	3919.5	4024.2	32
33	2908.4	2991.9	3076.9	3163.3	3251.4	3341.1	3432.7	3526.1	3621.6	3719.2	3819.0	3921.2	4025.9	33
34	2909.7	2993.3	3078.3	3164.8	3252.9	3342.7	3434.2	3527.7	3623.2	3720.8	3820.7	3922.5	4027.7	34
35	2911.2	2994.7	3079.7	3166.2	3254.4	3344.2	3435.8	3529.3	3624.8	3722.4	3822.3	3924.6	4029.5	35
36	2912.6	2996.1	3081.1	3167.7	3255.8	3345.7	3437.3	3530.9	3626.4	3724.1	3824.0	3926.4	4031.2	36
37	2914.0	2997.5	3082.6	3169.1	3257.3	3347.2	3438.9	3532.4	3628.0	3725.7	3825.7	3928.1	4033.0	37
38	2915.3	2998.9	3084.0	3170.6	3258.8	3348.7	3440.4	3534.0	3629.6	3727.4	3827.4	3929.8	4034.8	38
39	2916.7	3000.3	3085.4	3172.1	3260.3	3350.1	3442.0	3535.6	3631.3	3729.0	3829.1	3931.5	4036.6	39
40	2918.1	3001.8	3086.9	3173.5	3261.8	3351.7	3443.5	3537.2	3632.9	3730.7	3830.8	3933.3	4038.3	40
41	2919.5	3003.2	3088.3	3175.0	3263.3	3353.2	3445.0	3538.8	3634.5	3732.3	3832.5	3935.0	4040.1	41
42	2920.9	3004.6	3089.7	3176.4	3264.7	3354.8	3446.6	3540.3	3636.1	3734.0	3834.2	3936.7	4041.9	42
43	2922.3	3006.0	3091.2	3177.9	3266.2	3356.3	3448.1	3541.9	3637.7	3735.6	3835.8	3938.5	4043.6	43
44	2923.6	3007.4	3092.6	3179.3	3267.7	3357.8	3449.7	3542.5	3639.3	3737.3	3837.5	3940.2	4045.4	44
45	2925.0	3008.8	3094.0	3180.8	3269.2	3359.3	3451.2	3545.1	3640.9	3738.9	3839.2	3941.9	4047.2	45
46	2926.4	3010.2	3095.5	3182.3	3270.7	3360.8	3452.8	3546.7	3642.5	3740.6	3840.9	3943.7	4049.0	46
47	2927.8	3011.6	3096.9	3183.7	3272.2	3362.3	3454.3	3548.2	3644.2	3742.2	3842.6	3945.4	4050.8	47
48	2929.2	3013.0	3098.3	3185.2	3273.7	3363.9	3455.9	3549.8	3645.8	3743.9	3844.3	3947.1	4052.5	48
49	2930.6	3014.4	3099.8	3186.6	3275.2	3365.4	3457.4	3551.4	3647.4	3745.6	3846.0	3948.9	4054.3	49
50	2932.0	3015.8	3101.2	3188.1	3276.6	3366.9	3459.0	3553.0	3649.0	3747.2	3847.7	3950.6	4056.1	50
51	2933.3	3017.2	3102.6	3189.6	3278.1	3368.4	3460.5	3554.6	3650.6	3748.9	3849.4	3952.3	4057.9	51
52	2934.7	3018.7	3104.1	3191.0	3279.6	3369.9	3462.1	3556.1	3652.3	3750.5	3851.1	3954.1	4059.7	52
53	2936.1	3020.1	3105.6	3192.5	3281.1	3371.5	3463.6	3557.7	3653.9	3752.2	3852.8	3955.8	4061.4	53
54	2937.5	3021.5	3107.0	3194.0	3282.6	3373.0	3465.2	3559.3	3655.5	3753.8	3854.5	3957.6	4063.2	54
55	2038.9	3022.9	3108.4	3195.4	3284.1	3374.5	3466.7	3560.9	3657.1	3755.5	3856.2	3959.3	4065.0	55
56	2940.3	3024.3	3109.8	3196.9	3285.6	3376.0	3468.3	3562.5	3658.7	3757.2	3857.9	3961.0	4066.8	56
57	2941.7	3025.7	3111.2	3198.4	3287.1	3377.6	3469.8	3564.1	3660.4	3758.8	3859.6	3962.8	4068.6	57
58	2943.1	3027.1	3112.7	3199.8	3288.6	3379.1	3471.4	3565.7	3662.0	3760.5	3861.3	3964.5	4070.4	58
59	2944.4	3028.5	3114.1	3201.3	3290.1	3380.6	3473.0	3567.3	3663.6	3762.2	3863.0	3966.3	4072.2	59
M	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	M
L.	43	44	45	46	47	48	49	50	51	52	53	54	55	L.

L. M	56 Min.	57 Min.	58 Min.	59 Min.	60 Min.	61 Min.	62 Min.	63 Min.	64 Min.	65 Min.	66 Min.	67 Min.	68 Min.	L. M
0	4073.9	4182.7	4294.3	4409.2	4527.4	4649.3	4775.0	4905.0	5039.5	5178.8	5323.6	5474.0	5630.9	0
1	4075.7	4184.5	4296.2	4411.1	4529.4	4651.3	4777.1	4907.2	5041.7	5181.2	5326.0	5476.6	5633.5	1
2	4077.5	4186.3	4298.1	4413.1	4531.4	4653.4	4779.3	4909.4	5044.0	5183.6	5328.5	5479.2	5636.2	2
3	4079.3	4188.2	4300.0	4415.0	4533.4	4655.5	4781.4	4911.6	5046.3	5186.0	5330.9	5481.7	5638.9	3
4	4081.1	5190.0	4301.9	4417.0	4535.4	4657.5	4783.5	4913.8	5048.6	5188.3	5333.4	5484.3	5641.5	4
5	4082.9	4191.8	4303.8	4418.9	4537.4	4659.6	4785.7	4916.0	5050.9	5190.7	5335.9	5486.9	5644.2	5
6	4084.7	4193.7	4305.7	4420.8	4539.4	4661.7	4787.8	4918.2	5053.2	5193.1	5338.3	5489.4	5646.9	6
7	4086.5	4195.5	4307.6	4422.8	4541.4	4663.7	4790.0	4920.4	5055.5	5195.4	5340.8	5492.0	5649.0	7
8	4088.3	4197.4	4309.5	4424.7	4543.4	4665.8	4792.1	4922.6	5057.7	5197.8	5343.3	5494.6	5652.3	8
9	4090.1	4199.2	4311.4	4426.7	4545.4	4667.9	4794.2	4924.8	5060.0	5200.2	5345.7	5497.1	5655.0	9
10	4091.9	4201.1	4313.2	4428.6	4547.5	4669.9	4796.4	4927.1	5062.3	5202.6	5348.2	5499.7	5657.6	10
11	4093.7	4202.9	4315.1	4430.6	4549.5	4672.0	4798.5	4929.3	5064.6	5205.0	5350.7	5502.3	5660.3	11
12	4095.5	4204.7	4317.0	4432.5	4551.5	4674.1	4800.7	4931.5	5066.9	5207.3	5353.2	5504.9	5663.0	12
13	4097.3	4206.6	4318.9	4434.5	4553.5	4676.2	4802.8	4933.7	5069.2	5209.7	5355.6	5507.5	5665.7	13
14	4099.1	4208.4	4320.8	4436.4	4555.5	4678.2	4804.9	4935.9	5071.5	5212.1	5358.1	5510.0	5668.4	14
15	4100.9	4210.3	4322.7	4438.4	4557.5	4680.3	4807.1	4938.1	5073.8	5214.5	5360.6	5512.6	5671.1	15
16	4102.7	4212.1	4324.6	4440.4	4559.5	4682.4	4809.2	4940.4	5076.1	5216.9	5363.1	5515.2	5673.8	16
17	4104.5	4214.0	4326.5	4442.3	4561.5	4684.5	4811.4	4942.6	5078.4	5219.3	5365.6	5517.8	5676.5	17
18	4106.3	4215.8	4328.4	4444.3	4563.6	4686.6	4813.5	4944.8	5080.7	5221.7	5368.1	5520.4	5679.2	18
19	4108.1	4217.7	4330.3	4446.2	4565.6	4688.6	4815.7	4947.0	5083.0	5224.1	5370.5	5523.0	5681.9	19
20	4109.9	4219.5	4332.2	4448.2	4567.6	4690.7	4817.8	4949.3	5083.3	5226.5	5373.0	5525.6	5684.6	20
21	4111.7	4221.4	4334.2	4450.2	4569.6	4692.8	4820.0	4951.5	5087.7	5228.9	5375.5	5528.2	5687.3	21
22	4113.5	4223.2	4336.1	4452.1	4571.6	4694.9	4822.2	4953.7	5090.0	5231.3	5378.0	5530.8	5690.0	22
23	4115.3	4225.1	4338.0	4454.1	4573.7	4697.0	4824.3	4956.0	5092.3	5233.7	5380.5	5533.4	5692.8	23
24	4117.1	4227.0	4339.9	4456.0	4575.7	4699.1	4826.5	4958.2	5094.6	5236.1	5383.0	5536.0	5695.5	24
25	4118.9	4228.8	4341.8	4458.0	4577.7	4701.2	4828.6	4960.4	5099.9	5238.5	5385.5	5538.6	5698.2	25
26	4120.7	4230.7	4343.7	4460.0	4579.7	4703.2	4830.8	4962.7	5096.2	5240.9	5388.0	5541.2	5700.9	26
27	4122.5	4232.5	4345.6	4461.9	4581.8	4705.3	4832.9	4964.9	5101.5	5243.3	5399.5	5543.8	5703.6	27
28	4124.3	4234.4	4347.5	4463.9	4583.8	4707.4	4835.1	4967.1	5103.9	5245.7	5393.0	5546.4	5706.3	28
29	4126.1	4236.2	4349.4	4466.0	4585.8	4709.5	4837.3	4969.4	5106.2	5248.1	5395.5	5549.0	5709.1	29
30	4127.9	4238.1	4351.3	4467.8	4587.8	4711.6	4839.4	4971.6	5108.5	5250.5	5398.0	5551.6	5711.8	30
31	4129.7	4240.0	4353.3	4469.8	4589.9	4713.7	4841.6	4973.9	5110.8	5252.9	5400.5	5554.2	5714.5	31
32	4131.6	4241.8	4355.2	4471.8	4591.9	4715.8	4843.8	4976.1	5113.1	5255.3	5403.0	5556.8	5717.3	32
33	4133.4	4243.7	4357.1	4473.8	4593.9	4717.9	4845.9	4978.3	5115.5	5257.7	5405.6	5559.5	5720.0	33
34	4135.2	4245.6	4359.0	4475.7	4596.0	4720.0	4848.1	4980.6	5117.8	5260.1	5408.1	5562.1	5722.7	34
35	4137.0	4247.4	4360.9	4477.7	4598.0	4722.6	4850.3	4982.8	5120.1	5262.6	5410.6	5564.7	5725.5	35
36	4138.8	4249.3	4362.8	4479.7	4600.1	4724.2	4852.5	4985.1	5122.5	5265.0	5413.1	5567.3	5728.2	36
37	4140.6	4251.2	4364.8	4481.7	4602.1	4726.3	4854.6	4987.3	5124.8	5267.4	5415.6	5569.9	5731.0	37
38	4142.5	4253.0	4366.7	4483.6	4604.1	4728.4	4856.8	4989.6	5127.1	5269.8	5418.1	5572.6	5733.7	38
39	4144.3	4254.9	4368.6	4485.6	4606.2	4730.5	4859.0	4991.8	5129.5	5272.3	5420.7	5575.2	5736.4	39
40	4146.1	4256.8	4370.5	4487.6	4608.2	4732.6	4861.2	4994.1	5131.8	5274.7	5423.2	5577.8	5739.2	40
41	4147.9	4258.6	4372.5	4489.6	4610.3	4734.7	4863.3	4996.3	5134.1	5277.1	5425.7	5580.5	5741.9	41
42	4149.7	4260.5	4374.4	4491.6	4612.3	4736.9	4865.5	4998.6	5136.5	5279.5	5428.2	5583.1	5744.7	42
43	4151.6	4262.4	4376.3	4493.5	4614.3	4739.0	4867.7	5000.9	5138.8	5282.0	5430.8	5585.7	5747.5	43
44	4153.4	4264.3	4378.2	4495.5	4616.4	4741.1	4869.9	5003.1	5141.2	5284.4	5433.3	5588.4	5750.2	44
45	4155.2	4266.1	4380.2	4497.5	4618.4	4743.2	4872.1	5005.4	5143.5	5286.8	5435.8	5591.0	5753.0	45
46	4157.0	4268.0	4382.1	4499.5	4620.5	4745.3	4874.3	5007.6	5145.9	5289.3	5438.4	5593.7	5755.7	46
47	4158.8	4269.9	4384.0	4501.5	4622.5	4747.4	4876.4	5009.9	5148.2	5291.7	5440.9	5596.3	5758.5	47
48	4160.7	4271.8	4385.9	4503.5	4624.6	4749.5	4878.6	5012.2	5150.6	5294.2	5443.5	5599.0	5761.3	48
49	4162.5	4273.6	4387.9	4505.5	4626.6	4751.7	4880.8	5014.4	5152.9	5296.6	5446.0	5601.6	5764.0	49
50	4164.3	4275.5	4389.8	4507.5	4628.7	4753.8	4882.0	5016.7	5155.3	5299.0	5448.5	5604.3	5766.8	50
51	4166.2	4277.4	4391.7	4509.4	4630.7	4755.9	4885.2	5019.0	5157.6	5301.5	5451.1	5606.9	5769.6	51
52	4168.0	4279.3	4393.7	4511.4	4632.8	4758.0	4887.4	5021.2	5160.0	5303.9	5453.6	5609.6	5772.3	52
53	4169.8	4281.1	4395.6	4513.4	4634.8	4760.1	4889.6	5023.5	5162.3	5306.4	5456.2	5612.2	5775.1	53
54	4171.7	4283.0	4397.5	4515.4	4636.9	4762.3	4891.8	5025.8	5164.7	5308.8	5458.7	5614.9	5777.9	54
55	4173.5	4284.9	4399.5	4517.4	4639.0	4764.4	4894.0	5028.1	5167.0	5311.3	5461.3	5617.5	5780.7	55
56	4175.3	4286.8	4401.4	4519.4	4641.0	4766.5	4896.2	5030.3	5169.4	5313.7	5463.8	5620.2	5783.5	56
57	4177.2	4288.7	4403.4	4521.4	4643.1	4768.6	4998.4	5032.6	5171.8	5316.2	5466.4	5622.9	5786.2	57
58	4179.0	4290.6	4405.3	4523.4	4645.1	4770.8	4900.6	5034.9	5174.1	5318.6	5468.9	5625.5	5789.0	58
59	4180.8	4292.5	4407.2	4525.4	4647.2	4772.9	4902.8	5037.2	5176.5	5321.1	5471.5	5628.2	5791.8	59
M L.	Min. 56	Min. 57	Min. 58	Min. 59	Min. 60	Min. 61	Min. 62	Min. 63	Min. 64	Min. 65	Min. 66	Min. 67	Min. 68	M L.

L.	69	70	71	72	73	74	75	76	77	78	79	80	81	L.
M	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	M
0	5794.6	5966.0	6145.7	6334.9	6534.5	6745.7	6970.3	7210.1	7467.2	7744.6	8045.7	8375.3	8739.1	0
1	5797.4	5968.9	6148.8	6338.1	6537.9	6749.4	6974.2	7214.2	7471.7	7749.4	8051.0	8381.0	8745.5	1
2	5800.2	5971.8	6151.9	6341.4	6541.3	6753.0	6978.1	7218.3	7476.1	7754.2	8056.2	8386.8	8751.9	2
3	5803.0	5974.7	6155.0	6344.6	6544.7	6756.6	6980.9	7222.5	7480.6	7759.0	8061.5	8392.6	8758.3	3
4	5805.8	5977.7	6158.0	6347.8	6548.2	6760.3	6985.8	7226.6	7485.0	7763.9	8066.8	8398.3	8764.8	4
5	5808.6	5980.6	6161.1	6351.1	6551.6	6763.9	6989.7	7230.8	7489.5	7768.7	8072.0	8404.1	8771.2	5
6	5811.4	5983.5	6164.2	6354.3	6555.0	6767.6	6993.6	7234.9	7494.0	7773.5	8077.3	8409.9	8777.7	6
7	5814.2	5986.5	6167.3	6357.6	6558.5	6771.2	6997.5	7239.1	7498.5	7778.4	8082.6	8415.8	8784.1	7
8	5817.0	5989.4	6170.4	6360.9	6561.9	6774.9	7001.4	7243.3	7502.9	7783.2	8087.9	8421.6	8790.6	8
9	5819.8	5992.4	6173.5	6364.1	6565.4	6778.5	7005.3	7247.5	7507.4	7788.1	8093.2	8427.4	8797.1	9
10	5822.6	5995.3	6176.6	6367.4	6568.8	6782.2	7009.2	7251.6	7511.9	7793.0	8098.5	8433.3	8803.6	10
11	5825.4	5998.3	6179.7	6370.6	6572.3	6785.8	7013.1	7255.8	7516.4	7797.8	8103.8	8439.1	8810.1	11
12	5828.2	6001.2	6182.8	6373.9	6575.7	6789.5	7017.0	7260.0	7520.9	7802.7	8109.2	8445.0	8816.6	12
13	5831.0	6004.2	6185.9	6377.2	6579.2	6793.2	7020.9	7264.2	7525.4	7807.6	8114.5	8450.9	8823.2	13
14	5833.9	6007.1	6189.0	6380.5	6582.6	6796.9	7024.8	7268.4	7530.0	7812.5	8119.8	8456.8	8829.7	14
15	5836.7	6010.1	6192.1	6383.7	6586.1	6800.5	7028.7	7272.6	7534.5	7817.4	8125.2	8462.6	8836.3	15
16	5839.5	6013.0	6195.2	6387.0	6589.5	6804.2	7032.7	7276.8	7539.0	7822.3	8130.6	8468.6	8842.8	16
17	5842.3	6016.0	6198.3	6390.3	6593.0	6807.9	7036.6	7281.0	7543.6	7827.2	8135.9	8474.5	8849.4	17
18	5845.2	6019.0	6201.4	6393.6	6596.5	6811.6	7040.5	7285.2	7548.1	7832.2	8141.3	8480.4	8856.0	18
19	5848.0	6021.9	6204.6	6396.9	6600.0	6815.8	7044.5	7289.4	7552.7	7837.1	8146.7	8486.3	8862.6	19
20	5850.8	6024.9	6207.7	6400.2	6603.4	6819.0	7048.4	7293.7	7557.2	7842.0	8152.1	8492.3	8869.3	20
21	5853.7	6027.9	6210.8	6403.5	6606.9	6822.7	7052.4	7297.9	7561.8	7847.0	8157.5	8498.2	8875.9	21
22	5856.5	6030.8	6213.9	6406.8	6610.4	6826.4	7056.3	7302.1	7566.3	7851.9	8162.9	8504.2	8882.6	22
23	5859.3	6033.8	6217.1	6410.1	6613.9	6830.1	7060.3	7306.4	7570.9	7856.9	8168.3	8510.2	8889.2	23
24	5862.2	6036.8	6220.2	6413.4	6617.4	6833.8	7064.2	7310.6	7575.5	7861.9	8173.7	8516.2	8895.9	24
25	5865.0	6039.8	6223.3	6416.7	6620.9	6837.6	7068.2	7314.9	7580.1	7866.8	8179.2	8522.2	8902.6	25
26	5867.9	6042.7	6226.7	6420.0	6624.4	6841.3	7072.2	7319.1	7584.7	7871.8	8184.6	8528.2	8909.3	26
27	5870.7	6045.7	6229.6	6423.3	6627.9	6845.0	7076.2	7323.4	7589.3	7876.8	8190.1	8534.2	8916.0	27
28	5873.5	6048.7	6232.7	6426.6	6631.4	6848.7	7080.1	7327.7	7593.9	7881.8	8195.5	8540.2	8922.7	28
29	5876.4	6051.7	6235.9	6429.9	6635.0	6852.5	7084.1	7332.0	7598.3	7886.8	8201.0	8546.2	8929.5	29
30	5879.3	6054.7	6239.0	6433.2	6638.5	6856.2	7088.1	7336.2	7603.1	7891.8	8206.5	8552.3	8936.2	30
31	5882.1	6057.7	6242.2	6436.6	6642.0	6860.0	7092.1	7340.5	7607.7	7896.8	8212.0	8558.4	8943.0	31
32	5885.0	6060.7	6245.3	6439.9	6645.5	6863.7	7096.1	7344.8	7612.3	7901.9	8217.5	8564.4	8949.8	32
33	5887.8	6063.7	6248.5	6443.2	6649.1	6867.5	7100.1	7349.1	7617.0	7906.9	8223.0	8570.5	8956.6	33
34	5890.7	6066.7	6251.7	6446.6	6652.6	6871.2	7104.1	7353.4	7621.6	7911.9	8228.5	8576.6	8963.4	34
35	5893.6	6069.7	6254.8	6449.9	6656.1	6875.0	7108.2	7357.7	7626.3	7917.0	8234.1	8582.7	8970.2	35
36	5896.4	6072.7	6258.0	6453.3	6659.7	6878.7	7112.2	7362.0	7630.9	7921.1	8239.6	8588.9	8977.1	36
37	5899.3	6075.7	6261.2	6456.6	6663.2	6882.5	7116.2	7366.4	7635.6	7927.1	8245.1	8595.0	8983.9	37
38	5902.2	6078.8	6264.4	6460.0	6666.8	6886.3	7120.2	7370.7	7640.2	7932.2	8250.7	8601.1	8990.8	38
39	5905.1	6081.8	6267.5	6463.3	6670.3	6890.1	7124.3	7375.0	7644.9	7937.3	8256.3	8607.3	8997.7	39
40	5907.9	6084.8	6270.7	6466.7	6673.9	6893.8	7128.3	7379.4	7649.6	7942.4	8261.8	9613.5	9004.6	40
41	5910.8	6087.8	6273.9	6470.0	6677.4	6897.6	7132.3	7383.7	7654.3	7947.5	8267.4	8619.6	9011.5	41
42	5913.7	6090.8	6277.1	6473.4	6681.0	6901.4	7136.4	7388.0	7659.0	7952.6	8273.0	8625.8	9018.4	42
43	5916.6	6093.9	6280.3	6476.8	6684.6	6905.3	7140.4	7392.4	7663.7	7957.7	8278.6	8632.0	9025.4	43
44	5919.5	6096.9	6283.5	6480.1	6688.1	6909.0	7144.5	7396.8	7668.4	7962.8	8284.2	8638.2	9032.3	44
45	5922.4	6099.9	6286.6	6483.5	6691.7	6912.8	7148.6	7401.1	7673.1	7968.0	8289.9	8644.5	9039.3	45
46	5925.2	6103.0	6289.8	6486.9	6695.3	6916.6	7152.6	7405.5	7677.8	7973.1	8295.5	8650.7	9046.3	46
47	5928.1	6106.0	6293.0	6490.2	6698.9	6920.4	7156.7	7409.9	7682.6	79 8.2	8301.1	8656.9	9053.3	47
48	5931.0	6109.1	6296.2	6493.6	6702.4	6924.2	7160.8	7414.2	7687.3	7983.4	8306.8	8663.2	9060.3	48
49	5933.9	6112.1	6299.4	6497.0	6706.0	6928.1	7164.9	7418.6	7692.0	7988.5	8312.4	8669.5	9067.3	49
50	5936.8	6115.1	6302.7	6500.4	6709.6	6931.9	7169.0	7423.0	7696.8	7993.7	8318.1	8675.7	9074.4	50
51	5939.7	6118.2	6305.9	6503.8	6713.2	6935.7	7173.0	7427.4	7701.5	7998.9	8323.8	8682.0	9081.4	51
52	5942.6	6121.2	6309.1	6507.2	6716.8	6939.5	7177.1	7431.8	7706.3	8004.0	8329.4	8688.3	9088.5	52
53	5945.5	6124.3	6312.3	6510.6	6720.4	6943.4	7181.2	7436.2	7711.0	8009.2	8335.1	8694.6	9095.6	53
54	5948.5	6127.4	6315.5	6514.0	6724.0	6947.2	7185.3	7440.6	7715.8	8014.4	8340.8	8701.0	9102.7	54
55	5951.4	6130.4	6318.7	6517.4	6727.6	6951.1	7189.5	7445.0	7720.6	8019.6	8346.6	8707.3	9109.8	55
56	5954.3	6133.5	6322.0	6520.8	6731.2	6954.9	7193.6	7449.5	7725.4	8024.8	8352.3	8713.6	9116.9	56
57	5957.2	6136.5	6325.2	6524.2	6734.9	6958.8	7197.7	7453.9	7730.2	8030.0	8358.0	8720.0	9124.0	57
58	5960.1	6139.6	6328.4	6527.6	6738.5	6962.6	7201.8	7458.3	7735.0	8035.3	8363.7	8726.4	9131.2	58
59	5963.0	6142.7	6331.7	6531.0	6742.1	6966.5	7205.9	7462.8	7739.8	8040.5	8369.5	8732.7	9138.4	59
M	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	M
L.	60	70	71	72	73	74	75	76	77	78	79	80	81	L.

L.	82	83	84	85	86	87	88	89	L.
M	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	M
0	9145.6	9605.9	10137.0	10764.7	11532.6	12521.3	13916.6	16299.8	0
1	9152.7	9614.1	10146.6	10776.2	11547.0	12541.4	13945.4	16357.5	1
2	9159.9	9622.4	10156.2	10787.7	11561.4	12560.7	13974.4	16416.3	2
3	9167.2	9630.6	10165.8	10799.3	11575.9	12580.0	14003.7	16476.1	3
4	9174.4	9638.9	10175.4	10810.0	11590.5	12599.5	14033.2	16537.0	4
5	9181.6	9647.2	10185.1	10822.5	11619.0	12619.1	14063.0	16594.9	5
6	9188.9	9655.5	10194.8	10834.2	11605.8	12638.9	14093.0	16562.0	6
7	9196.2	9663.8	10204.6	10845.9	11634.5	12658.6	14123.3	16726.2	7
8	9203.5	9672.2	10214.4	10857.7	11649.3	12678.6	14153.9	16791.7	8
9	9210.8	9680.6	10224.2	10869.6	11664.1	12698.6	14184.7	16858.5	9
10	9218.1	9689.0	10234.0	10881.4	11679.1	12718.8	14215.8	16926.5	10
11	9225.4	9697.4	10243.8	10893.3	11694.0	12739.1	14247.2	16990.6	11
12	9232.8	9705.8	10253.7	10905.2	11709.1	12759.5	14278.9	17066.9	12
13	9240.2	9714.2	10263.6	10917.2	11724.2	12780.0	14310.9	17130.3	13
14	9247.6	9722.7	10273.5	10929.1	11739.4	12800.7	14343.2	17213.2	14
15	9255.0	9731.2	10283.5	10941.2	11754.7	12821.5	14375.8	17288.7	15
16	9262.4	9739.7	10293.5	10953.3	11770.0	12842.5	14408.7	17366.0	16
17	9269.9	9748.3	10303.5	10965.5	11785.4	12863.5	14441.9	17445.0	17
18	9277.3	9756.8	10313.6	10977.7	11800.9	12884.7	14475.4	17525.9	18
19	9284.8	9765.4	10323.7	10989.9	11816.4	12906.0	14509.3	17608.7	19
20	9292.3	9774.0	10333.8	11002.2	11832.0	12927.4	14543.5	17693.6	20
21	9299.8	9782.7	10344.0	11014.5	11847.6	12948.9	14578.1	17780.7	21
22	9307.3	9791.3	10354.1	11026.9	11863.4	12970.6	14613.0	17869.9	22
23	9314.8	9800.0	10364.3	11039.3	11879.1	12992.5	14648.3	17961.6	23
24	9322.4	9808.6	10374.5	11051.7	11895.3	13014.4	14683.9	18055.8	24
25	9330.0	9817.3	10384.8	11064.2	11911.0	13036.6	14719.9	18152.6	25
26	9337.5	9826.1	10395.1	11076.8	11927.2	13058.8	14756.3	18252.3	26
27	9345.2	9834.8	10405.4	11089.3	11943.1	13081.2	14793.0	18354.9	27
28	9352.8	9843.6	10415.8	11102.0	11959.4	13103.8	14830.2	18460.7	28
29	9360.4	9852.4	10426.2	11114.6	11975.6	13126.5	14867.8	18569.8	29
30	9368.1	9861.3	10436.6	11127.4	11992.0	13149.3	14905.8	18682.5	30
31	9375.8	9870.1	10447.1	11140.1	12008.4	13172.3	14944.2	18799.1	31
32	9383.5	9879.0	10457.5	11152.9	12024.9	13195.6	14983.0	18919.7	32
33	9391.2	9887.8	10468.0	11165.8	12041.5	13218.8	15022.3	19044.7	33
34	9398.9	9896.7	10478.5	11178.7	12058.2	13242.1	15062.1	19174.4	34
35	9406.6	9905.7	10489.1	11191.7	12074.9	13265.6	15102.3	19309.2	35
36	9414.4	9914.6	10499.7	11204.7	12091.7	13289.7	15143.0	19449.5	36
37	9422.1	9923.6	10510.4	11217.7	12108.6	13313.7	15184.2	19595.8	37
38	9429.9	9932.7	10521.1	11230.9	12125.6	13337.8	15225.8	19748.6	38
39	9437.8	9941.7	10531.8	11244.0	12142.7	13362.1	15268.0	19908.5	39
40	9445.6	9950.8	10542.6	11257.2	12159.9	13386.6	15310.7	20075.2	40
41	9453.4	9959.8	10553.3	11270.5	12177.1	13411.2	15354.0	20252.5	41
42	9461.3	9968.9	10564.1	11283.8	12194.4	13436.1	15397.8	20438.3	42
43	9469.1	9978.0	10574.9	11297.1	12211.8	13461.1	15442.1	20634.8	43
44	9477.0	9987.2	10585.8	11310.6	12229.3	13486.3	15487.0	20843.1	44
45	9484.9	9996.3	10596.7	11324.0	12246.9	13511.6	15532.6	21064.9	45
46	9492.9	10005.5	10607.7	11337.6	12264.6	13537.2	15578.7	21302.0	46
47	9500.8	10014.8	10618.7	11351.1	12282.4	13563.0	15625.5	21556.6	47
48	9508.8	10024.0	10629.7	11364.8	12300.2	13588.9	15673.0	21831.7	48
49	9516.8	10033.3	10640.8	11378.4	12318.2	13615.1	15721.0	22130.6	49
50	9524.8	10042.6	10651.9	11392.2	12336.3	13641.4	15769.8	22458.0	50
51	9532.9	10051.9	10663.0	11406.0	12354.4	13668.0	15819.3	22819.9	51
52	9540.9	10061.3	10674.1	11419.8	12372.7	13694.7	15869.5	23224.3	52
53	9548.9	10070.6	10685.3	11433.7	12391.0	13721.7	15920.4	23682.0	53
54	9557.0	10080.0	10696.5	11447.7	12409.5	13748.9	15972.1	24211.8	54
55	9565.1	10089.4	10707.7	11461.7	12428.0	13776.3	16024.6	24836.9	55
56	9573.2	10098.9	10719.1	11475.8	12446.7	13803.9	16077.9	25600.8	56
57	9581.4	10108.4	10730.4	11489.9	12465.3	13831.7	16132.0	26582.9	57
58	9589.5	10117.9	10741.8	11504.1	12484.2	13859.8	16187.0	27958.9	58
59	9597.7	10127.4	10753.3	11518.3	12543.1	13888.1	16242.9	30364.3	59
M	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	M
L.	82	83	84	85	86	87	88	80	L.

A

TABLE

OF

LOGARITHMS

For Numbers increasing in their Natural Order from Unite to 10000.

Num.	Logarith.	Num.	Logarith.	Num.	Logarith.
1	0.000000	34	1.531479	67	1.826075
2	0.301030	35	1.544068	68	1.832509
3	0.477121	36	1.556302	69	1.838849
4	0.602060	37	1.568202	70	1.845098
5	0.698970	38	1.579784	71	1.851258
6	0.778151	39	1.591065	72	1.857332
7	0.845098	40	1.602060	73	1.863323
8	0.903090	41	1.612784	74	1.869232
9	0.954242	42	1.623249	75	1.875061
10	1.000000	43	1.633468	76	1.880814
11	1.041393	44	1.643453	77	1.886491
12	1.079181	45	1.653212	78	1.892095
13	1.113943	46	1.662758	79	1.897627
14	1.146128	47	1.672098	80	1.903090
15	1.176091	48	1.681241	81	1.908485
16	1.204120	49	1.690196	82	1.913814
17	1.230449	50	1.698970	83	1.919078
18	1.255272	51	1.707570	84	1.924279
19	1.278754	52	1.716003	85	1.929419
20	1.301030	53	1.724276	86	1.934498
21	1.322219	54	1.732394	87	1.939519
22	1.342423	55	1.740363	88	1.944483
23	1.361728	56	1.748188	89	1.949390
24	1.380211	57	1.755875	90	1.954242
25	1.397940	58	1.763428	91	1.959041
26	1.414973	59	1.770852	92	1.963788
27	1.431364	60	1.778151	93	1.968483
28	1.447158	61	1.785330	94	1.973128
29	1.462398	62	1.792392	95	1.977724
30	1.477121	63	1.799340	96	1.982271
31	1.491362	64	1.806180	97	1.986772
32	1.505150	65	1.812913	98	1.991226
33	1.518514	66	1.819544	99	1.995635

No.	0	1	2	3	4	5	6	7	8	9
100	000000	000434	000868	001301	001734	002166	002598	003029	003460	00389
101	004321	004751	005180	005609	006038	006466	006894	007321	007748	00817
102	008600	009026	009451	009876	010300	010724	011147	011570	011993	01241
103	012837	013259	013680	014100	014520	014940	015360	015779	016197	01661
104	017033	017451	017868	018284	018700	019947	019532	019947	020361	02077
105	021189	021603	022016	022428	022841	023252	023664	024075	024486	02489
106	025306	025715	026124	026533	026942	027350	027757	028164	028571	028978
107	029384	029789	030195	030600	031004	031408	031812	032216	032619	033021
108	033424	033826	034227	034628	035029	035430	035830	036229	036629	037028
109	037426	037825	038223	038620	039017	039414	039811	040207	040602	040998
110	041393	041787	042182	042575	042969	043362	043755	044148	044540	044931
111	045323	045714	046105	046495	046885	047275	047664	048053	048442	048830
112	049218	049606	049993	050380	050766	051152	051538	051924	052309	052694
113	053078	053463	053846	054230	054613	054996	055378	055760	056142	056524
114	056905	057286	057666	058046	058426	058805	059185	059563	059942	060320
115	060698	061072	061452	061829	062206	062582	062958	063333	063708	064083
116	064458	064832	065206	065580	065953	066326	066698	067071	067443	067814
117	068186	068557	068928	069298	069668	070038	070407	070776	071145	071514
118	071882	072250	072617	072985	073352	073718	074085	074451	074816	075182
119	075547	075912	076276	076640	077004	077368	077731	078094	078457	078819
120	079181	079543	079904	080266	080626	080987	081347	081707	082067	082426
121	082785	083144	083503	083861	084219	084576	084934	085291	085647	086004
122	086360	086716	087071	087426	087781	088136	088490	088845	089198	089552
123	089905	090258	090611	090963	091315	091667	092018	092370	092721	093071
124	093422	093772	094122	094471	094820	095169	095518	095866	096215	096562
125	096910	097257	097604	097951	098297	098644	098990	099335	099681	100026
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916	961895	961943	961990	962038	962085	962132	962180	962227	962275	962322
917	962369	962417	962464	962511	962559	962606	962653	962701	962748	962795
918	962843	962890	962937	962985	963032	963079	963126	963174	963221	963268
919	963315	963363	963410	963457	963504	963552	963599	903646	963693	963741
920	963788	963835	963882	963929	963977	964024	964071	964118	964165	964212
921	964260	964307	964354	964401	964448	964495	964542	964590	964637	964684
922	964731	964778	964825	964872	964919	964966	965013	965060	965108	965155
923	965202	965249	965296	965343	965390	965437	965484	965531	965578	965625
924	965672	965719	965766	965813	965860	965907	965954	966001	966048	966095
925	966142	966189	966236	966283	966329	966376	966423	966470	966517	966564
926	966611	966658	966705	966752	966798	966845	966892	966939	966986	967033
927	967080	967127	967173	967220	967267	967314	967361	967408	967454	967501
928	967548	967595	967642	967688	967735	967782	967829	967875	967922	967969
929	968016	968062	968109	968156	968203	968249	968296	968343	968389	968436
930	968483	968530	968576	968623	968670	968716	968763	968810	968856	968903
931	968950	968996	969043	969090	969136	969183	969229	969276	969323	969369
932	969416	969462	969509	969556	969602	969649	969695	969742	969788	969835
933	969882	969928	969975	970021	970068	970114	970161	970207	970254	970300
934	970347	970393	970440	970486	970533	970579	970626	970672	970719	970765
935	970812	970858	970904	970951	970997	971044	971090	971137	971183	971229
936	971276	971322	971369	971415	971461	971508	971554	971600	971647	971693
937	971740	971786	971832	971879	971925	971971	972018	972064	972110	972156
938	972203	972249	972295	972342	972388	972434	972480	972527	972573	972619
939	972666	972712	972758	972804	972851	972897	972943	972989	973035	973082
940	973128	973174	973220	973266	973313	973359	973405	973451	973497	973543
941	973590	973636	973682	973728	973774	973820	973866	973913	973959	974005
942	974051	974097	974143	974189	974235	974281	974327	974373	974420	974466
943	974512	974558	974604	994650	974696	974742	974788	974834	974880	974926
944	974972	975018	975064	975110	975156	975202	975248	975294	975340	975386
945	975432	975478	975524	975570	975616	975661	975707	975753	975799	975845
946	975891	975937	975983	976029	976075	976121	976166	976212	976258	976304
947	976350	976396	976442	976487	976533	976579	976625	976671	976717	976762
948	976808	976854	976900	976946	976991	977037	977083	977129	977175	977220
949	977266	977312	977358	977403	977449	977495	977541	977586	977632	977678
950	977724	977769	977815	977861	977906	977952	977998	978043	978089	978135
951	978180	978226	978272	978317	978363	978409	978454	978500	978546	978591
952	978637	978683	978728	978774	978819	978865	978911	978956	979002	979047
953	979093	979138	979184	979230	979275	979321	979366	979412	979457	979503
954	979548	979594	979639	979685	979730	979776	979821	979867	979912	979958
	0	1	2	3	4	5	6	7	8	9

No.	0	1	2	3	4	5	6	7	8	9
955	980003	980049	980094	980140	980185	980231	980276	980322	980367	980412
956	980458	980503	980549	980594	980640	980685	980730	980776	980821	980867
957	980912	980957	981003	981048	981093	981139	981184	981229	981275	981320
958	981365	981411	981456	981501	981547	981592	981637	981683	981728	981773
959	981819	981864	981909	981954	982000	982045	982090	982135	982181	982226
960	982271	982316	982362	982407	982452	982497	982543	982588	982633	982678
961	982723	982769	982814	982859	982904	982949	982994	983040	983085	983130
962	983175	983220	983265	983310	983356	983401	983446	983491	983536	983581
963	983626	983671	983716	983762	983807	983852	983897	983942	983987	984032
964	984077	984122	984167	984212	984257	984302	984347	984392	984437	984482
965	984527	984572	984617	984662	984707	984752	984797	984842	984887	984932
966	984977	985022	985067	985112	985157	985202	985247	985292	985337	985382
967	985426	985471	985516	985561	985606	985651	985696	985741	985786	985830
968	985875	985920	985965	986010	986055	986100	986144	986189	986234	986279
969	986324	986369	986413	986458	986503	986548	986593	986637	986682	986727
970	986772	986816	986861	986906	986951	986995	987040	987085	987130	987174
971	987219	987264	987309	987353	987398	987443	987587	987532	987577	987622
972	987666	987711	987756	987800	987845	987890	987934	987979	988024	988068
973	988113	988157	988202	988247	988291	988336	988381	988425	988470	988514
974	988559	988603	988648	988693	988737	988782	988826	988871	988915	988960
975	989005	989049	989094	989138	989183	989227	989272	989316	989361	989405
976	989450	989494	989539	989583	989628	989672	989717	989761	989806	989850
977	989895	989939	989983	990028	990072	990117	990161	990206	990250	990294
978	990339	990383	990428	990472	990516	990561	990605	990650	990694	990738
979	990783	990827	990871	990916	990960	991004	991049	991093	991137	991182
980	991226	991270	991315	991359	991403	991448	991492	991536	991580	991625
981	991669	991713	991757	991802	991846	991890	991934	991979	992023	992067
982	992111	992156	992200	992244	992288	992333	992377	992421	992465	992509
983	992553	992598	992642	992686	992730	992774	992818	992863	992907	992951
984	992995	993039	993083	993127	993172	993216	993260	993304	993348	993392
985	993436	993480	993524	993568	993613	993657	993701	993745	993789	993833
986	993877	993921	993965	994009	994053	994097	994141	994185	994229	994273
987	994317	994361	994405	994449	994493	994537	994581	994625	994669	994713
988	994757	994801	994845	994889	994933	994977	995021	995064	995108	995152
989	995196	995240	995284	995328	995372	995416	995460	995504	995547	995591
990	995635	995679	995723	995767	995811	995854	995898	995942	995986	996030
991	996074	996117	996161	996205	996249	996293	996336	996380	996424	996468
992	996512	996555	996599	996643	996687	996730	996774	996818	996862	996905
993	996949	996993	997037	997080	997124	997168	997212	997255	997299	997343
994	997386	997430	997474	997517	997561	997605	997648	997629	997736	997779
995	997823	997867	997910	997954	997998	998041	998085	998128	998172	998216
996	998259	998303	998346	998390	998434	998477	998521	998564	998608	998652
997	998695	998739	998782	998826	998869	998913	998956	999000	999043	999087
998	999130	999174	999218	999261	999305	999348	999392	999435	999478	999522
999	999565	999609	999652	999696	999739	999783	999826	999870	999913	999957
.	0	1	2	3	4	5	6	7	8	9

A Triangular Canon Logarithmical: or, A TABLE of Artificial Sines, Tangents and Secants, the Radius 10.000000; and to every Degree of the Quadrant.

0 Degree.

Min.	Sine		Tang.		Secant.		
0	0.000000	10.00000	0.000000	Infinite.	10.000000	Infinite.	60
1	6.463726	9.999999	6.463726	13.536274	10.000000	13.536274	59
2	6.764756	9.999999	6.764756	13.235244	10.000000	13.235244	58
3	6.940847	9.999999	6.940847	13.059152	10.000000	13.059153	57
4	7.065786	9.999999	7.065786	12.934214	10.000000	12.934214	56
5	7.162696	9 999999	7.162696	12.837304	10.000000	12.837304	55
6	7.241877	9.999999	7.241878	12.758122	10.000001	12.758123	54
7	7.308824	9.999999	7.308825	12.691175	10.000001	12.691176	53
8	7.366816	9.999999	7.366817	12.633183	10.000001	12.633184	52
9	7.417968	9.999998	7.417970	12.582030	10.000001	12.582032	51
10	7.463725	9.999998	7.463727	12.536273	10.000002	12.536274	50
11	7.505118	9.999998	7.505120	12.494880	10.000002	12.494882	49
12	7.542906	9.999997	7.542909	12.457091	10.000003	12.457093	48
13	7.577668	9.999997	7.577671	12.422328	10.000003	12.422332	47
14	7.609853	9.999996	7.609857	12.390143	10.000004	12.390147	46
15	7.639816	9.999996	7.639820	12.360180	10.000004	12.360184	45
16	7.667844	9.999995	7.667849	12.332151	10.000005	12.332155	44
17	7.694173	9.999995	7.694179	12.305821	10.000005	12.305827	43
18	7.718997	9.999994	7.719003	12.280997	10.000006	12.281003	42
19	7.742477	9.999993	7.742484	12.257516	10.000007	12.257522	41
20	7.764754	9.999993	7.764761	12.235239	10.000007	12.235246	40
21	7.785943	9.999992	7.785951	12.214049	10.000008	12.214057	39
22	7.806146	9.999991	7.806155	12,193845	10 000009	12.193854	38
23	7.825451	9.999990	7.825460	12.174540	10.000010	12.174549	37
24	7.843934	9 999989	7.843944	12.156056	10.000011	12.156066	36
25	7.861662	9.999988	7.861674	12.138326	10.000011	12.138338	35
26	7.878695	9.999988	7.878708	12.121292	10.000012	12.121305	34
27	7.895085	9.999987	7.895099	12.104901	10.000013	12.104915	33
28	7.910879	9.999986	7.910894	12.089106	10.000014	12.089121	32
29	7.926119	9.999984	7.926134	12.073866	10.000015	12.073881	31
30	7.940842	9.999983	7.940858	12.059142	10.000016	12.059158	30
31	7.955082	9.999982	7.955100	12.044900	10.000018	12.044918	29
32	7.968870	9.999981	7.968889	12.031111	10.000019	12.031130	28
33	7.982233	9.999980	7.982253	12.017747	10.000020	12.017767	27
34	7.995198	9.999979	7.995219	12.004781	10.000021	12.004802	26
35	8.007787	9.999977	8.007809	11.992191	10.000022	11.992213	25
36	8.020021	9.999976	8.020044	11.979955	10.000024	11.979979	24
37	8.031919	9.999975	8.031945	11.968055	10.000025	11.968080	23
38	8.043501	9.999973	8.043527	11.956473	10.000026	11.956499	22
39	8.054781	9.999972	8.054809	11.945191	10.000028	11.945219	21
40	8.065776	9.999971	8 065806	11.934194	10.000029	11.934224	20
41	8.076500	9.999969	8.076531	11.923469	10.000031	11.923500	19
42	8.086965	9.999968	8.086997	11.913003	10.000032	11.913035	18
43	8.097183	9.999966	8.097217	11.902783	10.000034	11.902817	17
44	8.107167	9.999964	8.107202	11.892797	10 000036	11.892833	16
45	8.116926	9.999963	8.116963	11.883037	10.000037	11.883074	15
46	8.126471	9.999961	8.126510	11.873490	10.000039	11.873529	14
47	8.135810	9.999959	8.135851	11.864149	10.000041	11.864190	13
48	8.144953	9.999958	8.144996	11.855004	10.000042	11.855047	12
49	8.153907	9.999956	8.153952	11.846048	10.000044	11.846092	11
50	8.162681	9.999954	8.162727	11.837273	10.000046	11.837319	10
51	8.171280	9.999952	8.171328	11.828672	10.000048	11.828720	9
52	8.179713	9.999950	8.179763	11.820237	10.000050	11.820287	8
53	8.187985	9.999948	8.188036	11.811964	10.000052	11.812015	7
54	8,196102	9.999946	8.196156	11.803844	10.000054	11.803898	6
55	8.204070	9.999944	8 204126	11.795874	10.000056	11.795930	5
56	8.211895	9.999942	8.211953	11.788047	10.000058	11.788105	4
57	8.219581	9.999940	8.219641	11.780359	10.000060	11.780419	3
58	8.227133	9.999938	8 227195	11.772805	10.000062	11.772866	2
59	8.234557	9.999936	8 234641	11.765379	10.000064	11.765443	1
60	8.241855	9.999934	8.241921	11.758078	10.000066	11.758145	0
		Sine		Tang.		Secant.	M

89 Degrees.

1 Degree.

Min.	Sine		Tang.		Secant.		
0	8.241855	9,999934	8.241921	11.758078	10.000066	11.758145	60
1	8.249033	9.999932	8.249101	11.750898	10.000068	11.750967	59
2	8.256094	9.999929	8.256165	11.743835	10.000071	11.743906	58
3	8.263042	9.999927	8.263115	11.736885	10.000073	11.730958	57
4	8.269881	9.999925	8.269956	11.730044	10.000075	11.730119	56
5	8.276614	9.999922	8.276691	11.723309	10.000078	11.723386	55
6	8.283243	9.999920	8.283323	11.716677	10.000080	11.716757	54
7	8.289773	9.999917	8.289856	11.710144	10.000082	11.710227	53
8	8.296207	9.999915	8.296292	11.703708	10.000085	11.703793	52
9	8.302546	9.999912	8.302633	11.697366	10.000087	11.697454	51
10	8.308794	9.999910	8.308884	11.691116	10.000090	11.691206	50
11	8.314954	9.999907	8.315046	11.684954	10.000093	11.685046	49
12	8.321027	9.999905	8.321122	11.678878	10.000095	11.678973	48
13	8.327016	9.999902	8.327114	11.672886	10.000098	11.672984	47
14	8.332924	9.999899	8.333025	11.666975	10.000101	11.667076	46
15	8.338753	9.999897	8.338856	11.661144	10.000103	11.661247	45
16	8.344504	9.999894	8.344610	11.655389	10.000106	11.655496	44
17	8.350180	9.999891	8.350289	11.649710	10.000109	11.649819	43
18	8.355783	9.999888	8.355895	11.644105	10.000112	11.644216	42
19	8.361315	9.999885	8.361430	11.638570	10.000115	11.638685	41
20	8.366777	9.999882	8.366894	11.633105	10.000118	11.633223	40
21	8.372171	9.999879	8.372291	11.627708	10.000121	11.627829	39
22	8.377499	9.999876	8.377622	11.622378	10.000124	11 622501	38
23	8.382762	9.999873	8.382889	11.617111	10.000127	11.617238	37
24	8.387962	9.999870	8.388092	11.611908	10.000130	11.612038	36
25	8.393101	9.999867	8.393234	11.606766	10.000133	11.606899	35
26	8.398179	9.999864	8.398315	11.601685	10.000136	11.601821	34
27	8.403199	9.999861	8.403338	11.596662	10.000139	11.596801	33
28	8.408161	9.999858	8.408304	11.591696	10.000142	11.591839	32
29	8.413068	9.999854	8.413213	11.586787	10.000146	11.586931	31
30	8.417919	9.999851	8.418068	11.581932	10.000149	11.582081	30
31	8.422717	9.999848	8.422869	11.577131	10.000152	11.577283	29
32	8.427462	9.999844	8.427618	11.572382	10.000155	11.572538	28
33	8.432156	9.999841	8.432315	11.567685	10.000159	11.567844	27
34	8.436800	9.999838	8.436962	11.563038	10 000162	11.563200	26
35	8.441394	9.999834	8.441560	11.558440	10.000166	11.558606	25
36	8.445941	9.999831	8.446110	11.553890	10.000169	11.554059	24
37	8.450440	9.999827	8.450613	11.549387	10.000173	11.549560	23
38	8.454893	9.999823	8.455070	11.544930	10 000176	11 545107	22
39	8.459301	9.999820	8.459481	11.540519	10.000180	11.540699	21
40	8.463665	9.999816	8.463849	11.536151	10 000184	11.536335	20
41	8.467985	9.999812	8.468172	11.531827	10.000187	11.532015	19
42	8.472263	9.999809	8.472454	11.527546	10.000191	11.527737	18
43	8.476498	9.999805	8.476693	11.523307	10.000195	11.523502	17
44	8.480693	9.999801	8.480892	11 519108	10.000199	11.519307	16
45	8.484848	9.999797	8.485050	11.514949	10.000203	11.515152	15
46	8.488963	9.999793	8.489170	11.510830	10.000206	11.511037	14
47	8.493040	9.999790	8.493250	11.506750	10.000210	11.506960	13
48	8.497078	7.999786	8.497293	11.502707	10.000214	11.502922	12
49	8.501080	9.999782	8.501298	11.498702	10.000218	11.498920	11
50	8.505045	9.999778	8.505267	11.494733	10.000222	11.494955	10
51	8.508974	9.999774	8.509200	11.490800	10.000226	11.491026	9
52	8.512867	9.999769	8.513098	11.486902	10.000230	11.487133	8
53	8.516726	9.999765	8.516961	11.483039	10.000235	11.483274	7
54	8.520551	9.999761	8.520790	11.479210	10.000239	11.479449	6
55	8.524343	9.999757	8.524586	11 475414	10.000243	11.475657	5
56	8.528102	9.999753	8.528349	11.471651	10.000247	11.471898	4
57	8.531828	9.999748	8.532080	11.467920	10.000252	11.468172	3
58	8.535523	9.999744	8.535779	11 464221	10.000256	11.464477	2
59	8.539186	9.999740	8 539447	11.460553	10.000260	11.460814	1
60	8.542819	9.999735	8.543084	11.456916	10.000265	11.457181	0
		Sine		Tang.		Secant.	M

88 Degrees.

A Table of Artificial Sines, Tangents and Secants.

2 Degrees.

Min.	Sine.		Tang.		Secant.		
0	8.542819	9.999735	8.543084	11.456916	10.000265	11.457181	60
1	8.546422	9.999731	8.546691	11.453309	10.000269	11.453578	59
2	8.549595	9.999726	8.550268	11.449732	10.000273	11.450005	58
3	8.553539	9.999722	8.553817	11.446183	10.000278	11.446461	57
4	8.557054	9.999717	8.557336	11.442664	10.000283	11.442946	56
5	8.560540	9.999713	8.560828	11.439172	10.000287	11.439460	55
6	8.563999	9.999708	8.564291	11.435709	10.000292	11.436001	54
7	8.567431	9.999704	8.567727	11.432272	10.000296	11.432569	53
8	8.570836	9.999699	8.571137	11.428863	10.000301	11.429164	52
9	8.574214	9.999694	8.574520	11.425480	10.000306	11.425786	51
10	8.577566	9.999689	8.577877	11.422123	10.000311	11.422434	50
11	8.580892	9.999685	8.581208	11.418792	10.000315	11.419108	49
12	8.584193	9.999680	8.584514	11.415486	10.000320	11.415807	48
13	8.587469	9.999675	8.587794	11.412206	10.000325	11.412531	47
14	8.590721	9.999670	8.591051	11.408949	10.000330	11.409279	46
15	8.593918	9.999665	8.594283	11.405717	10.000335	11.406052	45
16	8.597152	9.999660	8.597492	11.402508	10.000340	11.402848	44
17	8.600332	9.999655	8.600677	11.399323	10.000345	11.399668	43
18	8.603189	6.999650	8.603839	11.396161	10.000350	11.396511	42
19	8.606623	9.999645	8.606978	11.393022	10.000355	11.393377	41
20	8.609734	9.999640	8.6 0004	11.389906	10.000360	11.390266	40
21	8.612823	9.999635	8.613189	11.386811	10.000365	11.387176	39
22	8.615891	9.999629	8.616262	11.383738	10.000371	11.384109	38
23	8.618937	9.999624	8.619313	11.380687	10.000376	11.381063	37
24	8.621962	9.999619	8.622343	11.377657	10.000381	11.378038	36
25	8.624965	9.999614	8.625352	11.374648	10.000386	11.375035	35
26	8.627948	9.999608	8.628340	11.371660	10.000392	11.372052	34
27	8.630911	9.999603	9.631308	11.368692	10.000397	11.369089	33
28	8.633854	9.999597	8.634256	11.365744	10.000403	11.366146	32
29	8.636776	9.999592	8.637184	11.362815	10.000408	11.353224	31
30	8.639680	9.999586	8.640093	11.359907	10.000413	11.360320	30
31	8.642563	9.999581	8.642982	11.357017	10.000419	11.357437	20
32	8.645428	9.999575	8.645853	11.354147	10.000425	11.354572	28
33	8.648274	9.999570	8.648704	11.351296	10.000430	11.351726	27
34	8.651102	9.999564	8.651537	11.348462	10.000436	11.348898	26
35	8.653911	9.999558	8.654532	11.345648	10.000442	11.346089	25
36	8.656702	9.999553	8.657149	11.342851	10.000447	11.343298	24
37	8.659475	9.999547	8.659928	11.340072	10.000453	11.340525	23
38	8.662230	9.999541	8.662689	11.337311	10.000459	11.337770	22
39	8.664968	9.999535	8.665433	11.334567	10.000465	11.335032	21
40	8.667689	9.999529	8.668160	11.331840	10.000471	11.332311	20
41	8.670393	9.999524	8.670870	11.329130	10.000476	11.329607	19
42	8.673080	9.999518	8.673563	11.326437	10.000482	11.326920	18
43	8.675751	9.999512	8.676239	11.323761	10.000488	11.324249	17
44	8.678405	9.999506	8.678900	11.321100	10.000494	11.321595	16
45	8.681043	9.999500	8.681544	11.318456	10.000500	11.318957	15
46	8.683665	9.999403	8.684172	11.315828	10.000506	11.316338	14
47	8.686272	9.999487	8.686784	11.313216	10.000513	11.313727	13
48	8.683863	9.999481	8.689381	11.310619	10.000519	11.311135	12
49	8.691438	9.999475	8.691963	11.308037	10.000525	11.308562	11
50	8.693998	9.999469	8.694529	11.305471	10.000531	11.306002	10
51	8.696543	9.999462	8.697081	11.302919	10.000537	11.303457	9
52	8.699073	9.999456	8.699617	11.300283	10.000544	11.300927	8
53	8.701589	9.999450	8.702130	11.297861	10.000550	11.298411	7
54	8.704090	9.999443	8.704646	11.295353	10.000556	11.295910	6
55	8.706577	9.999437	8.707139	11.292860	10.000563	11.293423	5
56	8.709049	9.999431	8.709618	11.290381	10.000560	11.290951	4
57	8.711507	9.999424	8.712083	11.287917	10.000576	11.288492	3
58	8.713952	9.999418	8.714534	11.285465	10.000582	11.286048	2
50	8.716283	9.999411	8.716972	11.283028	10.000589	11.283617	1
60	8.718800	9.999404	8.719396	11.280604	10.000596	11.281200	0
		Sine.		Tang.		Secant.	M

87 *Degrees.*

3 Degrees.

Min.	Sine		Tang.		Secant.		
0	8.718800	9.999404	8.719396	11.280604	10.000596	11.281200	60
1	8.721204	9.999398	8.721806	11.278194	10.000602	11.278796	59
2	8.723595	9.999391	7.724203	11.275796	10.000609	11.276405	58
3	8.725972	9.999384	8.726588	11.273412	10.000616	11.274028	57
4	8.728337	9.999378	8.728959	11.271041	10.000622	11.271663	56
5	8.730688	9.999371	8.731317	11.268683	10.000629	11.269312	55
6	8.733027	9.999364	8.733663	11.266337	10.000636	11.266973	54
7	8.735353	9.999357	8.735996	11.264004	10.000643	11.264646	53
8	8.737667	9.999350	8.738317	11.261683	10.000650	11.262332	52
9	8.739969	9.999343	8.740626	11.259374	10.000657	11.260031	51
10	8.742259	9.999336	8.742922	11.257078	10.000664	11.257741	50
11	8.744536	9.999329	8.745207	11.254793	10.000671	11.255464	49
12	8.746801	9.999322	8.747479	11.252521	10.000678	11.253158	48
13	8.749055	9.999315	8.749740	11.250260	10.000685	11.250945	47
14	8.751297	9.999308	8.751989	11.248011	10.000692	11.248703	46
15	8.753528	9.999301	8.754227	11.245773	10.000699	11.246472	45
16	8.755747	9.999294	8.756453	11.243547	10.000706	11.244253	44
17	8.757955	9.999286	8.758668	11.241332	19.000713	11.242045	43
18	8.760151	9.999279	8.760872	11.239128	10.000721	11.239849	42
19	8.762337	9.999272	8.763065	11.236935	10.000728	11.237663	41
20	8.764511	9.999265	8.765246	11.234753	10.000735	11.235489	40
21	8.766675	9.999257	8.767417	11.232582	10.000743	11.233325	39
22	8.768827	9.999250	8.769578	11.230422	10.000750	11.231172	38
23	8.770970	9.999242	8.771727	11.228273	10.000758	11.229030	37
24	8.773101	9.999235	8.773866	11.226133	10.000765	11.226899	36
25	8.775223	9.999227	8.775995	11.224005	10.000773	11.224777	35
26	8.777333	9.999220	8.778114	11.221886	10.000780	11.222667	34
27	8.779434	9.999212	8.780222	11.219778	10.000788	11.220566	33
28	8.781524	9.999205	8.782320	11.217680	10.000795	11.218476	32
29	8.783605	9.999197	8.784408	11.215592	10.000803	11.216395	31
30	8.785675	9.999180	8.786486	11.213514	10.000811	11.214325	30
31	8.787736	9.999181	8.788554	11.211446	10.000818	11.212264	29
32	8.789787	9.999174	8.790613	11.209387	10.000826	11.210213	28
33	8.791828	9.999166	8.792662	11.207338	10.000834	11.218172	27
34	8.793859	9.999158	8.794701	11.205299	10.000842	11.206141	26
35	8.795881	9.999150	8.796732	11.203269	10.000850	11.204110	25
36	8.797894	9.999142	8.798752	11.201248	10.000858	11.202106	24
37	8.799897	9.999134	8.800763	11.199237	10.000866	11.200103	23
38	8.801891	9.999126	8.802765	11.197235	10.000174	11.198108	22
39	8.803876	9.999118	8.804758	11.195242	10.000882	11.1 6124	21
40	8.805852	9.999110	8.806742	11.193258	10.000890	11.194148	20
41	8.807819	9.999102	8.808717	11.191283	10.000898	11.192181	19
42	8.809777	9.999104	8.810683	11.189317	10.000906	11.190223	18
43	8.811726	9.999086	8.812641	11.187359	10.000914	11.188 74	17
44	8.813667	9.999077	8.814589	11.185411	10.000923	11.186333	16
45	8.815598	9.999069	8.816529	11.183471	10.000931	11.184401	15
46	8.817522	9.999061	8.818461	11.181539	10.000939	11.182478	14
47	8.819436	9.999052	8.820384	11.179616	10.000947	11.180564	13
48	8.821342	9.999044	8.822298	11.177702	10.000956	11.178657	12
49	8.823240	9.999036	8.824205	11.175795	10.000964	11.176760	11
50	8.825130	9.999027	8.826103	11.173897	10.000973	11.174870	10
51	8.827011	9.999019	8.827992	11.172008	10.000981	11.172989	9
52	8.828884	9.999010	8.829874	11.170126	10.000990	11.171116	8
53	8.830749	9.999002	8.83 748	11.168252	10.000998	11.169250	7
54	8.832607	9.998993	8.833613	11.166387	10.001007	11.167393	6
55	8.834456	9.998984	8.835471	11.164529	10.001015	11.165544	5
56	8.836297	9.998976	8.837321	11.162679	10.001024	11.163703	4
57	8.838130	9.998967	8.839163	11.160837	10.001033	11.161870	3
58	8.839956	9.998958	8.840098	11.159002	10.001042	11.160044	2
59	8.841774	9.998950	8.842824	11.157175	10.001050	11.158216	1
60	8.843584	9.998941	8.844644	11.155356	10.001059	11.156415	0
		Sine.		Tang.		Secant.	M

86 *Degrees.*

4 *Degrees.*

Min.	Sine		Tang.		Secant.		
0	8.843584	9.998941	8.844644	11.155356	10.001059	11.156415	60
1	8.845387	9.998932	8.846455	11.153545	10.001068	11.154613	59
2	8.847183	9.998923	8.848260	11.151740	10.001077	11.152817	58
3	8.848971	9.998914	8.850056	11.149943	10.001086	11.151029	57
4	8.850751	9.998905	8.851846	11.148154	10.001095	11.149249	56
5	8.852524	9.998896	8.853628	11.146372	10.001104	11.147475	55
6	8.854290	9.998887	8.855403	11.144597	10.001113	11.145709	54
7	8.856049	9.998878	8.857171	11.142829	10.001122	11.143951	53
8	8.857801	9.998869	8.858932	11.141068	10.001131	11.142199	52
9	8.859946	9.998860	8.860686	11.139314	10.001140	11.140554	51
10	8.861283	9.998851	8.862433	11.137567	10.001149	11.138717	50
11	8.863014	9.998841	8.864172	11.135827	10.001159	11.136986	49
12	8.864738	0.998832	8.865905	11.134094	10.001168	11.135262	48
13	8.866454	9.998823	8.867632	11.132368	10.001177	11.133545	47
14	8.868165	9.998813	8.869351	11.130649	10.001186	11.131835	46
15	8.869869	9.998804	8.871064	11.128936	10.001196	11.130132	45
16	8.871565	9.998795	8.872770	11.127230	10.001205	11.128435	44
17	8.873255	9.998785	8.874469	11.125531	10.001215	11.126745	43
18	8.874938	9.998776	8.876162	11.123838	10.001224	11.125062	42
19	8.876615	9.998766	8.877849	11.122152	10.001234	11.123385	41
20	8.878285	9.998757	8.879529	11.120471	10.001243	11.121715	40
21	8.879949	9.998747	8.881202	11.118798	10.001253	11.120051	39
22	8.881607	9.998737	8.882869	11.117131	10.001262	11.118393	38
23	8.883258	9.998728	8.884530	11.115470	10.001272	11.116742	37
24	8.884903	9.998718	8.886185	11.113815	10.001282	11.115097	36
25	8.886542	9.998708	8.887833	11.112167	10.001292	11.113458	35
26	8.888174	9.998699	8.889476	11.110524	10.001301	11.111826	34
27	8.889801	9.998689	8.891112	11.108888	10.001311	11.110199	33
28	8.891421	9.998679	8.892742	11.107258	10.001321	11.108579	32
29	8.893035	9.998669	8.894366	11.105634	10.001331	11.106965	31
30	8.894643	9.998659	8.895984	11.104016	10.001341	11.105357	30
31	8.896245	9.998649	8.897596	11.102404	10.001351	11.103754	29
32	8.897842	9.998639	8.899203	11.100797	10.001361	11.102158	28
33	8.899432	9.998629	8.900803	11.099197	10.001371	11.100568	27
34	8.901017	9.998619	8.902398	11.097602	10.001381	11.098983	26
35	8.902595	9.998609	8.903987	11.096013	10.001391	11.097404	25
36	8.904168	9.998599	8.905570	11.094430	10.001401	11.095831	24
37	8.905736	9.998589	8.907147	11.092853	10.001411	11.094264	23
38	8.907297	9.998578	8.908719	11.091281	10.001422	11.092702	22
39	8.908853	9.998568	8.910285	11.089715	10.001432	11.091146	21
40	8.910404	9.998558	8.911846	11.088154	10.001442	11.089596	20
41	8.911949	9.998547	8.913401	11.086599	10.001452	11.088051	19
42	8.913488	9.998537	8.914951	11.085049	10.001463	11.086512	18
43	8.915022	9.998527	8.916495	11.083505	10.001473	11.084978	17
44	8.916550	9.998516	8.918034	11.081966	10.001484	11.083450	16
45	8.918073	9.998506	8.919567	11.080432	10.001494	11.081927	15
46	8.919591	9.998494	8.621096	11.078904	10.001505	11.080409	14
47	8.921103	9.998485	8.922619	11.077381	10.001515	11.078897	13
48	8.922610	9.998474	8.924136	11.075864	10.001526	11.077389	12
49	8.924112	9.998464	8.925649	11.074351	10.001536	11.075888	11
50	8.925609	9.998453	8.927156	11.072854	10.001547	11.074391	10
51	8.927100	9.998442	8.928658	11.071342	10.001558	11.072900	9
52	8.928587	9.998431	8.930155	11.069845	10.001568	11.071413	8
53	8.930068	9.998421	8.931647	11.068353	10.001579	11.069932	7
54	8.931544	9.998410	8.933134	11.066866	10.001590	11.068456	6
55	8.933015	9.998399	8.934616	11.065384	10.001601	11.066985	5
56	8.934481	9.968388	8.936093	11.063907	10.001612	11.065519	4
57	8.935942	9.998377	8.937565	11.062435	10.001623	11.064058	3
58	8.937398	9.998366	8.939032	11.060968	10.001634	11.062602	2
59	8.938850	9.998355	8.940494	11.059506	10.001645	11.061150	1
60	8.940296	9.998344	8.941952	11.058048	10.001656	11.059704	0
		Sine		Tang.		Secant.	M

85 *Degrees.*

5 *Degrees.*

Min.	Sine		Tang		Secant.		
0	8.940296	9.998344	8.941952	11.058048	10.001656	11.059704	60
1	8.941738	9.998333	8.943404	11.056595	10.001667	11.058262	59
2	8.943174	9.998322	8.944852	11.055148	10.001678	11.056826	58
3	8.944606	9.998311	8.946295	11.053705	10.001689	11.055394	57
4	8.946033	9.998300	8.947734	11.052266	10.001700	11.053966	56
5	8.947456	9.998288	8.949168	11.050832	10.001711	11.052544	55
6	8.948874	9.998277	8.950597	11.049403	10.001723	11.051126	54
7	8.950287	9.998266	8.952021	11.047979	10.001734	11.049713	53
8	8.951696	9.998255	8.953441	11.046559	10.001745	11.048304	52
9	8.953100	9.998243	8.954856	11.045144	10.001757	11.046900	51
10	8.954499	9.998232	8.956267	11.043733	10.001768	11.045501	50
11	8.955896	9.998220	8.957673	11.042326	10.001780	11.044106	49
12	8.957284	9.998209	8.959075	11.040925	10.001791	11.042716	48
13	8.958670	9.998197	8.960473	11.039527	10.001803	11.041330	47
14	8.960052	9.998186	8.961866	11.038134	10.001814	11.039948	46
15	8.961429	9.998174	8.963254	11.036745	10.001826	11.038571	45
16	8.962801	9.998163	8.964639	11.035361	10.001837	11.037199	44
17	8.964170	9.998151	8.966019	11.033981	10.001849	11.035830	43
18	8.965534	9.998139	8.967394	11.032606	10.001861	11.034466	42
19	8.966893	9.998127	8.968766	11.031234	10.001872	11.033107	41
20	8.968249	9.998116	8.970133	11.029867	10.001884	11.031751	40
21	8.969600	9.998104	8.971496	11.028504	10.001896	11.030400	39
22	8.970947	9.998092	8.972855	11.027145	10.001908	11.029053	38
23	8.972289	9.998080	8.974209	11.025791	10.001920	11.027710	37
24	8.973628	9.998068	8.975560	11.024440	10.001932	11.026372	36
25	8.974962	9.998056	8.976906	11.023094	10.001944	11.025038	35
26	8.976293	9.998044	8.978248	11.021752	10.001956	11.023707	34
27	8.977619	9.998032	8.979586	11.020413	10.001968	11.022381	33
28	8.978941	9.998020	8.980921	11.019079	10.001980	11.021059	32
29	8.980259	9.998008	8.982251	11.017749	10.001992	11.019741	31
30	8.981573	9.997996	8.983577	11.016423	10.002004	11.018427	30
31	8.982883	9.997984	8.984899	11.015101	10.002016	11.017117	29
32	8.984189	9.997972	8.986217	11.013783	10.002028	11.015811	28
33	8.985491	9.997959	8.987532	11.012468	10.002041	11.014509	27
34	8.986789	9.997947	8.988842	11.011158	10.002053	11.013211	26
35	8.988083	9.997935	8.990149	11.009851	10.002065	11.011917	25
36	8.989374	9.997922	8.991451	11.008549	10.002078	11.010626	24
37	8.990660	9.997910	8.992750	11.007250	10.002090	11.009340	23
38	8.991943	9.997897	8.994045	11.005955	10.002102	11.008057	22
39	8.993222	9.997885	8.995337	11.004663	10.002115	11.006778	21
40	8.994497	9.997872	8.996624	11.003376	10.002127	11.005503	20
41	8.995768	9.997859	8.997908	11.002092	10.002141	11.004232	19
42	8.997036	9.997847	8.999188	11.000812	10.002153	11.002964	18
43	8.998299	9.997835	9.000465	10.999535	10.002165	11.001701	17
44	8.999559	9.997822	9.001737	10.998262	10.002178	11.000440	16
45	9.000816	9.997809	9.003007	10.996993	10.002191	10.999184	15
46	9.002069	9.997797	9.004272	10.995728	10.002203	10.997931	14
47	9.003318	9.997784	9.005534	10.994466	10.002216	10.996682	13
48	9.004563	9.997771	9.006792	10.993208	10.002229	10.995437	12
49	9.005805	9.997758	9.008047	10.991953	10.002242	10.994195	11
50	9.007044	9.997745	9.009298	10.990702	10.002255	10.992956	10
51	9.008278	9.997732	9.010546	10.989454	10.002267	10.991722	9
52	9.009510	9.997719	9.011790	10.988210	10.002281	10.990490	8
53	9.010737	9.997706	9.013031	10.986969	10.002294	10.989263	7
54	9.011962	9.997693	9.014268	10.985732	10.002307	10.988038	6
55	9.013182	9.997680	9.015502	10.984498	10.002320	10.986818	5
56	9.014400	9.997667	9.016732	10.983267	10.002333	10.985600	4
57	9.015613	9.997654	9.017959	10.982041	10.002346	10.984386	3
58	9.016824	9.997641	9.019183	10.980817	10.002359	10.983176	2
59	9.018031	9.997628	9.020403	10.979597	10.002372	10.981969	1
60	9.019235	9.997614	9.021620	10.978380	10.002386	10.980765	0
		Sine		Tang.		Secant.	M

84 *Degrees.*

6 Degrees.

Min.	Sine.		Tang.		Secant.		
0	9.019235	9.997614	9.021620	10.978380	10.002386	10.980765	60
1	9.020435	9.997601	9.022834	10.977166	10.002399	10.979565	59
2	9.021632	9.997588	9.024044	10.975956	10.002412	10.978368	58
3	9.022825	9.997574	9.025251	10.974749	10.002426	10.977175	57
4	9.024016	9.997561	9.026455	10.973545	10.002439	10.975984	56
5	9.025203	9.997547	9.027655	10.972345	10.002452	10.974797	55
6	9.026386	9.997534	9.028852	10.971148	10.002466	10.973613	54
7	9.027567	9.997520	9.030046	10.969954	10.002479	10.972433	53
8	9.028744	9.997507	9.031237	10.968763	10.002493	10.971256	52
9	9.029918	9.997493	9.032425	10.967575	10.002507	10.970082	51
10	9.031089	9.997480	9.033609	10.966391	10.002520	10.968911	50
11	9.032257	9.997466	9.034791	10.965209	10.002534	10.967743	49
12	9.033421	9.997452	9.035969	10.964031	10.002548	10.966579	48
13	9.034582	9.997439	9.037144	10.962856	10.002561	10.965417	47
14	9.035741	9.997425	9.038316	10.961684	10.002575	10.964259	46
15	9.036896	9.997411	9.039485	10.960515	10.002589	10.963104	45
16	9.038048	9.997397	9.040651	10.959349	10.002603	10.961952	44
17	9.039197	9.997383	9.041813	10.958187	10.002617	10.960803	43
18	9.040342	9.997369	9.042973	10.957027	10.002631	10.959658	42
19	9.041485	9.997355	9.044130	10.955870	10.002645	10.958515	41
20	9.042625	9.997341	9.045284	10.954716	10.002659	10.957375	40
21	9.043762	9.997327	9.046434	10.953566	10.002673	10.956238	39
22	9.044895	9.997313	9.047582	10.952418	10.002687	10.955105	38
23	9.046026	9.997299	9.048727	10.951273	10.002701	10.953974	37
24	9.047154	9.997285	9.049869	10.950131	10.002715	10.952846	36
25	9.048279	9.997271	9.051008	10.948992	10.002729	10.951721	35
26	9.049400	9.997257	9.052144	10.947856	10.002743	10.950599	34
27	9.050519	9.997242	9.053277	10.946723	10.002758	10.949481	33
28	9.051635	9.997228	9.054407	10.945593	10.002772	10.648365	32
29	9.052748	9.997214	9.055535	10.944465	10.002786	10.947251	31
30	9.053859	9.997199	9.056659	10.943340	10.002801	10.946141	30
31	9.054966	9.997185	9.057781	10.942219	10.002815	10.945034	29
32	9.056071	9.997170	9.058900	10.941100	10.002830	10.943929	28
33	9.057172	9.997156	9.060016	10.939984	10.002844	10.942828	27
34	9.058271	9.997141	9.061130	10.938870	10.002859	10.941729	26
35	9.059367	9.997127	9.062240	10.937760	10.002873	10.940633	25
36	9.060460	9.997112	9.063348	10.936652	10.002888	10.930540	24
37	9.061551	9.997098	9.064453	10.935547	10.002902	10.938449	23
38	9.062639	9.997083	9.065556	10.934444	10.002917	10.937361	22
39	9.063723	9.997068	9.066655	10.933345	10.002932	10.936276	21
40	9.064806	9.997053	9.067752	10.932248	10.002946	10.935194	20
41	9.065885	9.997039	9.068846	10.931153	10.002961	10.934115	19
42	8.066962	9.997024	9.069938	10.930062	10.002976	10.933018	18
43	9.068036	9.997009	9.071027	10.928973	10.002991	10.931964	17
44	9.069107	9.996994	9.072113	10.927887	10.003006	10.930893	16
45	9.070176	9.996979	9.073197	10.926803	10.003021	10.929824	15
46	9.071242	9.996964	9.074278	10.925722	10.003036	10.928758	14
47	9.072305	9.996949	9.075356	10.924644	10.003051	10.927694	13
48	6.073366	9.996934	9.076432	10.923568	10.003066	10.926634	12
49	9.074424	6.996919	9.077505	10.922450	10.003081	10.925576	11
50	9.075480	9.996904	9.078576	10.921424	10.003096	10.924520	10
51	9.076523	9.996889	9.079644	10.920356	10.003111	10.923467	9
52	9.077583	9.996874	9.080710	10.919290	10.003126	10.922417	8
53	9.078631	9.996858	9.081773	10.918227	10.003142	10.921369	7
54	9.079676	9.996843	9.082834	10.917167	10.003157	10.920324	6
55	9.080719	9.996828	9.083891	10.916109	10.003172	10.919281	5
56	0.081759	9.996812	9.084947	10.915053	10.003187	10.918241	4
57	9.082797	9.996797	9.086000	10.914000	10.003203	10.917203	3
58	[illegible]	9.996782	9.087050	10.912950	10.003218	10.916168	2
59	9.084864	9.996766	9.088098	10.911902	10.003234	10.915136	1
60	9.085894	9.996751	9.089144	10.910856	10.003249	10.914105	0
		Sine.		Tang.		Secant.	M

83 Degrees.

7 Degrees.

Min.	Sine		Tang.		Secant.		
0	9.085894	9.996751	9.089144	10.910856	10.003249	10.914105	60
1	9.086922	9.996735	9.090187	10.909813	10.003265	10.913078	59
2	9.087947	9.996720	9.091228	10.908772	10.003280	10.912053	58
3	9.088970	9.996704	9.092266	10.907734	10.003296	10.911030	57
4	9.089990	9.996688	9.093302	10.906698	10.003312	10.910010	56
5	9.091008	9.996673	9.094335	10.905664	10.003327	10.908992	55
6	9.092024	9.996657	9.095367	10.904633	10.003343	10.907976	54
7	9.093037	9.996641	9.096396	10.903604	10.003359	10.906963	53
8	9.094047	9.996625	9.097422	10.902578	10.003375	10.905953	52
9	9.095056	9.996610	9.098446	10.901554	10.003390	10.904944	51
10	9.096061	9.996594	9.099468	10.900532	10.003406	10.903938	50
11	9.097065	9.996578	9.100487	10.899513	10.003422	10.902935	49
12	9.098066	9.996562	9.101504	10.898496	10.003438	10.901934	48
13	9.099065	9.996546	9.102519	10.897481	10.003454	10.900935	47
14	9.100062	9.996530	9.103532	10.896468	10.003470	10.899938	46
15	9.101056	9.996514	9.104542	10.895458	10.003486	10.898944	45
16	9.102048	9.996498	9.105550	10.894450	10.003502	10.897952	44
17	9.103037	9.996482	9.106556	10.893444	10.003518	10.896963	43
18	9.104025	9.996465	9.107559	10.892441	10.003534	10.895975	42
19	9.105010	9.996449	9.108560	10.891440	10.003551	10.894990	41
20	9.105992	9.996433	9.109559	10.890441	10.003567	10.894008	40
21	9.106973	9.996417	9.110556	10.889444	10.003583	10.893027	39
22	9.107951	9.996400	9.111551	10.888449	10.003600	10.892049	38
23	9.108927	9.996384	9.112543	10.887457	19.003616	10.891073	37
24	9.109901	9.996368	9.113533	10.886467	10.003632	10.890099	36
25	9.110873	9.996351	9.114521	10.885479	10.003649	10.889127	35
26	9.111842	9.996335	9.115507	10.884493	10.003665	10.888158	34
27	9.112809	9.996318	9.116491	10.883509	10.003682	10.887191	33
28	9.113774	9.996302	9.117472	10.882528	10.003698	10.886226	32
29	9.114738	9.996285	9.118452	10.881548	10.003715	10.885263	31
30	9.115698	9.996269	9.119429	10.880571	10.003731	10.884302	30
31	9.116656	9.996252	9.120404	10.879596	10.003748	10.883344	29
32	9.117612	9.996235	9.121377	10.878623	10.003765	10.882387	28
33	9.118567	9.996218	9.122348	10.877652	10.003781	10.881433	27
34	9.119519	9.996202	9.123317	10.876683	10.003789	10.880481	26
35	9.120469	9.996185	9.124284	10.875716	10.003815	10.879531	25
36	9.121417	9.996168	9.125249	10.874751	10.003832	10.878583	24
37	9.122362	9.996151	9.126201	10.873789	10.003849	10.877638	23
38	9.123306	9.996134	9.127172	10.872828	10.003866	10.876694	22
39	9.124248	9.996117	9.128130	10.871870	10.003883	10.875752	21
40	9.125187	9.996100	9.129087	10.870913	10.003900	10.874813	20
41	9.126125	9.996083	9.130041	10.869959	10.003917	10.873875	19
42	9.127060	9.996066	9.130994	10.869006	10.003934	10.872949	18
43	9.127993	9.996049	9.131944	10.868056	10.003951	10.872007	17
44	9.128925	9.996032	9.132893	10.867107	10.003968	10.871075	16
45	9.129854	9.996015	9.133839	10.866161	10.003985	10.870146	15
46	9.130781	9.995998	9.134783	10.865216	10.004002	10.869210	14
47	9.131706	9.995980	9.135726	10.864274	10.004020	10.868294	13
48	9.132630	9.995963	9.136666	10.863333	10.004037	10.867370	12
49	9.133551	9.995946	9.137605	10.862395	10.004054	10.866449	11
50	9.134470	9.995928	9.138542	10.861458	10.004072	10.865530	10
51	9.135387	9.995911	9.139476	10.860524	10.004089	10.864612	9
52	9.136303	9.995894	9.140409	10.859591	10.004106	10.863697	8
53	6.137216	9.995876	9.141340	10.858660	10.004124	10.862784	7
54	8.138127	9.995859	9.142269	10.857731	10.004141	10.861872	6
55	9.139037	9.995841	9.143196	10.856804	10.004159	10.860963	5
56	9.139904	9.995823	9.144121	10.855879	10.004176	10.860055	4
57	9.140850	9.995806	9.145044	10.854956	10.004194	10.859150	3
58	9.141754	9.995788	9.145965	10.854034	10.004212	10.858246	2
59	9.142655	9.995770	9.146885	10.853115	10.004229	10.857344	1
60	9.143555	9.995753	9.147802	10.852197	10.004247	10.856445	0
		Sine.		Tang.		Secant.	M

82 Degrees.

8 *Degrees.*

Min.	Sine		Tang.		Secant.		
0	9.143555	9.995753	9.147802	10.852197	10.004247	10.856445	60
1	9.144453	9.995735	9.148718	10.851282	10.004265	10.855547	59
2	9.145349	9.995717	9.149632	10.850368	10.004283	10.854651	58
3	9.146243	9.995699	9.150544	10.849456	10.004301	10.853756	57
4	9.147136	9.995681	9.151454	10.848546	10.004318	10.852864	56
5	9.148026	9.995663	9.152363	10.847637	10 004336	10.851974	55
6	9.148915	9.995646	9.153269	10.846731	10.004354	10.851085	54
7	9.149801	9.995628	9.154174	10.845826	10.004372	10.850198	53
8	9.150686	9.995609	9.155077	10.844923	10.004390	10.849314	52
9	9.151569	9.995591	9.155978	10.844022	10.004408	10 848431	51
10	9.152451	9.995573	9.156887	10.843123	10.004427	10.847549	50
11	9.153330	9.995555	9.157775	10.842225	10.004445	10.846670	49
12	9.154208	9.995537	9.158671	10.841329	10.004463	10.845792	48
13	9.155083	9.995519	9.159565	10.840435	10.004481	10.844917	47
14	9.155957	9.995500	9.160457	10.839543	10.004499	10.844043	46
15	9.156830	9.995482	9.161347	10 838653	10.004518	10.843170	45
16	9.157700	9.995464	9.162236	10.837764	10.004536	10.842300	44
17	9.158569	9.995445	9.163123	10.836877	10.004554	10.841431	43
18	9.159435	9.995427	9.164008	10.835992	10.004573	10.840565	42
19	9.160300	9.995409	9.164892	10.835108	10.004591	10.839699	41
20	9.161164	9.995390	9.165774	10.834226	10.004610	10.838836	40
21	9.162025	9.995372	9.166654	10.833346	10.004628	10.837974	39
22	9.162885	9.995353	9.167532	10.832468	10.004647	10.837115	38
23	9.163743	9.995334	9.168499	10.831591	10.004665	10.836257	37
24	9.164600	9.995316	9.169284	10.830716	10.004684	10.835400	36
25	9.165454	9.995297	9.170157	10.829843	10.004703	10.834546	35
26	9.166307	9.995278	9.171029	10.828971	10.004721	10.833693	34
27	9.567159	9.995260	9.171899	10.828101	10.004740	10.832841	33
28	9.168008	9.995241	9.172767	10.827233	10.004759	10 831992	32
29	9.168856	9.995222	9.173634	10.826366	10.004778	10.831144	31
30	9.169702	9.995203	9.174499	10.825501	10.004797	10.830298	30
31	9.170546	9.995184	9.175362	10.824638	10.004816	10.829453	29
32	9.171389	9.995165	9.176224	10.823776	10.004835	10.828611	28
33	9.172230	9.995146	9.177084	10.822916	10.004854	10.827769	27
34	9.173070	9.995127	9.177942	10.822057	10.004873	10.826930	26
35	9.173908	9.995108	9.178799	10.821201	10.004892	10.826092	25
36	9.174744	9.995089	9.179655	10.820345	10.004911	10.825256	24
37	9.175578	9.995070	9.180508	10.819492	10.004930	10.824422	23
38	9.176411	9.995051	9.181360	10.818640	10.004949	10.823589	22
39	9.177242	9.995032	9.182211	10.817789	10.004968	10.822757	21
40	9.178072	9.995013	9.183059	10.816940	10.004987	10.821928	20
41	9.178900	9.994993	9.183907	10.816093	10.005007	10.821100	19
42	9.179726	9.994974	9.184752	10.815247	10.005026	10.820273	18
43	9.180551	9.994955	9.185597	10.814403	10.005045	10.819489	17
44	9.181374	9.994935	9.186439	10.813561	10.005065	10 818626	16
45	9.182196	9.994916	9.187280	10.812720	10.005084	10.817804	15
46	9.183016	9.994896	9.188120	10.811880	10.005104	10.816984	14
47	9.183834	9.994877	9.188957	10.811042	10.005123	10.816166	13
48	9.184651	9.994857	9.189795	10.810206	10.005143	10.815349	12
49	9.185466	9.994838	9.190629	10.809371	10.005162	10.814533	11
50	9.186280	9.994818	9.191462	10.808538	10.005182	10.813720	10
51	9.187092	9.994798	9.192294	10.807706	10.005201	10.812908	9
52	9.187903	9.994779	9.193124	10.806876	10.005221	10.812007	8
53	9.188712	9.994759	9.193953	10.806047	10.005241	10.811288	7
54	9.189519	9.994739	9.194780	10 805220	10 005261	10.810480	6
55	9.190325	9.994719	9.195606	10.804394	10.005280	10.809675	5
56	9.191130	9.994700	9.196430	10.803570	10.005309	10.808870	4
57	9.191933	9.994680	9.197253	10.802747	10.005320	10.808067	3
58	9.192734	9.994660	9.198074	10.801926	10.005340	10.807266	2
59	9.193534	9.994640	9.198894	10.801106	10.005360	10.806466	1
60	9.194332	9.994620	9.199712	10.800287	10.005380	10.805668	0
		Sine		Tang.		Secant.	M

81 *Degrees.*

9 *Degrees.*

Min.	Sine		Tang.		Secant.		
0	9.194332	9.994620	9.199712	10.800287	10.005380	10.805668	60
1	9.195129	9.994600	9.200529	10.799471	10.005400	10.804871	59
2	9.195925	9.994580	9.201345	10.798655	10.005420	10.804075	58
3	9.196719	9.994560	9.202159	10.797841	10.005440	10.803281	57
4	9.197511	9.994540	9.202971	10.797029	10.005460	10.802489	56
5	9.198302	9.994519	9.203782	10.796217	10.005481	10.801689	55
6	9.199091	9.994499	9.204592	10.795408	10.005501	10.800909	54
7	9.199879	9.994479	9.205400	10.794600	10.005521	10.800121	53
8	9.200666	9.994459	9.206207	10.793793	10.005541	10.799334	52
9	9.201451	9.994438	9.207013	10.792987	10.005562	10.798549	51
10	9.202234	9.994418	9.207816	10.792183	10.005581	10.797765	50
11	9.203017	9.994397	9.208619	10.791381	10.005602	10.756983	49
12	9.203797	9.994377	9.209420	10.790580	10.005623	10.796203	48
13	9.204577	9.994357	9.210220	10.789780	10.005643	10.795423	47
14	9.205354	9.994336	9.211018	10.788982	10.005664	10.794645	46
15	9.206131	9.994316	9.211815	10.788185	10.005684	10.793869	45
16	9.206906	9.994295	9.212611	10.787389	10.005705	10.793094	44
17	9.207679	9.994274	9.213405	10.786595	10.005723	10.792320	43
18	9.208452	9.994254	9.214198	10.785802	10.005746	10.791548	42
19	9.209222	9.994233	9.214989	10.785011	10.005767	10.790778	41
20	9.209992	9.994212	9.215779	10.784220	10.005788	10.790008	40
21	9.210760	9.994191	9.216568	10.783432	10.005809	10.789240	39
22	9.211526	9.994171	9.217356	10.782644	10.005829	10.788474	38
23	9.212291	9.994150	9.218142	10.781858	10.005850	10.787709	37
24	9.213055	9.994129	9.218926	10.781074	10.005871	10.786645	36
25	9.213818	9.994108	9.219710	10.780290	10.005892	10.786182	35
26	9.214579	9.994087	9.220492	10.779508	10.005913	10.785421	34
27	9.215338	9.994066	9.221272	10.778728	10.005934	10.784662	33
28	9.216097	9.994045	9.222052	10.777948	10.005955	10.783903	32
29	9.216854	9.994024	9.222830	10.777170	10.005976	10.783146	31
30	9.217609	9.994003	9 223606	10.776393	10.005997	10.782301	30
31	9.218363	9.993981	9.224328	10.775618	10.005018	10.781636	2c
32	9.219116	9.993960	9.225156	10.774844	10.006040	10.780884	28
33	9.219868	9.993939	9.225929	10.774071	10.006061	10.780132	27
34	9.220618	9.993918	9.226700	10.773300	10 006082	10.779382	26
35	9.221367	9.993896	9.227471	10.772529	10.006103	10.778633	25
36	9.222115	9.993875	9.228239	10.771761	10.006125	10.777885	24
37	9.222861	9.993854	9.229007	10.770993	10.006146	10 777139	23
38	9.223606	9.993832	9.229773	10.770226	10.006168	10.776394	22
39	9.224349	9.993811	9.230539	10.769461	10.006189	10 775650	21
40	9.225092	9.993789	9.231302	10.768698	10 006211	10.774908	20
41	9.225833	9.993768	9.232065	10.767935	10.006232	10.774167	19
42	9.226571	9.993746	9.232826	10.767174	10.006254	10.773427	18
43	9.227311	9.993725	9.233586	10.766414	10.006275	10.772689	17
44	9.228048	9.993703	9.234345	10.765655	10.006297	10.771952	16
45	9 228784	9.993681	9.235103	10.764897	10.006319	10.771216	15
46	9.229518	9.993660	9.235859	10.764141	10.006340	10.770481	14
47	9.230252	9.993638	9.236614	10.763386	10.006362	10.769748	13
48	9.230948	7.993616	9.237368	10.762632	10.006384	10.769016	12
49	9.231714	9.993594	9.238120	10.761880	10.006406	10.768285	11
50	9.232444	9.993572	9.238872	10.761128	10.006428	10.767556	10
51	9.233172	9.993550	9.239622	10.760378	10.006450	10.766828	9
52	9.233899	9.993528	9.240371	10.759629	10.006471	10.766101	8
53	9.234625	9.993506	9.241118	10.758881	10.006493	10.765375	7
54	9.235349	9.993484	9.241865	10.758135	10.006516	10.764651	6
55	9.236073	9.993462	9.242610	10 757390	10.006528	10.763927	5
56	9.236795	9.993440	9.243354	10.756646	10.006560	10.763205	4
57	9.237515	9.993418	9.244097	10.755903	10.006582	10.762485	3
58	9 238235	9.993396	9.244839	10.755161	10.006604	10.761765	2
59	9 238953	9.993374	9.245579	10 754421	10.006626	10.761047	1
60	9.239670	9.993351	9.246319	10.753681	10 006648	10.760330	0
		Sine		Tang.		Secant.	M

80 *Degrees.*

10 *Degrees.*

Min.	Sine.		Tang.		Secant.		
0	9.239670	9.993351	9.246319	10.753681	10.006648	10.760330	60
1	9.240386	9.993329	9.247057	10.752943	10.006671	10.759614	59
2	9.241101	9.993307	9.247794	10.752206	10.006693	10.758899	58
3	9.241814	9.993284	9.248530	10.751470	10.006715	10.758186	57
4	9.242526	9.993262	9.249264	10.750736	10.006738	10.757474	56
5	9.243237	9.993240	9.249998	10.750:01	10.006760	10.756763	55
6	9.243947	9.993217	9.25073	10.749270	10.006783	10.756053	54
7	9.244656	9.993195	9.251461	10.748539	10.006805	10.755344	53
8	9.245363	9.993172	9.252191	10.747809	10.006828	10.754637	52
9	9.246069	9.993149	9.252920	10.747080	10.006851	10.753930	51
10	9.246775	9.993127	9.253648	10.746352	10.006873	10.753225	50
11	9.247478	9.993104	9.254374	10.745626	10.006896	10.752522	49
12	9.248181	9.993081	9.255100	10.744900	10.006919	10.751819	48
13	9.248883	9.993059	9.255824	10.744176	10.006941	10.751117	47
14	9.249583	9.993036	9.256547	10.743453	10.006964	10.750417	46
15	9.250282	9.993013	9.257269	10.742731	10.006987	10.749718	45
16	9.250980	9.992990	9.257990	10.742010	10.007010	10.749020	44
17	9.251677	9.992967	9.258710	10.741290	10.007033	10.748323	43
18	9.252373	9.992944	9.259428	10.740571	10.007056	10.747627	42
19	9.253067	9.992921	9.260146	10.739854	10.007079	10.746932	41
20	9.253761	9.992898	9.260862	10.739137	10.007102	10.746239	40
21	9.254453	9.992875	9.261578	10.738422	10.007125	10.745547	39
22	9.255144	9.992852	9.262292	10.737708	10.007148	10.744856	38
23	9.255834	9.992829	9.263005	10.736995	10.007171	10.744166	37
24	9.256523	9.992806	9.263717	10.736283	10.007194	10.743477	36
25	9.257211	9.992783	9.264428	10.735572	10.007217	10.742789	35
26	9.257898	9.992759	9.265138	10.734862	10.007240	10.742102	34
27	9.258583	9.992736	9.265847	10.734153	10.007264	10.741417	33
28	9.259268	9.992713	9.266555	10.733445	10.007287	10.740732	32
29	9.259951	9.992689	9.267261	10.732739	10.007310	10.740049	31
30	9.260633	9.992666	9.267967	10.732033	10.007334	10.739367	30
31	9.261314	9.992643	9.268671	10.731329	10.007357	10.738686	29
32	9.261994	9.992619	9.269375	10.730625	10.007381	10.738006	28
33	9.262673	9.992596	9.270077	10.729923	10.007404	10.737327	27
34	9.263351	9.992572	9.270779	10.729221	10.007428	10.736649	26
35	9.264027	9.992549	9.271479	10.728521	10.007451	10.735973	25
36	9.264703	9.992525	9.272178	10.727822	10.007475	10.735297	24
37	9.265377	9.992501	9.272876	10.727124	10.007499	10.734622	23
38	9.266051	9.992478	9.273573	10.726427	10.007522	10.733949	22
39	9.266723	9.992454	9.274269	10.725731	10.007546	10.733277	21
40	9.267394	9.992430	9.274964	10.725036	10.007570	10.732605	20
41	9.268065	9.992406	9.275658	10.724342	10.007594	10.731935	19
42	9.268734	9.992382	9.276351	10.723649	10.007618	10.731266	18
43	9.269402	9.992358	9.277043	10.722957	10.007641	10.730598	17
44	9.270069	9.992335	9.277734	10.722266	10.007665	10.729931	16
45	9.270735	9.992311	9.278424	10.721576	10.007689	10.729265	15
46	9.271400	9.992287	9.279113	10.720887	10.007713	10.728600	14
47	9.272063	9.992263	9.279801	10.720199	10.007737	10.727936	13
48	9.272726	9.992238	9.280488	10.719512	10.007761	10.727274	12
49	9.273388	9.992214	9.281174	10.718826	10.007786	10.726612	11
50	9.274049	9.992190	9.281858	10.718141	10.007810	10.725951	10
51	9.274708	9.992166	9.282542	10.717458	10.007834	10.725292	9
52	9.275367	9.992142	9.283225	10.716775	10.007858	10.724633	8
53	9.276024	9.992117	9.283907	10.716093	10.007882	10.723975	7
54	9.276681	9.992093	9.284588	10.715412	10.007907	10.723319	6
55	9.277337	9.992069	9.285268	10.714732	10.007931	10.722663	5
56	9.277991	9.992044	9.285947	10.714053	10.007955	10.722009	4
57	9.278644	9.992020	9.286624	10.713375	10.007980	10.721355	3
58	9.279297	9.991996	9.287301	10.712699	10.008004	10.720703	2
59	9.289948	9.991971	9.287977	10.712023	10.008029	10.720052	1
60	9.280599	9.991947	9.288652	10.711348	10.008053	10.719401	0
		Sine.		Tang.		Secant.	M

79 *Degrees.*

11 *Degrees.*

Min.	Sine		Tang.		Secant.	
0	9.280593	9.991947	9.288652	10.711348	10.008053	10.7194 1
1	9.281248	9.991922	9.289326	10.710674	10.008078	[illegible]
2	9.281897	9.991897	9.289999	10.710001	10.008103	[illegible]
3	9.282544	9.991873	9.290671	10.709329	10.008127	[illegible]
4	9.283190	9.991848	9.291342	10.708658	10.008152	[illegible]
5	9.283836	9.991823	9.292013	10.707987	10.008177	[illegible]
6	9.284480	9.991799	9.292682	10.707318	10.008201	[illegible]
7	9.285124	9.991774	9.293350	10.706650	10.008226	[illegible]
8	9.285766	9.991749	9.294017	10.705983	10.008251	[illegible]
9	9.286408	9.991724	9.294684	10.705316	10.008276	[illegible]
10	9.287048	9.991699	9.295349	10.704651	10.008301	[illegible]
11	9.287687	9.991674	9.296013	10.703987	10.008326	[illegible]
12	9.288326	9.991649	9.296677	10.703323	10.008351	[illegible]
13	9.288964	9.991624	9.297339	10.702661	10.008376	[illegible]
14	9.289600	9.991599	9.298001	10.701999	10.008401	[illegible]
15	9.290236	9.991574	9.298662	10.701338	10.008426	[illegible]
16	9.290870	9.991549	9.299322	10.700678	10.008451	[illegible]
17	9.291504	9.991524	9.299980	10.700020	10.008476	[illegible]
18	9.292137	9.991498	9.300638	10.699362	10.008502	[illegible]
19	9.292768	9.991473	9.301295	10.698705	10.008527	[illegible]
20	9.293399	9.991448	9.301951	10.698049	10.008552	[illegible]
21	9.294029	9.991422	9.302607	10.697393	10.008577	[illegible]
22	9.294658	9.991397	9.303261	10.696739	10.008603	[illegible]
23	9.295286	9.991372	9.303914	10.696086	10.008628	[illegible]
24	9.295913	9.991346	9.304567	10.695433	10.008654	[illegible]
25	9.296539	9.991321	9.305218	10.694782	10.008679	[illegible]
26	9.297164	9.991295	9.305869	10.694131	10.008705	[illegible]
27	9.297788	9.991270	9.306519	10.693481	10.008730	[illegible]
28	9.298412	9.991244	9.307167	10.692832	10.008756	[illegible]
29	9.299034	9.991218	9.307815	10.692184	10.008782	[illegible]
30	9.299655	9.991193	9.308463	10.691537	10.008807	[illegible]
31	9.300267	9.991167	9.309109	10.690891	10.008833	[illegible]
32	9.300895	9.991141	9.309754	10.690246	10.008859	[illegible]
33	9.301514	9.991115	9.310398	10.6896 1	10.008885	[illegible]
34	9.302132	9.991090	9.311042	10.688958	10.008910	[illegible]
35	9.302748	9.991064	9.311685	10.688315	10.008936	[illegible]
36	9.303364	9.991038	9.312327	10.687673	10.008962	[illegible]
37	9.303979	9.991012	9.312967	10.687032	10.008988	[illegible]
38	9.304593	9.990986	9.313608	10.686392	10.009014	[illegible]
39	9.305207	9.990960	9.314247	10.685753	10.009041	[illegible]
40	9.305819	9.990934	9.314885	10.685115	10.009066	[illegible]
41	9.306430	9.990908	9.315523	10.684477	10.009092	[illegible]
42	9.307041	9.990881	9.316159	10.683841	10.009118	[illegible]
43	9.307650	9.990855	9.316795	10.683205	10.009145	[illegible]
44	9.308259	9.990829	9.317430	10.682570	10.009171	[illegible]
45	9.308867	9.990803	9.318064	10.681936	10.009197	[illegible]
46	9.309474	9.990777	9.318697	10.681303	10.009223	[illegible]
47	9.310080	9.990750	9.319329	10.680670	10.009250	[illegible]
48	9.310685	9.990724	9.319961	10.680039	10.009276	[illegible]
49	9.311289	9.990697	9.320592	10.679408	10.009303	10.688711
50	9.311893	9.990671	9.321222	10.678778	10.009329	10.688107
51	9.312495	9.990644	9.321851	10.678149	10.009355	10.687505
52	9.313097	9.990618	9.322479	10.677521	10.009382	10.686903
53	9.313698	9.990591	9.323106	10.676894	10.009409	10.686302
54	9.314297	9.990565	9.323733	10.676267	10.009435	10.685702
55	9.314896	9.990538	9.324358	10.675642	10.009462	10.685103
56	9.315495	9.990511	9.324983	10.675017	10.009488	10.684505
57	9.316092	9.990485	9.325607	10.674393	10.009515	10.683908
58	9.316688	9.990458	9.326230	10.673769	10.009542	10.683311
59	9.317284	9.990431	9.326853	10.673147	10.009569	10.682716
60	9.317879	9.990404	9.327474	10.672525	10.009596	10.682121
		Sine.		Tang.		Secant.

78 *Degrees.*

12 Degrees.

Min.	Sine		Tang.		Secant.		
0	9.317879	9.990464	9.327474	10.672525	10.009596	10.682121	60
1	9.318473	9.990377	9.328095	10.671905	10.009622	10.681527	59
2	9.319066	9.990351	9.328715	10.671285	10.009649	10.680934	58
3	9.319658	9.990324	9.329334	10.670665	10.009676	10.680342	57
4	9.320249	9.990997	9.329953	10.670047	10.009703	10.679750	56
5	9.320840	9.990270	9.330570	10.669943	10.009730	10.679160	55
6	9.321430	9.990243	9.331187	10.668813	10.009757	10.678570	54
7	9.322019	9.990215	9.331803	10.668197	10.009784	10.677981	53
8	9.322607	9.990188	9.332418	10.667582	10.009812	10.677393	52
9	9.323194	9.990161	9.333033	10.666967	10.009839	10.676806	51
10	9.323780	9.990134	9.333646	10.666354	10.009866	10.676220	50
11	9.324366	9.990107	9.334259	10.665741	10.009893	10.675634	49
12	9.324950	9.990079	9.334871	10.665129	10.009921	10.675049	48
13	9.325534	9.990052	9.335482	10.664518	10,009948	10.674466	47
14	9.326117	9.990025	9.336093	10.663907	10.009975	10.673883	46
15	9.326700	9.989997	9.336702	10.663297	10.010003	10.673300	45
16	9.327281	9.989970	9.337311	10.662689	10.010030	10.672719	44
17	9.327862	9.989942	9.337919	10.662081	10.010058	10.672138	43
18	9.328442	9.989915	9.338527	10.661473	10.010085	10.671558	42
19	9.329021	9.989887	9.339133	10.660867	10.010113	10.670979	41
20	9.329599	9.989860	9.339739	10.660261	10.010140	10.670401	40
21	9.330176	9.989832	9.340344	10.659656	10.010169	10.669824	39
22	9.330753	9.989804	9.340948	10.659052	10.010196	10.669247	38
23	9.331328	9.989777	9.341552	10.658448	10.010223	10.668671	37
24	9.331903	9.989749	9.342155	10.657845	10.010251	10.668096	36
25	9.332478	9.989721	9.342757	10.657243	10.010279	10 667522	35
26	9.333051	9.989693	9.343358	10.656642	10.010307	10.666949	34
27	9.333624	9.989665	9.343958	10.656042	10.010335	10.666376	33
28	9.334195	9.989637	9.344558	10.655442	10.010363	10.665804	32
29	9.334766	9.989609	9.345157	10.654843	10.010390	10.665233	31
30	9.335337	9.989581	9.345755	10.654245	10.010418	10.664663	30
31	9.335906	9.989553	9.346353	10.653647	10.010446	10.664094	29
32	9.336475	9.989525	9.346949	10.653051	10.010475	10.663525	28
33	9.337043	9.989497	9.347545	10.652455	10.010503	10.662957	27
34	9.337610	9.989469	9.348141	10.651859	10.010531	10.662390	26
35	9.338176	9.989441	9.348735	10.651265	10.010559	10.661827	25
36	9.338742	9.989413	9.349329	10.650671	10.010587	10.661258	24
37	9.339306	9.989384	9.349922	10.650078	10.010615	10.660693	23
38	9.339871	9.989356	9.350514	10.649486	10.010644	10.660129	22
39	9.340434	9.989328	9.351106	10.648894	10.010672	10.659566	21
40	9.340996	9.989299	9.351697	10.648303	10.010700	10.659004	20
41	9.341558	9.989271	9.352287	10.647713	10.010729	10.658442	19
42	9.342119	9.989243	9.352876	10.647124	10.010757	10.657881	18
43	9.342679	9.989214	9.353465	10.646535	10.010786	10.657321	17
44	9.343239	9.989186	9.354053	10.645947	10.010814	10.656761	16
45	9.343797	9.989157	9.354640	10.645360	10.010843	10.656203	15
46	9.344355	9.989128	9.355227	10.644773	10.010872	10.655645	14
47	9.344912	9.989100	9.355813	10.644187	10.010900	10.655089	13
48	9.345469	9.989071	9.356398	10.643602	10.010929	10.654531	12
49	9.346024	9.989042	9.356982	10.643018	10.010958	10.653975	11
50	9.346579	9.989014	9.357566	10.642434	10.010986	10.653421	10
51	9.347134	9.988985	9.358149	10.641851	10.011015	10.652866	9
52	9.347687	9.988956	9.358731	10.641269	10.011044	10.652313	8
53	9.348240	9.988927	9.359313	10.640687	10.011073	10.651760	7
54	9.348792	9.988898	9.359893	10.640106	10.011102	10.651208	6
55	9.349343	9.988869	9.360474	10.639526	10.011131	10.650656	5
56	9.349893	9.988840	9.361053	10.638947	10.011160	10 650107	4
57	9.350443	9.988811	9.361632	10.638368	10 011189	10.649557	3
58	9.350992	9.988782	9.362210	10.637790	10.011218	10.649008	2
59	9.351540	9.988753	9.362787	10.637213	10.011247	10.648459	1
60	9.352088	9.988724	9.363364	10.636636	10.011276	10.647912	0
		Sine		Tang.		Secant.	M

77 Degrees.

13 Degrees.

Min.	Sine		Tang.		Secant.		
0	9.352088	9.988724	9.363364	10.636636	10.011276	10.647912	60
1	9.352635	9.988695	9.363940	10.636060	10 011305	10.647365	59
2	9.353181	9.988665	9.364515	10.635484	10.011334	10.646819	58
3	9.353726	9.988636	9.365090	10.634910	10.011364	10.646274	57
4	9.354272	9.988607	9.365664	10.634336	10.011393	10.645729	56
5	9.354815	9.988578	9.366237	10.633763	10.011422	10.645185	55
6	9.355358	9.988548	9.366810	10.633190	10.011452	10.644642	54
7	9.355901	9.988519	9.367382	10.632618	10.011481	10.644099	53
8	9.356443	9.988489	9.367953	10.632047	10 011511	10.643557	52
9	9.356984	9.988460	9.368524	10.631476	10.011540	10.643016	51
10	9.357524	9.988430	9.369094	10.630906	10.011570	10.642476	50
11	9.358064	9,988401	9.369663	10.630337	10.011599	10.641936	49
12	9.358603	9.988371	9.370231	10.629768	10.011629	10.641397	48
13	9.359141	9.988341	9.370799	10.629201	10.011658	10.640859	47
14	9.359678	9.988312	9.371367	10.628633	10.011688	10.640321	46
15	6.360215	9.988282	9.371933	10.628067	10.011718	10.639785	45
16	9.360751	9.988252	9.372499	10.627501	10.011748	10.639248	44
17	9.361287	9.988222	9.373064	10.626935	10.011777	10.638713	43
18	9.361822	9.988193	9.373629	10.626371	10.011807	10.638178	42
19	9.362356	9.988163	9.374193	10.625807	10.011837	10.637644	41
20	9.362889	9.988133	9.374756	10.625244	10.011867	10.637111	40
21	9.363422	9.988103	9.375319	10.624681	10.011897	10.636578	39
22	9.363954	9.988073	9.375881	10.624119	10.011927	10.636046	38
23	9.364485	9.988043	9.376442	10.623558	10.011957	10.635515	37
24	9.365016	9.988013	9.377003	10.622947	10.011987	10.634984	36
25	9.365546	9.987983	9.377563	19.622437	10.012017	10.634454	35
26	9.366075	9.987952	9.378122	10.621877	10.012047	10.633925	34
27	9.366604	9.987922	9.378681	10.621319	10.012078	10.633396	33
28	9.367131	9.987892	9.379239	10.620761	10.012108	10.632868	32
29	9.367659	9.987862	9.379797	10.620203	10.012138	10.632341	31
30	9.368185	9.987831	9.380354	10.619646	10.012168	10.631815	30
31	9.368711	9.987801	9.380910	10.619090	10.012199	10.631289	29
32	9.369236	9.987771	9.381465	10.618534	10.012229	10.630764	28
33	9.369761	9.987740	9.382020	10.617979	10.012260	10.630239	27
34	9.370285	9.987710	9.382575	10,617425	10.012390	10.622715	26
35	9.370808	9.987679	9.383128	10.616871	10.012321	10.629192	25
36	9.371330	9.987649	9.383682	10.616318	10.012351	10.628670	24
37	9.371852	9.987618	9.384234	10.615766	10.012382	10.628148	23
38	9.372373	9.987588	9.384786	10.615214	10.012412	10.627626	22
39	9.372894	9.987557	9.385337	10.614663	10.012443	10.627106	21
40	9.373414	9.987526	9.385888	10.614112	10.012474	10.626586	20
41	9.373933	9.987495	9.386438	10.613562	10.012504	10.626067	19
42	9.374452	9.987465	9.386987	10.613013	10.012535	10.925548	18
43	9.374970	9.987434	9.387536	10.612464	10.012566	10.625030	17
44	9.375487	9.987403	9.388084	10.611916	10.012597	10.624513	16
45	9.376003	9.987372	9.388631	10.611369	10.012628	10.623997	15
46	9.376519	9.987341	9.389178	10.610822	10.012659	10.623481	14
47	9.377035	9.987310	9.389824	10.610276	10.012690	10.922965	13
48	9.377549	9.987279	9.390270	10.609730	10.012721	10.622451	12
49	9.378063	9.987248	9.390815	10.609185	10.012752	10.621937	11
50	9.378577	9.987217	9.391359	10.608640	10.012783	10.621423	10
51	9.379089	9.987186	9.391903	10.608097	10.012814	10.620911	9
52	9.379601	9.987155	9.392447	10.607553	10.012845	10.620398	8
53	9.380113	9.987124	9.392989	10.607011	10.012876	10.619887	7
54	9.380624	9.987092	9.393531	10.606449	10.012908	10.619376	6
55	6.381134	9.987061	9.394073	10.605927	10.012939	10.618866	5
56	9.381643	9,987030	9.394614	10.605386	10.012970	10.618357	4
57	9,382152	9.986998	9.395154	10.604846	10.013002	10.617848	3
58	9.382660	9.986967	9.395693	10.604306	10.013033	10.617339	2
59	9.383168	9.986936	9.396233	10.603767	10.013064	10.616832	1
60	9.383675	9.986904	9.396771	10.603229	10.013096	10.616325	0
		Sine		Tang.		Secant.	M

76 Degrees.

14 *Degrees.*

Min.	Sine.		Tang.		Secant.		
0	9.383675	9.986904	9.396771	10.603229	10.013096	10.616325	60
1	9.384181	9.986873	9.397309	10.602691	10.013127	10.615818	59
2	9.384687	9.986841	9.397846	10.602154	10.013159	10.615313	58
3	9.385192	9.986809	9.398383	10.601617	10.013191	10.614808	57
4	9.385697	9.986778	9.398919	10.601081	10.013222	10.614303	56
5	9.386201	9.986746	9.399455	10.600545	10.013254	10.613799	55
6	9.386704	9.986714	9.399990	10.600010	10.013286	10.613296	54
7	9.387207	9.986683	9.400524	10.599479	10.013317	10.612793	53
8	9.387709	9.986651	9.401058	10.598942	10.013349	10.612291	52
9	9.388210	9.986619	9.401591	10.598409	10.013381	10.611790	51
10	9.388711	9.986587	9.402124	10.597876	10.013413	10.611289	50
11	9.389211	9.986555	9.402656	10.597344	10.013445	10.610789	49
12	9.389711	9.986523	9.403187	10.596813	10.013477	10.610289	48
13	9.390210	9.986491	9.403718	10.596282	10.013509	10.609790	47
14	9.390708	9.986459	9.404249	10.595751	10.013541	10.609292	46
15	9.391206	9.986427	9.404778	10.595222	10.013573	10.608794	45
16	9.391703	9.986395	9.405308	10.594692	10.013605	10.608297	44
17	9.392199	9.986363	9.405836	10.594164	10.013637	10.607801	43
18	9.392695	9.986331	9.406364	10.593636	10.013669	10.607305	42
19	9.393190	9.986299	9.406892	10.593108	10.013701	10.606809	41
20	9.393685	9.986266	9.407419	10.592581	10.013734	10.606315	40
21	9.394179	9.986234	9.407945	10.592055	10.013766	10.605821	39
22	9.394673	9.986202	9.408471	10.591529	10.01379	10.605327	38
23	9.395166	9.986169	9.408996	10.591003	10.013831	10.604834	37
24	9.395658	9.986137	9.409521	10.590479	10.013863	10.604342	36
25	9.396150	9.986104	9.410045	10.589955	10.013895	10.603850	35
26	9.396641	9.986072	9.410569	10.589431	10.013928	10.603359	34
27	9.397 31	9.986039	9.411092	10.588908	10.01396	10.602868	33
28	9.397621	9.986007	9.411615	10.588385	10.013993	10.602378	32
29	9.398111	9.985974	9.412137	10.587863	10.014026	10.601889	31
30	9.398600	9.985942	9.412658	10.587342	10.014058	10.601400	30
31	9.399088	9.985909	9.413179	10.586821	10.014091	10.600912	29
32	9.399575	9.985876	9.413699	10.586301	10.014124	10.600425	28
33	9.400062	9.985843	9.414219	10.585781	10.014157	10.599937	27
34	9.400549	9.985811	9.414738	10.585262	10.014189	10.599451	26
35	9.401035	9.985778	9.415257	10.584743	10.014222	10.598965	25
36	9.401520	9.985745	9.415775	10.584225	10.014255	10.598480	24
37	9.402005	9.985712	9.416293	10.583707	10.014288	10.597995	23
38	9.402489	9.985679	9.416810	10.583190	10.014321	10.597511	22
39	9.402973	9.985646	9.417326	10.582673	10.014354	10.597028	21
40	9.403455	9.985613	9.417842	10.582157	10.914387	10.596545	20
41	9.403938	9.985580	9.418358	10.581642	10.014420	10.596062	19
42	9.404410	9.985547	9.418873	10.581127	10.014453	10.595580	18
43	9.404901	9.985513	9.419387	10.580613	10.014486	10.595099	17
44	9.405382	9.985480	9.419901	10.580099	10.014520	10.594618	16
45	9.405862	9.985447	9.420415	10.579585	10.014553	10.594138	15
46	9.406341	9.985414	9.420927	10.579072	10.014586	10.593659	14
47	9.406820	9.985380	9.421440	10.578560	10.014619	10.593180	13
48	9.407299	9.985347	9.421951	10.578048	10.014653	10.592701	12
49	9.407777	9.985314	9.422463	10.577537	10.014686	10.592223	11
50	9.408254	9.985280	9.422973	10.577026	10.014720	10.591746	10
51	9.408731	9.985247	9.423484	10.576516	10.014753	10.591269	9
52	9.409207	9.985213	9.423993	10.576006	10.014787	10.590793	8
53	9.409682	9.985180	9.424503	10.575497	10.014820	10.590318	7
54	9.410157	9.985146	9.425011	10.574989	10.014854	10.589842	6
55	9.410632	9.985112	9.425519	10.574481	10.014887	10.589368	5
56	9.411106	9.985079	9.426027	10.573973	10.014921	10.588894	4
57	9.411579	9.985045	9.426534	10.573466	10.014955	10.588421	3
58	9.412052	9.985011	9.427041	10.572959	10.014989	10.587948	2
59	9.412524	9.984978	9.427547	10.572453	10.015022	10.587475	1
60	9.412996	9.984944	9.428052	10.571947	10.015056	10.587004	0
		Sine.		Tang.		Secant.	M

75 *Degrees.*

15 *Degrees.*

Min.	Sine		Tang.		Secant.		
0	9.412996	9.984944	9.428052	10.571947	10.015036	10.587004	60
1	9.413467	9.984910	9.428557	10.571442	10.015090	10.586533	59
2	9.413938	9.984876	9.429062	10.570938	10.015124	10.586062	58
3	9.414408	9.984842	9.429566	10.570434	10.015158	10.585592	57
4	9.414878	9.984808	9.430070	10.569930	10.015192	10.585122	56
5	9.415347	9.984774	9.430573	10.569427	10.015226	10.584653	55
6	9.415815	9.984740	9.431075	10.568925	10.015260	10.584185	54
7	9.416283	9.984706	9.431577	10.568423	10.015294	10.583717	53
8	9.416751	9.984672	9.432079	10.567921	10.015328	10.583249	52
9	9.417217	9.984637	9.432580	10.567420	10.015362	10.582783	51
10	9.417684	9.984603	9.433080	10.566920	10.015397	10.582316	50
11	9.418149	9.984569	9.433580	10.566419	10.015431	10.581850	49
12	9.418615	9.984535	9.434080	10.565920	10.015465	10.581385	48
13	9.419079	9.984500	9.434579	10.565421	10.015500	10.580920	47
14	9.419544	9.984466	9.435078	10.564922	10.015534	10.580456	46
15	9.420007	9.984432	9.435576	10.564424	10.015568	10.579993	45
16	9.420470	9.984397	9.436073	10.563927	10.015603	10.579530	44
17	9.420933	9.984363	9.436570	10.563430	10.015637	10.579067	43
18	9.421395	9.984328	9.437067	10.562933	10.015672	10.578605	42
19	9.421857	9.984293	9.437563	10.562437	10.015706	10.578143	41
20	9.422318	9.984259	9.438059	10.561941	10.015741	10.577682	40
21	9.422778	9.984224	9.438554	10.561446	10.015776	10.577222	39
22	9.423238	9.984189	9.439048	10.560951	10.015810	10.576762	38
23	9.423697	9.984155	9.439543	10.560457	10.015845	10.576303	37
24	9.424156	9.984120	9.440036	10.559964	10.015880	10.575844	36
25	9.424615	9.984085	9.440529	10.559470	10.015915	10.575385	35
26	9.425073	9.984050	9.441022	10.558978	10.015950	10.574927	34
27	9.425530	9.984015	9.441514	10.558485	10.015985	10.574470	33
28	9.425987	9.983980	9.442006	10.557994	10.016019	10.574013	32
29	9.426443	9.983945	9.442497	10.557502	10.016054	10.573557	31
30	9.426899	9.983910	9.442988	10.557012	10.016089	10.573101	30
31	9.427354	9.983875	9.443479	10.556521	10.016124	10.572646	29
32	9.427809	9.983840	9.443968	10.556031	10.016160	10.572191	28
33	9.428263	9.983805	9.444458	10.555542	10.016195	10.571737	27
34	9.428717	9.983770	9.444947	10.555053	10.016230	10.571283	26
35	9.429170	9.983735	9.445435	10.554565	10.016265	10.570830	25
36	9.429623	9.983700	9.445923	10.554077	10.016300	10.570377	24
37	9.430075	9.983664	9.446411	10.553589	10.016336	10.569925	23
38	9.430527	9.983629	9.446898	10.553102	10.016371	10.569473	22
39	9.430978	9.983594	9.447384	10.552616	10.016406	10.569022	21
40	9.431429	9.983558	9.447870	10.552130	10.016442	10.568571	20
41	9.431879	9.983523	9.448356	10.551644	10.016477	10.568121	19
42	9.432328	9.983487	9.448841	10.551159	10.016513	10.567671	18
43	9.432778	9.983452	9.449326	10.550674	10.016548	10.567222	17
44	9.433226	9.983416	9.449810	10.550190	10.016584	10.566774	16
45	9.433675	9.983380	9.450294	10.549706	10.016619	10.566325	15
46	9.434122	9.983345	9.450777	10.549223	10.016655	10.565878	14
47	9.434569	9.983309	9.451260	10.548740	10.016691	10.565431	13
48	9.435016	9.983273	9.451743	10.548257	10.016726	10.564984	12
49	9.435462	9.983238	9.452225	10.547775	10.016762	10.564538	11
50	9.435908	9.983202	9.452706	10.547294	10.016798	10.564092	10
51	9.436353	9.983166	9.453187	10.546813	10.016834	10.563647	9
52	9.436798	9.983130	9.453668	10.546332	10.016870	10.563202	8
53	9.437242	9.983094	9.454148	10.545852	10.016906	10.562758	7
54	9.437686	9.983058	9.454628	10.545372	10.016942	10.562314	6
55	9.438129	9.983022	9.455107	10.544893	10.016978	10.561871	5
56	9.438572	9.982986	9.455586	10.544414	10.017014	10.561428	4
57	9.439014	9.982950	9.456064	10.543936	10.017050	10.560986	3
58	9.439456	9.982914	9.456542	10.543458	10.017086	10.560544	2
59	9.439897	9.982878	9.457019	10.542981	10.017122	10.560103	1
60	9.430338	9.982842	9.457496	10.542504	10.017159	10.569662	0
		Sine.		Tang.		Secant.	M

74 *Degrees.*

16 Degrees.

Min.	Sine		Tang.		Secant.		
0	9.440338	9.982842	9.457496	10.542504	10.017158	10.559662	60
1	9.440778	9.982805	9.457973	10.542027	10.017195	10.559222	59
2	9.441218	9.982769	9.458449	10.541551	10.017231	10.558782	58
3	9.441658	9.982733	9.458925	10.541075	10.017267	10.558342	57
4	9.442096	9.982696	9.459400	10.540600	10.017304	10.557903	56
5	9.442535	9.982660	9.469875	10.540125	10.017340	10.557465	55
6	9.442973	9.982623	9.460349	10.539651	10.017376	10.557027	54
7	9.443410	9.982587	9.460823	10.539177	10.017413	10.556590	53
8	9.443847	9.982551	9.461297	10.538703	10.017449	10.556153	52
9	9.444284	9.982514	9.461770	10.538230	10.017486	10.555716	51
10	9.444720	9.982477	9.462242	10.537758	10.017523	10.555280	50
11	9.445155	9.982441	9.462714	10.557285	10.017559	10.554845	49
12	9.445590	9.982404	9.463186	10.536814	10.017596	10.554410	48
13	9.446025	9.982367	9.463658	10.53634[illegible]	10.017633	10.553975	47
14	9.446459	9.982331	9.464128	10.535871	10.017669	10.553541	46
15	9.446893	9.982294	9.464599	10.535401	10 91[illegible]706	10.553107	45
16	9.447326	9.982257	9.465069	10.534931	10.017743	10 55[illegible]674	44
17	9.447759	9.982220	9.465539	10.534461	10.017780	10.552241	43
18	9.448191	9.982183	9.466008	10.533992	10.017817	10.551809	42
19	9.448623	9.982146	9.466476	10.533[illegible]23	10.017854	10.551377	41
20	9.449054	9.982109	9.466945	10.533055	10.017891	10.550946	40
21	9.449485	9.982072	9.467413	10.532587	10.917928	10.550515	39
22	9.449915	9.982035	9.467880	10.532120	10.017965	10.550085	38
23	9.450345	9.981998	9.468347	10.531653	10.018002	10.549655	37
24	9.450775	9.981961	9.468814	10.531186	10.018039	10.549225	36
25	9.451204	9.981924	9.469280	10.530720	10.018076	10.548796	35
26	9.451632	9.981886	9.469746	10.530254	10.018114	10.548368	34
27	9.452060	9.981849	9.470211	10.529789	10.018151	10.547940	33
28	9.452488	9.981812	9.470676	10.519324	10.018188	10.547512	32
29	9.452915	9.981774	9.471141	10.528859	10.018226	10.547085	31
30	9.453342	9.981737	9.471605	10.528395	10.018263	10.546658	30
31	9.453768	9.981699	9.472068	10.527931	10.018300	10.546233	29
32	9.454194	9.981662	9.472532	10.527468	10.018338	10.545806	28
33	9.454619	9.981624	9.472995	10.527005	10.018375	10.545381	27
34	9.455044	9.981587	9.473457	10.526543	10.018413	10 544956	26
35	9.455469	9.981549	9.473919	10.526081	10.0184[illegible]1	10 541531	25
36	9.455893	9.981512	9.474381	10.525619	10.018488	10.544107	24
37	9.456316	9.981474	9.474842	10.525158	10.018526	10.543684	23
38	9.456739	9.981436	9.475303	10.524697	10.018564	10.543261	22
39	9.457162	9.981399	9.475763	10.524137	10.018601	10.542838	21
40	9.457584	9.981361	9.476223	10.523777	10.018639	10.542416	20
41	9.458006	9.981323	9.476683	10.523317	10.018677	10.541994	19
42	9.458427	9.981285	9.477142	10.522858	20.018715	10.541573	18
43	9.458848	9.981247	9.477601	10.522399	10.018753	10.541152	17
44	9.459268	9.981209	9.478059	10.521941	10.018791	10.540732	16
45	9.459688	9.981171	9.478517	10.521483	10.018829	10.540312	15
46	9.460108	9.981133	9.478975	10.521025	10.018867	10.539892	14
47	9.460527	9.981095	9.479432	10.520568	10.018905	10 539473	13
48	9.460946	9.981057	9.479889	10.520111	10.018943	10.539054	12
49	9.461364	9.981019	9.480345	10.519655	10.0[illegible]8981	10.538636	11
50	9.461782	9.980980	9.480801	10.519199	10.019019	10.538218	10
51	9.462199	9.980942	9.481257	10.518743	10.019058	10.537801	9
52	9.462616	9.980904	9.481712	10.518288	10.019096	10.537384	8
53	9.463032	9.980866	9.482167	10.517833	10 019134	10.536968	7
54	9.463448	9.980827	9.482621	10.517379	10.019173	10 536552	6
55	9.463864	9.980789	9.483075	10.516925	10.019211	10.536136	5
56	9.464279	9.980750	9.483529	10.526471	10.019249	10.535721	4
57	9.464694	9.980712	9 483982	10.516018	10.019288	10.535306	3
58	9.465108	9.980673	9.48443[illegible]	10.515565	10.019326	10.534892	2
59	9.465552	9.980635	9.484887	10.515113	10.019365	10.534478	1
60	9.465935	9.980596	9.48533[illegible]	10.514661	10 019404	10.534065	0
		Sine.		Tang.		Secant.	M

73 Degrees

17 Degree[illegible].

Min.	Sine		Tang.		Secant.		
0	9.465935	9.980596	9.485339	10.514661	10 0194[illegible]4	10.[illegible]46[illegible]	60
1	9.466348	9.980558	9.485791	10.514209	10.019442	10 53365[illegible]	[illegible]
2	9.466761	9 980519	9.486242	10 513758	10.019481	10 533239	58
3	9.467[illegible]3	9.980480	9.486693	10.513307	10.019520	10.53282[illegible]	[illegible]
4	9.467585	9.980441	9.487143	10.512857	10 0195[illegible]8	10 5324[illegible]5	56
5	9.467996	9.980403	9.487593	10.512407	10 01959[illegible]	10.532004	55
6	9.468407	9.980364	9.488043	10.511957	10.019636	10 531593	54
7	9.468817	9.980325	9.488492	10 511508	10.01[illegible]675	10.5311[illegible]3	53
8	9.469227	9.980286	9.488941	10.511059	10.019714	10.530773	52
9	9.469637	9.980247	9.489390	10.[illegible]106 0	10.[illegible]19753	10.530363	51
10	9.470046	9.980208	9.499838	10.510162	10.019792	10.539954	50
11	9.470455	9.980169	9.490286	10.509714	10.019831	10.539545	49
12	9.470863	9.980130	9.490733	10.509267	10.019870	10.520137	48
13	9 471271	9.980091	9.491180	10.508820	10.019909	10.528729	47
14	9.471678	9.980052	9.49162[illegible]	10.508373	10 019948	10 528231	46
15	9.472086	9.980012	9.492073	10 507927	10.0199[illegible]8	10.527914	45
16	9.472492	9.979973	9.492519	10.507481	10.[illegible]027	10.527508	44
17	9.472898	9.979934	9 492965	10.507035	10 020056	10.527101	43
18	9.473304	9.979895	9.493410	10 506590	10.020105	10.526696	42
19	9.473710	9 979855	9.493854	10.506145	10.020145	10.526290	41
20	9.474115	9.979816	9.494299	10 505701	10.020184	10 525885	40
21	9.474519	9.979776	9.494743	10.505257	10.020224	10 525481	39
22	9.474923	9.979737	9.495186	10.504813	10 020263	10 525077	38
23	9.475327	9.979697	9.495630	10.504370	10.020303	10.524673	37
24	9.475730	9.979659	9.496073	10.503927	10.020342	10 [illegible]2427[illegible]	36
25	9 476133	9.979618	9.496515	10.503485	10 020382	10.523807	35
26	9.476536	9.979578	9.496957	10.503043	10 020421	10.523464	34
27	9.476938	9.979539	9.497399	10.502601	10.020461	10.523062	33
28	9.477340	9.979499	9.497841	10.502159	10.020501	10.522660	32
29	9.477741	9.979459	9.498282	10.501718	10.020541	10.522259	31
30	9.478142	9.979419	9.498722	10.501278	10.020580	10.521858	30
31	9.478542	9.979380	9 499163	10.500837	10.020620	10.521458	29
32	9.478942	9.979340	9 499603	10.500397	10.020660	10.521058	28
33	9.479342	9.979300	9.500042	10.499958	10.020700	10.520658	27
34	9.479741	9.979260	9.500481	10.499519	10.020740	10.520259	26
35	9 480140	9.979220	9.500920	10.499080	10.020780	10.519860	25
36	9.480538	9.979180	9.501359	10.498641	10.020820	10.519461	24
37	9.480937	9.979140	9.501797	10.498203	10.020860	10.519063	23
38	9.481334	9.979100	9.502235	10.497765	10.020900	10.518666	22
39	9.481731	9.979059	9.502672	10.497328	10.020941	10.518268	21
40	9.482128	9.979019	9.503109	10.496891	10 020981	10.517872	20
41	9.482525	9.978979	9.503546	10.496454	10.021021	10.517475	19
42	9.482921	9.978939	9 503982	10.496018	10.021061	10.517079	18
43	9.483316	9.978898	9.504418	10.495582	10.021102	10.516683	17
44	9.483712	9.978858	9.504854	10.495146	10.021142	10.516288	16
45	9.484107	9.978817	9.505289	10.494711	10.021182	10.515893	15
46	9.484501	9.978777	9.505724	10.494276	10.021223	10.515499	14
47	9.484895	9.978736	9 506159	10.493841	10.021263	10.515105	13
48	9.485289	9.978696	9.506593	10.493407	10.021304	10.514711	12
49	9.485682	9.978655	9.507027	10.492973	10.021345	10.514318	11
50	9.486075	9.978615	9.507460	10.492540	10.021385	10.513925	10
51	9.486467	9.978574	9.507893	10.492107	10.021426	10.513533	9
52	9.486850	9.978533	9.508326	10.491674	10 021467	10.513140	8
53	9.487251	9.978493	9.508759	10.491241	10.021507	10.512749	7
54	9.487643	9.978452	9.509191	10.490809	10.021548	10.512357	6
55	6.488033	9.978411	9.509682	10.490378	10.021589	10.511966	5
56	9.488424	9.978370	9.510054	10.489946	10.021630	10.511[illegible]76	4
57	9.488814	9.978329	9.510485	10.489515	10.021671	10.511186	3
58	9.489204	9.978288	9.510916	10 489084	10.021712	10 510796	2
59	9.48959[illegible]	9.978247	9.511346	10.488654	10.0217[illegible]3	10.510407	1
60	9 4099[illegible]	9.978206	9.501776	10.488224	10.021704	10.5100[illegible]8	0
		Sine		Tang.		Secant.	M

72 Degrees.

18 *Degrees.*

Min.	Sine.		Tang.		Secant.		
0	9.489982	9.978206	9.511776	10.488224	10.021794	10.510018	60
1	9.490371	9.978165	9.512206	10.487794	10.021835	10.509629	59
2	9.490759	9.978124	9.512635	10.487365	10.021876	10.509241	58
3	9.491147	9.978083	9.513064	10.486936	10.021917	10.508853	57
4	9.491534	9.978042	9.513493	10.486507	10.021958	10.508465	56
5	9.491922	9.978001	9.513921	10.486079	10.021999	10.508070	55
6	9.492308	9.977959	9.514349	10.485651	10.022041	10.507692	54
7	9.492695	9.977918	9.514777	10.485223	10.022082	10.507305	53
8	9.493081	9.977877	9.515204	10.484796	10.022123	10.506919	52
9	9.493466	9.977835	9.515631	10.484369	10.022165	10.506534	51
10	9.493851	9.977794	9.516057	10.483942	10.022206	10.506149	50
11	9.494236	9.977752	9.516484	10.483516	10.022248	10.505764	49
12	9.494620	9.977711	9.516910	10.483090	10.022289	10.505379	48
13	9.495005	9.977669	9.517335	10.482665	10.022331	10.504995	47
14	9.495388	9.977628	9.517761	10.482239	10.022372	10.504612	46
15	9.495772	9.977586	9.518185	10.481814	10.022414	10.504228	45
16	9.490154	9.977544	9.518610	10.481390	10.022456	10.503845	44
17	9.496537	9.977503	9.519034	10.480966	10.022497	10.503463	43
18	9.496919	9.977461	9.519458	10.480542	10.022539	10.503081	42
19	9.497301	9.977419	9.519882	10.480118	10.022581	10.502699	41
20	9.497682	9.977377	9.520305	10.479695	10.022623	10.502318	40
21	9.498063	9.977335	9.520728	10.479272	10.022665	10.501936	39
22	9.498444	9.977293	9.521151	10.478849	10.022707	10.501556	38
23	9.498824	9.977251	9.521573	10.478427	10.022748	10.501175	37
24	9.499204	9.977209	9.521995	10.478002	10.022790	10.500795	36
25	9.499584	9.977167	9.522417	10.477583	10.022833	10.500416	35
26	9.499963	9.977125	9.522838	10.477162	10.022875	10.500037	34
27	9.500342	9.977083	9.523259	10.476741	10.022917	10.499658	33
28	9.500721	9.977041	9.523679	10.476320	10.022959	10.499279	32
29	9.501099	9.976999	9.324100	10.475900	10.023001	10.498901	31
30	9.501476	9.976957	9.524520	10.475480	10.023043	10.498524	30
31	9.501854	9.976914	9.524939	10.475060	10.022086	10.498146	29
32	9.502231	9.976872	9.525359	10.474641	10.023128	10.497769	28
33	9.502607	9.976830	9.525778	10.474222	10.023170	10.497392	27
34	9.502984	9.976787	9.526197	10.473803	10.023213	10.497016	26
35	9.503360	9.976745	9.526615	10.473385	10.023255	10.496640	25
36	9.503735	9.976702	9.527033	10.472967	10.023298	10.496265	24
37	9.504110	9.976660	9.527451	10.472549	10.023340	10.495889	23
38	9.504485	9.976617	9.527868	10.472132	10.023383	10.495515	22
39	9.504860	9.976574	9.528285	10.471715	10.023425	10.495140	21
40	9.505234	9.976532	9.528702	10.471298	10.023468	10.494766	20
41	9.505608	9.976489	9.529119	10.470881	10.023511	10.494392	19
42	9.505981	9.976446	9.529535	10.470465	10.023554	10.494019	18
43	9.506354	9.976404	9.529950	10.470049	10.023596	10.493646	17
44	9.506727	9.976361	9.530366	10.469634	10.023639	10.493273	16
45	9.507099	9.976318	9.530781	10.469219	10.023682	10.492902	15
46	9.507471	9.976275	9.531196	10.468804	10.023725	10.492529	14
47	9.507842	9.976232	9.531611	10.468389	10.023768	10.492157	13
48	9.508214	9.976189	9.532025	10.467975	10.023811	10.491786	12
49	9.508585	9.976146	9.532439	10.467561	10.023854	10.491415	11
50	9.508956	9.976103	9.532853	10.467147	10.023897	10.491044	10
51	9.509326	9.976060	9.533266	10.466734	10.023940	10.490674	9
52	9.509696	9.976017	9.533679	10.466321	10.023983	10.490304	8
53	9.510065	9.975974	9.534092	10.465908	10.024026	10.489935	7
54	9.510434	9.975930	9.534504	10.465496	10.024070	10.489566	6
55	9.510803	9.975887	9.534916	10.465084	10.024113	10.489197	5
56	9.511172	9.975844	9.535328	10.464672	10.024156	10.488828	4
57	9.511540	9.975800	9.535739	10.464261	10.024200	10.488460	3
58	9.511907	9.975757	9.536150	10.463849	10.024243	10.188093	2
59	9.512275	9.975713	9.536561	10.463439	10.024287	10.487725	1
60	9.512642	9.975670	9.536977	10.463028	10.024330	10.487358	0
		Sine.		Tang.		Secant.	M.

71 *Degrees.*

19 *Degrees.*

Min.	Sine.		Tang.		Secant.		
0	9.512642	9.975670	9.536972	10.463028	10.024330	10.487358	60
1	9.513009	9.975626	9.537382	10.462618	10.024373	10.486991	59
2	9.513375	9.975583	9.537792	10.462208	10.024417	10.486625	58
3	9.513741	9.975539	9.538202	10.461798	10.024461	10.486259	57
4	9.514107	9.975496	9.538611	10.461389	10.024504	10.485893	56
5	9.514472	9.975452	9.539020	10.460980	10.024548	10.485528	55
6	9.514837	9.975408	9.539429	10.460571	10.024592	10.485163	54
7	9.515202	9.975365	9.539837	10.460163	10.024635	10.484798	53
8	9.515566	9.975321	9.540245	10.459755	10.024679	10.484434	52
9	9.515930	9.975277	9.540653	10.459347	10.024723	10.484070	51
10	9.516294	9.975233	9.541061	10.458939	10.024767	10.483706	50
11	9.516657	9.975189	9.541468	10.458532	10.024811	10.483343	49
12	9.517020	9.975145	9.541875	10.458125	10.024855	10.482980	48
13	9.517382	9.975101	9.542281	10.457719	10.024899	10.482618	47
14	9.517745	9.975057	9.542688	10.457312	10.024943	10.482255	46
15	9.518107	9.975013	9.543094	10.456906	10.524987	10.481893	45
16	9.518468	9.974969	9.543499	10.456501	10.025031	10.481532	44
17	9.518829	9.974925	9.543005	10.456095	10.025075	10.481170	43
18	9.519190	9.974880	9.544310	10.455690	10.025120	10.480810	42
19	9.519551	9.974836	9.544715	10.455285	10.025164	10.480449	41
20	9.519911	9.974792	9.545119	10.454881	10.025208	10.480089	40
21	9.520271	9.974747	9.545524	10.454476	10.025252	10.479729	39
22	9.320631	9.974703	9.545928	10.454072	10.025297	10.479369	38
23	9.520990	9.974659	9.546331	10.453669	10.025341	10.479010	37
24	9.521349	9.974614	9.546735	10.453265	10.025386	10.478651	36
25	9.521707	9.974570	9.547138	10.452862	10.025430	10.478293	35
26	9.522066	9.974525	9.547540	10.452459	10.025475	10.477934	34
27	9.522423	9.974481	9.547943	10.452057	10.025519	10.477576	33
28	9.522781	9.974436	9.548345	10.451655	10.025564	10.477219	32
29	9.523138	9.974391	9.548747	10.451253	10.025609	10.476862	31
30	9.523495	9.974347	9.549149	10.450851	10.025653	10.476505	30
31	9.523852	9.974302	9.549550	10.450450	10.025698	10.476148	29
32	9.524208	9.974257	9.549951	10.450049	10.025743	10.475792	28
33	2.524564	9.974212	9.550352	10.449648	10.025788	10.475436	27
34	9.524920	9.974167	9.550752	10.449248	10.025833	10.475080	26
35	9.525275	9.974122	9.551152	10.448847	10.025873	10.474725	25
36	9.525630	9.974077	9.551552	10.448448	10.025923	10.474370	24
37	9.525984	9.974032	9.551952	10.448048	10.025968	10.474016	23
38	9.526339	9.973987	9.552351	10.447649	10.026013	10.473661	22
39	9.526692	9.973942	9.552750	10.447250	10.026058	10.473307	21
40	9.527046	9.973897	9.553149	10.446851	10.026103	10.472954	20
41	9.527400	9.973852	9.553548	10.446452	10.026148	10.472600	19
42	9.527753	9.973807	9.553946	10.446054	10.026193	10.472247	18
43	9.528105	9.973761	9.554344	10.445656	10.026238	10.471895	17
44	9.528457	9.973716	9.554741	10.445258	10.026284	10.471542	16
45	9.528810	9.973671	9.555139	10.444861	10.026329	10.471190	15
46	9.529161	9.973625	9.555536	10.444464	10.026374	10.470839	14
47	9.529513	9.973580	9.555933	10.544067	10.026420	10.470487	13
48	9.529864	9.973535	9.556329	10.443671	10.026465	10.470136	12
49	9.530215	9.973489	9.556725	10.443274	10.026511	10.469785	11
50	9.530565	9.973443	9.557121	10.442879	10.026556	10.469435	10
51	9.530915	9.973398	9.557517	10.442483	10.026602	10.469085	9
52	9.531265	9.973352	9.557912	10.442087	10.026648	10.468735	8
53	9.531614	9.973307	9.558308	10.441692	10.026693	10.468386	7
54	9.531963	9.973261	9.558702	10.441297	10.026739	10.468036	6
55	9.532312	9.973215	9.559097	10.440903	10.026785	10.467688	5
56	9.532661	9.973169	9.559491	10.440509	10.026831	10.467339	4
57	9.533009	9.973124	9.559885	10.440115	10.026876	10.466991	3
58	9.533357	9.973078	9.560279	10.439721	10.026922	10.466643	2
59	9.533704	9.973032	9.560673	10.439327	10.026968	10.466296	1
60	9.534052	9.973986	9.561066	10.438934	10.027014	10.465948	0
		Sine		Tang.		Secant.	M.

70 *Degrees.*

20 *Degrees.*

Min.	Sine		Tang.		Secant.		
0	9.534052	9.972986	9.561066	10.438935	10.027014	10.465948	60
1	9.534399	9.972940	9.561459	10.438541	10.027060	10.465601	59
2	9.534745	9.972894	9.561851	10.438146	10.027106	10.465255	58
3	9.535091	9.972848	9.562244	10.437756	10.027152	10.464908	57
4	9.535437	9.972802	9.562636	10.437364	10.027198	10.464562	56
5	9.535783	9.972755	9.563028	10.436972	10.027245	10.464217	55
6	9.536129	9.972709	9.563419	10.436581	10.027291	10.463871	54
7	9.536474	9.972663	9.563811	10.436189	10.027337	10.463526	53
8	9.536818	9.972617	9.564202	10.435798	10.027383	10.463182	52
9	9.537163	9.972570	9.564592	10.435407	10.027430	10.462837	51
10	9.537507	9.972524	9.564983	10.435017	10.027476	10.462493	50
11	9.537851	9.972477	9.565373	10.434627	10.027522	10.462149	49
12	9.538194	9.972431	9.565763	10.434237	10.027569	10.461806	48
13	9.538537	9.972384	9.566153	10.433847	10.027615	10.461462	47
14	9.538880	9.972338	9.566542	10.433458	10.027662	10.461120	46
15	9.539223	9.972291	9.566932	10.433068	10.027709	10.460777	45
16	9.539565	9.972245	9.567320	10.432679	10,027755	10.460435	44
17	9.539907	9.972198	9.567709	10.432291	10.027802	10.460093	43
18	9.540249	9.972151	9.568097	10.431902	10.027849	10.459751	42
19	9.540590	9.972105	9.568486	10.431514	10.027895	10.459410	41
20	9.540931	9.972058	9.568873	10.431126	10.027942	10.459069	40
21	9.541272	9.972011	9.569261	10.430739	10.027989	10.458728	39
22	9.541613	9.971964	9.569648	10.430352	10.028036	10.458387	38
23	9.541953	9.571917	9.570035	10.429964	10.028083	10.458047	37
24	9.542293	9.971870	9.570422	10.429578	10.028130	10.457707	36
25	9.542632	9.971823	9.570809	10.429191	10.028177	10.457368	35
26	9.542971	9.971776	9.571195	10.428805	10.028224	10.457029	34
27	9.543310	9.971729	9.571581	10.428419	10.028271	10.456690	33
28	9.543649	9.971681	9.571967	10.428033	10.028318	10.456351	32
29	9.543987	9.971635	9.572352	10.427648	10.028365	10.456013	31
30	9.544325	9.971588	9.572738	10.427262	10.028412	10.455675	30
31	9.544663	9.971540	9.573123	10.426877	10.028460	10.455337	29
32	9.545000	9.971493	9.573507	10.426493	10.028507	10.454999	28
33	9.545338	9.971446	9.573892	10.426108	10.028554	10.454662	27
34	9.545674	9.971398	9.574276	10.425724	10.028602	10.454325	26
35	9.546011	9.971351	9.574660	10.425340	10.028649	10.453989	25
36	9.546347	9.971303	9.575044	10.424956	10.028696	10.453653	24
37	9.546683	9.971256	9.575429	10.424573	10.028744	10.453317	23
38	9.547019	9.971208	9.575810	10.424190	10.028792	10.452981	22
39	9.547354	9.971161	9.576193	10.423807	10.028839	10.452646	21
40	9.547689	9.971113	9.576576	10.423424	10.028887	10.452311	20
41	9.548024	9.971065	9.576958	10.423041	10.028934	10.451976	19
42	9.548358	9.971018	9.577341	10.422659	10.028982	10 451641	18
43	9.548693	9.970970	9.577723	10.422277	10.029030	10.451307	17
44	9.549027	9.970922	9.578104	10.421896	10.029078	10.450975	16
45	9.549360	9.970874	9.578486	10.421514	10.029126	10.450640	15
46	9.549693	9.970826	9.578867	10.421133	10.029173	10.450306	14
47	9.550026	9.970779	9.579248	10.420752	10.029221	10.449973	13
48	9.550359	9.970731	9.579629	10.420371	10.029269	10.449641	12
49	9.550692	9.970683	9.580009	10.419991	10.029317	10.449308	11
50	9.551024	9.970635	9.580389	10.419611	10.029365	10.448976	10
51	9.551356	9.970586	9.580769	10.419231	10.029413	10.448644	9
52	9.551687	9.970538	9.581149	10.418851	10.029462	10.448313	8
53	9.552018	9.970490	9.581528	10.418472	10.029510	10.447982	7
54	9.552349	9.970442	9.581907	10.418093	10.029558	10.447651	6
55	9.552680	9.970394	9.582286	10.417714	10.029606	10.447320	5
56	9.553010	9.980345	9.582665	10.417335	10.029655	10.446989	4
57	9.553341	9.970297	9.583043	10.416956	10.029703	10.446659	3
58	9.553670	9.970249	9.583422	10.416578	10.029751	10.446330	2
59	9.554000	9.970200	9.583802	10.416200	10.029800	10.446000	1
60	9.554329	9.970152	9.584177	10.415823	10.029848	10.445671	0
		Sine		Tang.		Secant.	M

69 *Degrees.*

21 *Degrees.*

Min.	Sine		Tang.		Secant.		
0	9.554329	9.970152	9.581177	10 415823	10.029848	10.445671	60
1	9.554658	9.970103	9.584555	10.415445	10.029897	10.445342	59
2	9.554987	9.970055	9.584932	10.415068	10.029945	10.445013	58
3	9.555315	9.970006	9.585309	10 414691	10.029994	10.444685	57
4	9.555643	9.969957	9.585686	10.414314	10.030043	10.444357	56
5	9.555971	9.969909	9.586062	10.413938	10.030090	10.444014	55
6	9.556299	9.969860	9.586439	10.413565	10.030140	10.443701	54
7	9.556626	9.969811	9.586815	10.413181	10.030189	10.443374	53
8	9.556953	9.969762	9.587190	10.412810	10.030238	10.443047	52
9	9.557280	9.969714	9.587566	10.412434	10.030286	10.442720	51
10	9.557606	9.969665	9.587941	10.412059	10.030335	10.442394	50
11	9.557932	9.969616	9.588316	10.411684	10.030384	10.442068	49
12	9.558258	9.969567	9.588691	10.411309	10.030433	10.441742	48
13	9.558583	9.969518	9.589066	10.410933	10.030482	10.441416	47
14	9.558909	9.969469	9.589440	10.410560	10.030531	10.441091	46
15	9.559234	9.969420	9.589814	10.410186	10.030580	10.440766	45
16	9.559558	9.969370	9.590158	10.409812	10.030630	10.440441	44
17	9.559883	9.969321	9.590562	10.409438	10.030679	10.440117	43
18	9.560207	9.969272	9.590935	10.409065	10.030728	10.439793	42
19	9.560531	9.969223	9.591308	10.408692	10 030777	10.439469	41
20	9.560855	9.969173	9.591681	10.408319	10.030827	10.439145	40
21	9.561178	9.969124	9.592054	10.407946	10.030876	10.438822	39
22	9.561501	9.969075	9.592426	10.407574	10.030925	10.438499	38
23	9.561824	9.969025	9.592798	10.407201	10.030975	10.438176	37
24	9.562146	9.968976	9.593170	10.406829	10.031024	10.437854	36
25	9.562468	9.968926	9.593542	10.406458	10.031074	10.437531	35
26	9.562795	9.968877	9.593914	10.406086	10.031123	10.437210	34
27	9.563112	9.968827	9.594285	10.405715	10.031173	10.436888	33
28	9.563433	9.968777	9.594656	10.405344	10.031223	10.436566	32
29	9.563755	9.968728	9.595027	10.404973	10.031272	10.436245	31
30	9.564075	9.968678	9.595397	10.404602	10.031322	10.435925	30
31	9.564396	9.968628	9.595768	10.404232	10.031372	10.435605	29
32	9.564716	9.968578	9.596138	10.403862	10.031422	10.435284	28
33	9.565036	9.968528	9.596508	10 403492	10.031472	10.434964	27
34	9.565356	9.968478	9.596878	10.403122	10.031521	10.434644	26
35	9.565676	9.968429	9.597247	10.402753	10.031571	10.434324	25
36	9.565995	9.968379	9.597616	10.402384	10.031621	10.434005	24
37	9.566314	9.968328	9.597985	10.402015	10.031671	10.433686	23
38	9.566632	9.968278	9.598354	10.401646	10.031722	10.433368	22
39	9.566951	9.968228	9.598722	10.401274	10.031772	10.433049	21
40	9.567269	9.968178	9.599091	10.400909	10.031822	10.432731	20
41	9.567587	9.968128	9.599459	10.400541	10.031872	10.432413	19
42	9.567904	9.968078	9.599827	10.400176	10.031922	10.432096	18
43	9.568222	9.968027	9.600194	10.399803	10.031973	10.431778	17
44	9.568539	9.967977	9.600562	10.399438	10.032023	10.431461	16
45	9.568835	9.967927	9.600929	10.399071	10.032073	10.431144	15
46	9.569172	9.967876	9.601296	10.398704	10.032124	10.430828	14
47	9.569488	9.967826	9.601662	10.398337	10.032174	10.430512	13
48	9.569804	9.967775	9.602029	10 397971	10.032225	10.430196	12
49	9.570120	9.967725	9.602395	10.397605	10.032275	10.429880	11
50	9.570435	9.967674	9.602761	10.397239	10.032326	10.429564	10
51	9.570751	9.967623	9.603127	10.396873	10.032376	10.429249	9
52	9.571066	9.967573	9.603493	10.396507	10.032427	10.428933	8
53	9.571380	9.967522	9.603858	10.396142	10.032478	10.428620	7
54	9.571695	9.967471	9.604223	10.395777	10.032529	10.428305	6
55	9.572009	9.967420	9.604588	10.395412	10.032579	10.427991	5
56	9.572323	9.967370	9.604953	10.395047	10.032630	10.427677	4
57	9.572636	9.967319	9.605317	10.394683	10.032681	10.427363	3
58	9.572949	9.967268	9.605682	10.394318	10.032732	10.427050	2
59	9.573263	9.967217	9.606046	10.393954	10.032783	10.426737	1
60	9.573575	9.967166	9.606410	10.393590	10 032834	10.426425	0
		Sine		Tang.		Secant.	M

68 *Degrees.*

22 Degrees.

Min.	Sine.		Tang.		Secant.		
0	9.573575	9.967166	9.606410	10.393590	10.032834	10.426425	60
1	9.573888	9.967115	9.606773	10.393227	10.032885	10.426112	59
2	9.574200	9.967064	9.607137	10.392863	10.032936	10.425800	58
3	9.574512	9.967012	9.607500	10.392500	10.032987	10.425488	57
4	9.574824	9.966961	9.607863	10.392137	10.033039	10.425176	56
5	9.575136	9.966910	9.608225	10.391775	10.033090	10.424864	55
6	9.575447	9.966859	9.608588	10.391412	10.033141	10.424553	54
7	9.575738	9.966807	9.608950	10.391050	10.033192	10.424242	53
8	9.576068	9.966756	9.609312	10.390688	10.033244	10.423931	52
9	9.576379	9.966704	9.609674	10 390326	10.033295	10.423621	51
10	9.576689	9.966653	9.610039	10.389964	10.033347	10.423311	50
11	9.576999	9.966602	9.610397	10.389602	10.033398	10.423001	49
12	9.577309	9.966550	9.610759	10.389241	10.033450	10.422691	48
13	9.577618	9.966499	9.611120	10.388880	10.033501	10.422382	47
14	9.577927	9.966447	9.611480	10.388520	10.033553	10.422072	46
15	9.578236	9.966395	9.611841	10.388159	10.033605	10.421764	45
16	9.578545	9.966344	9.612001	10.387799	10.033656	10.421455	44
17	9.578853	9.966292	9.612561	10.387438	10.033708	10.421146	43
18	9.579162	9.966240	9.612921	10.387079	10.033760	10.420838	42
19	9.579469	9.966188	9.613281	10.386719	10 033812	10.420530	41
20	9.579777	9.966136	9.613641	10.386359	10.033863	10.420233	40
21	9.580084	9.966085	9.614000	10.386000	10.033915	10.419915	39
22	9.580392	9.966033	9.614359	10.385641	10.033967	10.419608	38
23	9.580699	9.965981	9.614718	10.385282	10.034019	10.419301	37
24	9.581005	9.965928	9.615077	10.384923	10.034071	10.418995	36
25	9.581312	9.965876	9.615435	10.384565	10.034124	10.418688	35
26	9.581618	9.965824	9.615793	10.384207	10.034176	10.418382	34
27	9.581924	9.965772	9.616151	10.383849	10.034228	10.418076	33
28	9.582229	9.965720	9.616509	10.383491	10.034280	10.417771	32
29	9.582534	9.965668	9.616867	10.383133	10.034332	10.417465	31
30	9.582840	9.965615	9.617224	10.382776	10.034385	10.417160	30
31	9.583144	9.965563	9.617581	10.382418	10.034437	10.416855	29
32	9.583449	9.965511	9.617938	10.382061	10.034489	10.416551	28
33	9.583753	9.965458	9.618295	10.381705	10.034542	10.416246	27
34	9.584058	9.965406	9.618652	10.381347	10.034594	10.415942	26
35	9.584361	9.965353	9.619008	10.380992	10.034647	10.415638	25
36	9.584665	9.965301	9.619364	10.380635	10.034699	10.415335	24
37	9.584968	9.965248	9.619720	10.380279	10.034752	10.415031	23
38	9.585272	9.965195	9.620076	10.379924	10.034805	10.414728	22
39	9.585574	9 965143	9.620432	10.379568	10.034857	10.414425	21
40	9.585877	9.965090	9.620787	10.379213	10.034910	10.414123	20
41	9.586179	9.965037	9.621142	10.378858	10.034963	10.413820	19
42	9.586482	9.964984	9.621497	10.378503	10.035016	10.413518	18
43	9.586783	9.964931	9.621852	10.378148	10.035069	10.413216	17
44	9.587085	9.964878	9.622207	10.377793	10.035121	10.412915	16
45	9.587386	9.964826	9.622561	10.377439	10.035174	10.412613	15
46	9.587688	9.964773	9.622915	10.377085	10.035227	10.412312	14
47	9.587988	9.964719	9.623269	10.376731	10.035280	10.412011	13
48	9.588289	9.964666	9.623623	10.376377	10.035333	10.411711	12
49	9.588591	9.964613	9.623976	10.376024	10.035387	10.411410	11
50	9.588890	9.964560	9.624330	10.375670	10.035440	10.411110	10
51	9.589190	9.964507	9.624683	10.375317	10.035493	10.410810	9
52	9.589489	9.964454	9.625036	10.374964	10.035546	10.410511	8
53	9.589789	9.964400	9.625388	10.374612	10.035600	10.410211	7
54	9.590088	9.964347	9.625741	10 374259	10.035659	10.409912	6
55	9.590387	9.964294	9.626093	10.373907	10.035706	10.409613	5
56	9.590686	9.964240	9.626445	10.373555	10.035760	10.409314	4
57	9.590984	9.964187	9.626707	10.373203	10.035813	10.409016	3
58	9.591282	9.964133	9.627149	10.372851	10.035867	10.408718	2
59	9.591580	9 964080	9.627501	10.372499	10.035920	10.408420	1
60	9.591878	9.964026	9.627852	10.372148	10.035074	10.408122	0
		Sine.		Tang.		Secant.	M

67 Degrees.

23 Degrees.

Min.	Sine		Tang.		Secant.		
0	9.591878	9.964026	9.627853	10.372148	10.035974	10.408122	60
1	9.592175	9.963974	9.628203	10.371797	10.036028	10 407824	59
2	9.592473	9.963919	9.628554	10.371448	10.036081	10.407527	58
3	9.592770	9.963865	9.628905	10.371095	10.036135	10.407230	57
4	9.593069	9.963811	9.629255	10.370745	10.036189	10.406933	56
5	9.592363	9.963757	9.629606	10.370394	10.036243	10.406637	55
6	9.593659	9.963704	9.629956	10.370044	10.036296	10.406341	54
7	9.593955	9.963650	9.630306	10.369694	10.036350	10.406044	53
8	9.594251	9.963596	9.630656	10 369344	10.036404	10.405749	52
9	9.594547	9.963543	9.631005	10.368995	10.036458	10.405453	51
10	9.594842	9.963488	9.631354	10.368645	10.036512	10.405158	50
11	9.595137	9.963434	9.631704	10.368296	10.036566	10.404863	49
12	9.595432	9.963379	9.632053	10.367947	10.036620	10.404568	48
13	9.595727	9.963325	9.632401	10.367598	10.036677	10.404273	47
14	9.596021	9.963281	9.632750	10.367250	10.036729	10.403979	46
15	9.596315	9.963217	9.633098	10.366901	10.036783	10.403685	45
16	9.596609	9.963162	9.633447	10.366553	10.036837	10.403391	44
17	9.596903	9.963108	9.633795	10.366205	10.036892	10.403097	43
18	9.597196	9.963054	9.634143	10.365857	10.036946	10.402805	42
19	9.597490	9.962999	9.634490	10.365510	10.037001	10.402510	41
20	9.597783	9.962945	9.634838	10.365162	10.037055	10.402217	40
21	9.598075	9.962890	9.635185	10.364815	10.037110	10.401925	39
22	9.598368	9.962836	9.635532	10.364468	10.037164	10.401632	38
23	9.598660	9.962781	9.635879	10.364121	10.037219	10.401340	37
24	9.598952	9.962727	9.636226	10.363774	10.037273	10.401048	36
25	9.599244	9.962672	9.636572	10.363428	10.037328	10.400756	35
26	9.599536	9.962617	9.636918	10.363081	10.037383	10.400464	34
27	9.599827	9.962562	9.637265	10.362735	10.037438	10.400173	33
28	9.600118	9.962509	9.637611	10.362389	10.037493	10.399882	32
29	9.600409	9.962453	9.637956	10.362044	10.037547	10.399591	31
30	9.600500	9.962398	9.638302	10.361698	10.037602	10.399300	30
31	9.600990	9.962343	9.638647	10.361353	10.037657	10.399010	29
32	9.601280	9.962288	9.638992	10.361007	10.037712	10.398720	28
33	9.601570	9.962233	9.639337	10.360662	10.037767	10.398430	27
34	9.601860	9.962178	9.639682	10.360318	10.037822	10.398140	26
35	9.602149	9.962123	9.640027	10.359973	10.037877	10.397850	25
36	9.602439	9.962067	9.640371	10.359629	10.037933	10.397561	24
37	9.602728	9.962012	9.640716	10.359284	10.037988	10.397272	23
38	9.603017	9.961957	9.641060	10.358940	10.038043	10.396983	22
39	9.603305	9.961902	9.641404	10.358596	10.038098	10.396695	21
40	9.603594	9.961846	9.641747	10.358253	10.038154	10.396406	20
41	9.603882	9.961791	9.642091	10.357909	10.038209	10.396118	19
42	9.604170	9.961735	9.642434	10.357566	10.038264	10.395830	18
43	9.604457	9.961680	9.642777	10.357223	10.538320	10.395543	17
44	9.604745	9.961624	9.643120	10.356880	10.038375	10.395255	16
45	9.605032	9.961569	9.643463	10.356537	10.038431	10.394968	15
46	9.605319	9.961513	9.643806	10 356194	10.038487	10.394681	14
47	9.605606	9.961458	9.644148	10.355852	10.038542	10.394394	13
48	9.605892	9.961402	9.644490	10.355510	10.038598	10.394108	12
49	9.606179	9.961346	9.644832	10.355168	10.038654	10.393821	11
50	9.606465	9.961290	9.645174	10.354826	10.038710	10.393535	10
51	9.606751	9.961235	9.645516	10.354484	10.038765	10.393249	9
52	9.607036	9.961179	9.645857	10.354142	10.038821	10.392964	8
53	9.607322	9.961123	9.646199	10.353801	10.038877	10.392678	7
54	9.607607	9.961067	9.646540	10.353460	10.038933	10.392393	6
55	9.607892	9.961011	9.646881	10.353119	10.038989	10.392108	5
56	9.608176	9.960955	9.647222	10.352778	10.039045	10.391823	4
57	9.608461	9.960899	9.647562	10.352438	10.039101	10.391539	3
58	9.608745	9.960843	9.647903	10.352097	10.039157	10.391255	2
59	9.609029	9.960786	9.648243	10.351757	10.039214	10.390971	1
60	9.609313	9.960730	9.648583	10.351417	10.039270	10.390687	0
		Sine.		Tang.		Secant.	M

66 Degrees.

24 Degrees.

Min.	Sine		Tang.		Secant.		
0	9.609313	9.960730	9.648583	10.351417	10.039270	10.390687	60
1	9.609597	9.960674	9.648923	10.351077	10.039326	10.390403	59
2	9.609880	9.960618	9 649263	10.350737	10.039382	10.390120	58
3	9.610163	9.960561	9.649602	10.350398	10.039439	10.389836	57
4	9.610446	9.960505	9.649942	10.350058	10.039495	10.389553	56
5	9.610729	9.960448	9.650281	10.349719	10.039552	10.389271	55
6	9.611012	9.960392	9.650620	10.349380	10.039608	10.388988	54
7	9.611294	9.960335	9.650959	10.349041	10.039665	10.388706	53
8	9.611576	9.960279	9.651297	10.348703	10.039721	10.388424	52
9	9.611858	9.960222	9.651636	10.348364	10.039778	10.388142	51
10	9.612140	9 960165	9.651974	10.348026	10.039834	10.387860	50
11	9.612421	9.960109	9.652312	10.347688	10.039891	10.387579	49
12	9.612702	9.960052	9.652650	10.347350	10.039948	10.387298	48
13	9.612983	9.959995	9.652988	10.347012	10.040005	10.387017	47
14	9.613264	9.959938	9.653326	10.346674	10.040062	10.386736	46
15	9.613545	9.959881	9.653663	10.346337	10.040118	10.386455	45
16	9.613825	9.959825	9.654000	10.346000	10.040175	10.386175	44
17	9.614105	9.959768	9.654337	10.345662	10.040232	10.385895	43
18	9.614385	9.959711	9.654674	10.345326	20.040289	10.385615	42
19	9.614665	9.959653	9.655011	10.344989	10.040346	10.385335	41
20	9.614944	9.959596	9.655348	10.344652	10.040404	10.385056	40
21	9.615223	9.959539	9.655684	10.344316	10.040461	10 384777	39
22	9.615502	9.959482	9.656020	10.343980	10.040518	10.384498	38
23	9.615781	9.959425	9.656356	10.343644	10.040575	10.384219	37
24	9.616060	9.959367	9.656692	10.343308	10.040633	10.383940	36
25	9.616338	9.959310	9.657028	10.342972	10.040690	10.383662	35
26	9.616616	9.959253	9.657364	10.342636	10.040747	10.383384	34
27	9.616894	9.959195	9.657699	10.342301	10.040805	10.383106	33
28	9.617172	9.959138	9.658034	10.341966	10.040862	10.382828	32
29	9.617450	9.959080	9.658369	10.341631	10.040919	10.382550	31
30	9.617727	9.959023	9.658704	10.341296	10.040977	10.382273	30
31	9.618004	9.958065	9.659039	10.340961	10.041035	10.381996	29
32	9.618281	9.958908	9.659303	10.340627	10.041092	10.381719	28
33	9.618558	9.958850	9.659778	10.340292	10.041150	10.381442	27
34	9.618834	9.958792	9.660042	10.339958	10.041208	10.381166	26
35	9.619110	9.958734	9.660376	10.339624	10.041265	10.380890	25
36	9.619386	9.958677	9.660710	10.339290	10.041323	10.380614	24
37	9.619662	9.958619	9.661043	10.338957	10.941381	10.380338	23
38	9.619938	9.958561	9.661377	10.338623	10.041439	10.380062	22
39	9.620213	9.958503	9.661710	10.338290	10.041497	10.379787	21
40	9.620488	9.958445	9.662043	10.337957	10.041555	10 379512	20
41	9.620763	9.958387	9.662376	10.337623	10 941613	10.379237	19
42	9.621038	9.958329	9.662709	10.337291	10.041671	10.378962	18
43	9.621313	9 958271	9.663042	10.336958	10.041729	10.378687	17
44	9.621587	9.958212	9.663374	10.336625	10.041787	10.378413	16
45	9.621861	9.958154	9.663707	10.336293	10.041846	10.378139	15
46	9.622135	9.958096	9.664039	10.335961	10.041904	10.377865	14
47	9.622409	9.958038	9.664371	10.335629	10.041962	10.377591	13
48	9.622682	9.957979	9.664703	10.335297	10.042021	10.377318	12
49	9.622956	9.957921	9.665035	10.334965	10.042079	10.377044	11
50	9.623229	9.957863	9.665366	10.334634	10.042137	10.376771	10
51	9.623502	9.957804	9.665697	10.334302	10.042196	10.376498	9
52	9.623777	9.957746	9.666029	10.333971	10.042254	10.376226	8
53	9.624044	9.957687	9.666360	10.333640	10.042313	10.375953	7
54	9.624319	9.957628	9.666691	10.333309	10.042372	10.375681	6
55	9.624591	9.957570	9.667021	10.332979	10.042430	10.375409	5
56	9.624863	9.957511	9.667352	10.332648	10.042489	10.375137	4
57	9.625135	9.957452	9.667682	10.332318	10.042548	10.374865	3
58	9.625406	9.957393	9.668013	10.331987	10.042607	10.374594	2
59	9.625677	9.957335	9.668343	10.331657	10.042665	10.374323	1
60	9.625948	9.957276	9.668672	10.331327	10.042724	10.374052	0
		Sine.		Tang.		Secant.	M

65 Degrees

25 Degrees.

Min.	Sine		Tang.		Secant.		
0	6.625948	9.957276	9.668672	10.331327	10.042724	10.374052	60
1	9.626219	9.957217	9.669002	10.330998	10.042783	10.373781	59
2	9.626490	9.957158	9.669332	10.330668	10.042842	10.373510	58
3	9.626760	9.957099	9.669661	10.330339	10.042901	10.373240	57
4	9.627030	9.957040	9.669991	10.330009	10.042960	10.372970	56
5	9.627300	9.956981	9.670320	10.329680	10.043019	10.372700	55
6	9.627570	9.956921	9.670649	10.329351	10.043078	10.372430	54
7	9.627840	9.956862	9.670977	10.329023	10.043138	10.372160	53
8	9.628109	9.956803	9.671306	10.328694	10 043197	10.371891	52
9	9.628378	9.956744	9.671634	10.328365	10.043256	10.371622	51
10	9.628647	9.956684	9.671963	10.328037	10.043316	10.371353	50
11	9.628916	9.956625	9.672291	10.327709	10.043375	10.371084	49
12	9.629184	9.956566	9.672619	10.327381	10.043434	10.370815	48
13	9.629453	9.956506	9 672947	10.327053	10.043494	10.370547	47
14	9.629721	9.956447	9.673274	10.326725	10.043553	10.370279	46
15	9.629989	9.956387	9.673602	10.326398	10.043613	10.370011	45
16	9.630251	9.956327	9.673929	10.326071	10.043673	10.369743	44
17	9.630524	9.956268	9.674257	10.325743	10.043732	10.369476	43
18	9 630792	9.956208	9.674584	10.325416	10.043792	10.369208	42
19	9.631059	9.956148	9.674910	10.325089	10.043852	10.368941	41
20	9.631326	9.956089	9.675237	10.324763	10.043911	10.368674	40
21	9.631593	9.956029	9.675564	10.324436	10.043971	10.368407	39
22	9.631859	9.955969	9.675890	10.324110	10.044031	10.368141	38
23	9.632125	9.955909	9.676216	10.323783	10.044091	10.367874	37
24	9.632392	9.955849	9.676543	10.323457	10.044151	10.367608	36
25	9.632658	9.955789	9.676869	10.323131	10.044211	10.367342	35
26	9.632923	9.955729	9.677194	10.322806	10.044271	10.367077	34
27	9.633189	9.955669	9.677520	10.322480	10.044331	10.366811	33
28	9.633454	9.955609	9.677846	10.322154	10.044391	10.366546	32
29	9.633719	9.955548	9.678171	10.321829	10.044452	10.366281	31
30	9.633984	9.955488	9.678496	10.321504	10.044512	10 366016	30
31	9.634249	9.955428	9.678821	10.321179	10.044572	10.365751	29
32	9.634514	9.955368	9.679146	10.320854	10.044632	10.365486	28
33	9.634778	9.955307	9.679471	10.320529	10.044693	10.365222	27
34	9.635042	9.955247	9.679795	10.320205	10.044753	10.364958	26
35	9.635306	9.955186	9.680120	10.319880	10.044814	10.364694	25
36	9.635570	9.955126	9.680444	10.519556	10.044874	10.364430	24
37	9.635833	9.955065	9.680768	10.319232	10.044935	10.364166	23
38	9.636097	9.955005	9.681092	10.318908	10 044995	10.363903	22
39	9.636360	9.954944	9.681416	10.318584	10.045056	10.363640	21
40	9.636623	9.954883	9.681740	10.318260	10.045117	10.363377	20
41	9.636886	9.954823	9.682063	10.317937	10.045177	10.363144	19
42	9.637148	9.954762	9.682386	10.317613	10.045238	10.362852	18
43	9.637411	9.954701	9.682710	10.317290	10.045299	10.362589	17
44	9.637673	9.954640	9.683033	10.316967	10.045359	10.362327	16
45	9.637935	9.954579	9.683356	10.316644	10.045421	10.362065	15
46	9.638197	9.954518	9.683678	10.316321	10.045482	10.361803	14
47	9.638458	9.954457	9.684001	10.315999	10.045543	10.361541	13
48	9.638720	9.954396	9.684324	10.315676	10.045604	10.361280	12
49	9.638981	9.954335	9.684646	10.315354	10.045665	10.361019	11
50	9.639242	9.954274	9.684968	10.315032	10.045726	10.360758	10
51	9.639503	9.954213	9.685290	10.314710	10.045787	10.360497	9
52	9.639764	9.954152	9.685612	10.314388	10.045848	10.360236	8
53	9.640024	9.954090	9.685934	10.314066	10.045910	10.359976	7
54	9.640284	9.954029	9.686255	10.313745	10.045971	10.359716	6
55	9.640544	9.953968	9.686577	10.313423	10.046032	10.359455	5
56	9.640804	9.953906	9.686898	10.313102	10.046094	10.359196	4
57	9.641064	9.953845	9.687219	10.312781	10.046155	10.358936	3
58	9.641323	9.953783	9.687540	10.312460	10.046217	10.358676	2
59	9.641583	9.953722	9.687861	10.312139	10.046278	10.358417	1
60	9.641842	9.953660	9.688182	10.311818	10.046340	10.358158	0
		Sine		Tang.		Secant.	M

64 Degrees.

26 *Degrees.*

Min.	Sine.		Tang.		Secant.		
0	9.641842	9.953660	9.688182	10.311818	10.046340	10.358158	60
1	9.642101	9.953598	9.688502	10.311498	10.046410	10.357899	59
2	9.642360	9.953537	9.688823	10.311177	10.046463	10.357640	58
3	9.642618	9.953475	9.689143	10.310857	10.046525	10.357382	57
4	9.642876	9.953412	9.689463	10.310537	10.046587	10.357123	56
5	9.643135	9.953351	9.689783	10.310217	10.046648	10.356865	55
6	9.643393	9.953290	9.690103	10.309897	10.046710	10.356607	54
7	9.643650	9.953228	9.690423	10.309577	10.046772	10.356350	53
8	9.643908	9.953166	9.690742	10.309258	10.046834	10.356092	52
9	9.644165	9.953104	9.691062	10.308938	10.046896	10.355835	51
10	9.644423	9.953042	9.691381	10.308619	10.046958	10.355577	50
11	9.644680	9.952980	9.691700	10.308300	10.047020	10.355320	49
12	9.643936	9.952917	9.692019	10.307981	10.047082	10.355063	48
13	9.645193	9.952855	9.692338	10.307662	10.047145	10.354807	47
14	9.945450	9.952793	9.692656	10.307343	10.047207	10.354550	46
15	9.645706	9.952731	9.692975	10.307025	10.047269	10.354294	45
16	9.645962	9.952668	9.693293	10.306707	10.047331	10.354038	44
17	9.646218	9.952606	9.693612	10.306388	10.047394	10.353782	43
18	9.646473	9.952544	9.693930	10.306070	10.047456	10.353526	42
19	9.646729	9.952481	9.694248	10.305752	10.047519	10.353271	41
20	9.646984	9.952419	9.694566	10.305434	10.047581	10.353016	40
21	9.647239	9.952356	9.694883	10.305117	10.047644	10.352760	39
22	9.647494	9.952294	9.695201	10.304799	10.047706	10.352505	38
23	9.647749	9.952231	9.695518	10.304482	10.047769	10.352251	37
24	9.648004	9.952168	9.695835	10.304164	10.047832	10.351996	36
25	9.648258	9.952105	9.696153	10.303847	10.047894	10.351742	35
26	9.648512	9.952043	9.696470	10.303530	10.047957	10.351488	34
27	9.648766	9.951980	9.696786	10.303213	10.048020	10.351233	33
28	9.649020	9.951917	9.697103	10.302897	10.048083	10.350980	32
29	9.649274	9.951854	9.697420	10.302580	10.048146	10.350726	31
30	9.649527	9.951791	9.697736	10.302264	10.048209	10.350473	30
31	9.649781	9.951728	9.698053	10.301947	10.048272	10.350219	29
32	9.650034	9.951665	9.698369	10.301631	10.048335	10.349966	28
33	9.650287	9.951602	9.698685	10.301315	10.048398	10.349713	27
34	9.650939	9.951539	9.699001	10.300999	10.048461	10.349460	26
35	9.650792	9.951476	9.699316	10.300684	10.048524	10.349208	25
36	9.651044	9.951412	9.699632	10.300368	10.048588	10.348956	24
37	9.651297	9.951349	9.699947	10.300053	10.048651	10.348703	23
38	9.651549	9.951286	9.700263	10.299737	10.048714	10.348451	22
39	9.651800	9.951222	9.700578	10.299422	10.048778	10.348200	21
40	9.652052	9.951159	9.700893	10.299107	10.048841	10.347948	20
41	9.652303	9.951096	9.701208	10.298792	10.048904	10.347696	19
42	9.652555	9.951032	9.701523	10.298477	10.048968	10.347445	18
43	9.652806	9.950968	9.701837	10.298163	10.049031	10.347194	17
44	9.653057	9.950905	9.702152	10.297848	10.049095	10.346943	16
45	9.653307	9.950841	9.702466	10.297534	10.049159	10.346692	15
46	9.653558	9.950777	9.702780	10.297219	10.049222	10.346442	14
47	9.653808	9.950714	9.703095	10.296905	10.049286	10.346192	13
48	9.654059	9.950650	9.703409	10.296591	10.049350	10.345941	12
49	9.654309	9.950586	9.703722	10.296277	10.049414	10.345691	11
50	9.654558	9.950522	9.704036	10.295964	10.049478	10.345442	10
51	9.654808	9.950458	9.704350	10.295650	10.049542	10.345192	9
52	9.655057	9.950394	9.704663	10.295337	10.049606	10.344442	8
53	9.655307	9.950330	9.704976	10.295023	10.049670	10.344693	7
54	9.655556	9.950266	9.705290	10.294710	10.049734	10.344444	6
55	9.655805	9.950202	9.705603	10.294397	10.049798	10.344195	5
56	9.656054	9.950138	9.705916	10.294084	10.049862	10.343946	4
57	9.656302	9.950074	9.706228	10.293772	10.049926	10.343698	3
58	9.656550	9.950000	9.706541	10.293459	10.049990	10.343449	2
59	9.656799	9.949945	9.706853	10.293146	10.050055	10.343201	1
60	9.657047	9.949881	9.707166	10.292834	10.051119	10.342953	0
		Sine.		Tang.		Secant.	M

63 *Degrees.*

27 *Degrees.*

Min.	Sine.		Tang.		Secant.		
0	9.657047	9.949881	9.707166	10.292834	10.050119	10.342953	60
1	9.657295	9.949816	9.707478	10.292522	10.050183	10.342705	59
2	9.657542	9.949754	9.707790	10.292210	10.050248	10.342458	58
3	9.657790	9.949688	9.708102	10.291898	10.050312	10.342210	57
4	9.658037	9.949623	9.708414	10.291586	10.050377	10.341963	56
5	9.658284	9.949558	9.708726	10.291274	10.050441	10.341716	55
6	9.658531	9.949494	9.709037	10.290963	10.050506	10.341469	54
7	9.658778	9.949429	9.709349	10.290651	10.050571	10.341222	53
8	9.659024	9.949364	9.709660	10.290340	10.050635	10.340975	52
9	9.659271	9.949300	9.709971	10.290027	10.050700	10.740729	51
10	9.659517	9.949235	9.710282	10.289718	10.050765	10.340483	50
11	9.659763	9.949170	9.710593	10.289407	10.050830	10.340237	49
12	9.660009	9.949105	9.710904	10.289096	10.050895	10.339991	48
13	9.660255	9.949040	9.711215	10.288785	10.050960	10.339745	47
14	9.660500	9.948975	9.711525	10.288475	10.051025	10.339499	46
15	9.660746	9.948910	9.711836	10.288164	10.051090	10.339254	45
16	9.660991	9.948845	9.712146	10.287854	10.051155	10.339009	44
17	9.661236	9.948780	9.712456	10.287544	10.051220	10.338764	43
18	9.661481	9.948715	9.712766	10.287234	10.051285	10.338519	42
19	9.661726	9.948649	9.713076	10.286924	10.051350	10.338274	41
20	9.661970	9.948584	9.713386	10.286614	10.051416	10.338030	40
21	9.662214	9.948519	9.713699	10.286304	10.051481	10.337785	39
22	9.662459	9.948453	9.714004	10.285995	10.051546	10.337541	38
23	9.662703	9.948388	9.714314	10.285685	10.051612	10.337297	37
24	9.662946	9.948323	9.714624	10.285376	10.051677	10.337054	36
25	9.663190	9.948257	9.714933	10.285067	10.051743	10.336810	35
26	9.663433	9.948192	9.715242	10.284758	10.051808	10.336566	34
27	9.663677	9.948126	9.715551	10.284449	10.051874	10.336323	33
28	9.663920	9.948060	9.715859	10.284140	10.051940	10.336080	32
29	9.664163	9.947995	9.716168	10.283832	10.052005	10.335837	31
30	9.664406	9.947929	9.716477	10.283523	10.052071	10.335594	30
31	9.664648	9.947863	9.716785	10.283215	10.052137	10.335352	29
32	9.664891	9.947797	9.717093	10.282907	10.052203	10.335109	28
33	9.665133	9.947731	9.717401	10.282599	10.052269	10.334867	27
34	9.665375	9.947665	9.717709	10.282291	10.052334	10.334625	26
35	9.665617	9.947599	9.718017	10.281983	10.052400	10.334383	25
36	9.665859	9.947533	9.718325	10.281675	10.052466	10.334141	24
37	9.666100	9.947467	9.718633	10.281367	10.052533	10.333900	23
38	9.666341	9.947401	9.718940	10.281060	10.052599	10.333658	22
39	9.666583	9.947335	9.719248	10.280752	10.052665	10.333417	21
40	9.666825	9.967269	9.719555	10.280445	10.052731	10.333176	20
41	9.667065	9.967203	9.719862	10.280138	10.052797	10.332935	19
42	9.667305	9.967136	9.720169	10.279831	10.052864	10.332695	18
43	9.667546	9.967070	9.720476	10.279524	10.052930	10.332454	17
44	9.667786	9.967004	9.720783	10.279217	10.052996	10.332214	16
45	9.668026	9.946937	9.721089	10.278911	10.053063	10.331973	15
46	9.668266	9.946871	9.721396	10.278604	10.053129	10.331733	14
47	9.668506	9.946804	9.721702	10.278298	10.053196	10.331494	13
48	9.668746	9.946738	9.722008	10.277991	10.053262	10.331254	12
49	9.668986	9.946671	9.722315	10.277685	10.053329	10.331014	11
50	9.669225	9.946604	9.722621	10.277379	10.053396	10.330775	10
51	9.669464	9.946538	9.722927	10.277073	10.053462	10.330536	9
52	9.669703	9.946471	9.723232	10.276768	10.053529	10.330297	8
53	9.669942	9.946404	9.723538	10.276462	10.053566	10.330058	7
54	9.670181	9.946337	9.723844	10.276156	10.053663	10.329819	6
55	9.670419	9.946270	9.724149	10.275851	10.053730	10.329581	5
56	9.670658	9.946203	9.724454	10.275546	10.053797	10.329342	4
57	9.670896	9.946136	9.724759	10.275240	10.053864	10.329104	3
58	9.671134	9.946069	9.725065	10.274935	10.053931	10.328866	2
59	9.671372	9.946002	9.725369	10.274630	10.053998	10.328628	1
60	9.671609	9.945935	9.725674	10.274326	10.054065	10.328391	0
		Sine		Tang.		Secant.	M

62 *Degrees.*

28 *Degrees.*

Min.	Sine.		Tang.		Secant.		
0	9.671609	9.945935	9.725674	10.274326	10.054065	10.328391	60
1	9.671847	9.945868	9.725979	10.274021	10.054132	10.328153	59
2	9.672084	9.945800	9.726284	10.273716	10.054199	10.327916	58
3	9.672321	9.945733	9.726588	10.273412	10.054267	10.327679	57
4	9.672558	9.945666	9.726892	10.273107	10.054334	10.327442	56
5	9.672795	9.945598	9.727197	10.272803	10.054401	10.327205	55
6	9.673032	9.945531	9.727501	10.272499	10.054469	10.326968	54
7	9.673268	9.945464	9.727805	10.272195	10.054536	10.326732	53
8	9.673505	9.945396	9.728109	10.271891	10.054604	10.326495	52
9	9.673741	9.945328	9.728412	10.271588	10.054671	10.326259	51
10	9.673977	9.945261	9.728716	10.271284	10.054739	10.326023	50
11	9.674213	9.945193	9.729020	10.270980	10.054807	10.325787	49
12	9.674448	9.945125	9.729323	10.270677	10.054874	10.325551	48
13	9.674684	9.945058	9.729626	10.270374	10.054942	10.325316	47
14	9.674919	9.944990	9.729929	10.270070	10.055010	10.325081	46
15	9.675155	9.944922	9.730232	10.269767	10.055078	10.324845	45
16	9.675390	9.944854	9.730535	10.269465	10.055146	10.324610	44
17	9.675624	9.944786	9.730838	10.269162	10.055214	10.324376	43
18	9.675859	9.944718	9.731141	10.268859	10.055282	10.324141	42
19	9.676094	9.944650	9.731444	10.268556	10.055350	10.323906	41
20	9.675328	9.944582	9.731746	10.268254	10.055418	10.323672	40
21	9.676562	9.944514	9.732048	10.267952	10.055486	10.323438	39
22	9.676796	9.944446	9.732351	10.267649	10.055554	10.323204	38
23	9.677030	9.944377	9.732653	10.267347	10.055622	10.322970	37
24	9.677264	9.944309	9.732955	10.267045	10.055691	10.322736	36
25	9.677497	9.944241	9.733257	10.266743	10.055759	10.322502	35
26	9.677731	9.944172	9.733558	10.266442	10.055827	10.322269	34
27	9.677964	9.944104	9.733860	10.266140	10.055896	10.322036	33
28	9.678197	9.944036	9.734162	10.265838	10.055964	10.321803	32
29	9.678430	9.943967	9.734463	10.265537	10.056033	10.321570	31
30	9.678663	9.943898	9.734764	10.265236	10.056101	10.321337	30
31	9.678895	9.943830	9.735066	10.264934	10.056170	10.321104	29
32	9.679128	9.943761	9.735367	10.264633	10.056239	10.320872	28
33	9.679360	9.943692	9.735668	10.264332	10.056307	10.320640	27
34	9.679592	9.943624	9.735968	10.264031	10.056376	10.320408	26
35	9.679824	9.943555	9.736269	10.263731	10.056445	10.320176	25
36	9.680056	9.943486	9.736570	10.263430	10.056514	10.319944	24
37	9.680288	9.943417	9.736870	10.263129	10.056583	10.319712	23
38	9.680519	9.943348	9.737171	10.262829	10.056652	10.319481	22
39	9.680750	9.943279	9.737471	10.262529	10.056721	10.319250	21
40	9.680982	9.943210	9.737771	10.262229	10.056790	10.319018	20
41	9.681213	9.943141	9.738071	10.261928	10.056859	10.318787	19
42	9.681443	9.943072	9.738371	10.261629	10.056928	10.318557	18
43	9.681674	9.943003	9.738671	10.261329	10.056995	10.318326	17
44	9.681905	9.942933	9.738971	10.261029	10.057066	10.318095	16
45	9.682135	9.942864	9.739271	10.260729	10.057136	10.317865	15
46	9.682365	9.942795	9.739570	10.260430	10.057205	10.317635	14
47	9.682595	9.942725	9.739870	10.260130	10.057274	10.317405	13
48	9.682825	9.942656	9.740169	10.259831	10.057344	10.317175	12
49	9.683055	9.942587	9.740468	10.259532	10.057413	10.316945	11
50	9.683284	9.942517	9.740767	10.259233	10.057483	10.316716	10
51	9.683514	9.942448	9.741066	10.258934	10.057552	10.316486	9
52	9.683743	9.942378	9.741365	10.258635	10.057622	10.316257	8
53	9.683972	9.942308	9.741664	10.258336	10.057692	10.316028	7
54	9.684201	9.942239	9.741962	10.258038	10.057761	10.315799	6
55	9.684430	9.942169	9.742261	10.257739	10.057831	10.315570	5
56	9.684658	9.942099	9.742559	10.257441	10.057901	10.315342	4
57	9.684887	9.942029	9.742858	10.257142	10.057971	10.315113	3
58	9.685115	9.941959	9.743156	10.256844	10.058041	10.314885	2
59	9.685343	9.941889	9.743454	10.256546	10.058111	10.314657	1
60	9.685571	9.941819	9.743752	10.256248	10.058181	10.314429	0
		Sine.		Tang.		Secant.	M

61 *Degrees.*

29 *Degrees.*

Min.	Sine.		Tang.		Secant.		
0	9.685571	9.941819	9.743752	10.256248	10.058181	10.314429	60
1	9.685799	9.941747	9.744050	10.255950	10.058251	10.314201	59
2	9.686027	9.941679	9.744348	10.255652	10.058321	10.313973	58
3	9.686254	9.941609	9.744645	10.255355	10.058391	10.313746	57
4	9.686482	9.941539	9.744943	10.255057	10.058461	10.313518	56
5	9.686709	9.941468	9.745240	10.254700	10.058531	10.313291	55
6	9.686936	9.941398	9.745538	10.254462	10.058602	10.313604	54
7	9.687163	9.941328	9.745835	10.254165	10.058672	10.312837	53
8	9.687389	9.941257	9.746132	10.253868	10.058742	10.312610	52
9	9.687616	9.941187	9.746429	10.253571	10.058813	10.312384	51
10	9.687842	9.941117	9.746726	10.253274	10.058883	10.312157	50
11	9.688069	9.941046	9.747023	10.252977	10.058954	10.311931	49
12	9.688295	9.940975	9.747319	10.252681	10.059024	10.311705	48
13	9.688521	9.940905	9.747616	10.252384	10.059095	10.311479	47
14	9.688747	9.940834	9.747912	10.252087	10.059166	10.311253	46
15	9.688972	9.940763	9.748209	10.251791	10.059237	10.311028	45
16	9.689198	9.940693	9.748505	10.251495	10.059307	10.310802	44
17	9.689423	9.940622	9.748801	10.251199	10.059378	10.310577	43
18	9.689648	9.940551	9.749097	10.250903	10.059449	10.310352	42
19	9.689873	9.940480	9.749393	10.250607	10.059520	10.310127	41
20	9.690098	9.940409	9.749689	10.250311	10.059591	10.309902	40
21	9.690323	9.940338	9.749985	10.250015	10.059662	10.309677	39
22	9.690548	9.940265	9.750281	10.249719	10.059733	10.309452	38
23	9.690772	9.940196	9.750576	10.249424	10.059804	10.309228	37
24	9.690996	9.940125	9.750872	10.249128	10.059875	10.309004	36
25	9.691220	9.940053	9.751167	10.248833	10.059946	10.308779	35
26	9.691444	9.939982	9.751462	10.248538	10.060018	10.308555	34
27	9.691668	9.939913	9.751757	10.248243	10.060089	10.308332	33
28	9.691892	9.939842	9.752052	10.247948	10.060160	10.308108	32
29	9.692115	9.939768	9.752347	10.247653	10.060232	10.307884	31
30	9.692339	9.939697	9.752642	10.247358	10.060303	10.307661	30
31	9.692562	9.939625	9.752937	10.247063	10.060375	10.307348	29
32	9.692785	9.939554	9.753231	10.246769	10.060446	10.307215	28
33	9.693009	9.939482	9.753526	10.246474	10.060518	10.306991	27
34	9.693231	9.939410	9.753820	10.246180	10.060589	10.306769	26
35	9.693453	9.939339	9.754115	10.245885	10.060661	10.306547	25
36	9.663676	9.939267	9.754409	10.245591	10.060733	10.306324	24
37	9.693898	9.939195	9.754703	10.245297	10.060805	10.306102	23
38	9.694120	9.939123	9.754997	10.245003	10.060877	10.305880	22
39	9.664342	9.939051	9.755291	10.244709	10.060948	10.305658	21
40	9.694564	9.938980	9.755585	10.244415	10.061020	10.305436	20
41	9.694786	9.938908	9.755878	10.244122	10.061092	10.305214	19
42	9.695007	9.938836	9.756172	10.243828	10.061164	10.304993	18
43	9.695229	9.938763	9.756465	10.243535	10.061236	10.304771	17
44	9.695450	9.938691	9.756759	10.243241	10.061309	10.304550	16
45	9.695671	9.938619	9.757052	10.242948	10.061381	10.304329	15
46	9.695892	9.938547	9.757345	10.242655	10.061453	10.304108	14
47	9.696113	9.938475	9.757638	10.242362	10.061525	10.303887	13
48	9.696334	9.938402	9.757931	10.242069	10.061598	10.303666	12
49	9.696554	9.938330	9.758224	10.241776	10.061670	10.303446	11
50	9.696774	9.938258	9.758517	10.241483	10.061742	10.303225	10
51	9.696995	9.938185	9.758810	10.241190	10.061815	10.303005	9
52	9.697215	9.938113	9.759102	10.240896	10.061887	10.302785	8
53	9.697435	9.938040	9.759395	10.240605	10.061960	10.302565	7
54	9.697654	9.937967	9.759687	10.240313	10.061033	10.302345	6
55	9.697874	9.937895	9.759979	10.240021	10.062105	10.302126	5
56	9.698094	9.937822	9.760272	10.239728	10.062178	10.301906	4
57	9.698313	9.937749	9.760564	10.239436	10.062251	10.301687	3
58	9.698532	9.937676	9.760856	10.239144	10.062324	10.301468	2
59	9.698751	9.937603	9.761148	10.238852	10.062396	10.301249	1
60	9.698970	9.937531	9.761439	10.238561	10.062469	10.301030	0
		Sine		Tang.		Secant.	M

60 *Degrees.*

30 *Degrees.*

Min.	Sine.		Tang.		Secant.		
0	9.698970	9.937531	9.761439	10.238561	10.062469	10.301030	60
1	9.699189	9.937458	9.761731	10.238269	10.062543	10.300811	59
2	9.699407	9.937385	9.762223	10.237977	10.062615	10.300593	58
3	9.699626	9.937312	9.762314	10.237686	10.062688	10.300374	57
4	9.699844	9.937238	9.762606	10.237395	10.062761	10.300156	56
5	9.700062	9.937165	9.762897	10.237103	10.062835	10.299938	55
6	9.700280	9.937092	9.763188	10.236812	10.062908	10.299720	54
7	9.700498	9.937019	9.763479	10.236521	10.062981	10.299502	53
8	9.700716	9.936946	9.763770	10.236230	10.063054	10.299284	52
9	9.700933	9.936872	9.764061	10.235939	10.063128	10.299067	51
10	9.701151	9.936799	9.764352	10.235648	10.063201	10.298849	50
11	9.701368	9.936725	9.764643	10.235357	10.063275	10.298634	49
12	9.701585	9.936652	9.764933	10.235067	10.063348	10.298415	48
13	9.701802	9.936578	9.765224	10.234776	10.063422	10.298198	47
14	9.702019	9.936505	9.765514	10.234486	10.063495	10.297981	46
15	9.702236	9.936431	9.765805	10.234195	10.063569	10.297764	45
16	9.702452	9.936357	9.766095	10.233905	10.063643	10.297548	44
17	9.702669	9.936284	9.766385	10.233615	10.063716	10.297331	43
18	9.702885	9.936210	9.766675	10.233325	10.063790	10.297115	42
19	9.703101	9.936136	9.766965	10.233035	10.063864	10.296899	41
20	9.703317	9.936062	9.767255	10.232745	10.063938	10.296683	40
21	9.703533	9.935988	9.767545	10.232455	10.064012	10.296467	39
22	9.703749	9.935914	9.767834	10.232166	10.064086	10.296251	38
23	9.703964	9.935840	9.768124	10.231876	10.064160	10.296036	37
24	9.704179	9.935766	9.768413	10.231586	10.064234	10.295820	36
25	9.704395	9.935692	9.768703	10.231297	10.064308	10.295605	35
26	9.704610	9.935618	9.768992	10.231008	10.064382	10.295390	34
27	9.704825	9.935543	9.769281	10.230719	10.064457	10.295175	33
28	9.705040	9.935469	9.769570	10.230429	10.064531	10.294960	32
29	9.705254	9.935395	9.769860	10.230140	10.064605	10.294746	31
30	9.705469	9.935320	9.770148	10.229851	10.064680	10.294531	30
31	9.705683	9.935246	9.770437	10.229563	10.064754	10.294317	29
32	9.705897	9.935171	9.770726	10.229274	10.064828	10.294102	28
33	9.706111	9.935097	9.771015	10.228985	10.064903	10.293888	27
34	9.706326	9.935022	9.771303	10.228697	10.064978	10.293674	26
35	9.706539	9.934948	9.771592	10.228408	10.065052	10.293461	25
36	9.706753	9.934873	9.771880	10.228120	10.065127	10.293247	24
37	9.706967	9.934798	9.772168	10.227832	10.065202	10.293033	23
38	9.707180	9.934723	9.772457	10.227543	10.065276	10.292820	22
39	9.707393	9.934649	9.772745	10.227255	10.065351	10.292607	21
40	9.707606	9.934574	9.773033	10.226967	10.065426	10.292393	20
41	9.707819	9.934499	9.773321	10.226679	10.065501	10.292181	19
42	9.708032	9.934424	9.773608	10.226392	10.065576	10.291968	18
43	9.708245	9.934349	9.773896	10.226104	10.065651	10.291755	17
44	9.708457	9.934274	9.774184	10.225816	10.065726	10.291542	16
45	9.708670	9.934199	9.774471	10.225529	10.065801	10.291330	15
46	9.708882	9.934123	9.774759	10.225241	10.065877	10.291118	14
47	9.709094	9.934048	9.775046	10.224954	10.065952	10.290906	13
48	9.709306	9.933973	9.775333	10.224667	10.066027	10.290694	12
49	9.709518	9.933898	9.775621	10.224379	10.066102	10.290482	11
50	9.709730	9.933822	9.775908	10.224092	10.066178	10.290270	10
51	9.709941	9.933747	9.776194	10.223805	10.066253	10.290058	9
52	9.710153	9.933671	9.776482	10.223518	10.066329	10.289847	8
53	9.710364	9.933596	9.776768	10.223231	10.066404	10.289636	7
54	9.710575	9.933520	9.777055	10.222945	10.066480	10.289429	6
55	9.710786	9.933444	9.777342	10.222658	10.066555	10.289214	5
56	9.710997	9.933369	9.777628	10.222372	10.066631	10.289003	4
57	9.711208	9.933293	9.777915	10.222085	10.066707	10.288792	3
58	9.711419	9.933217	9.778201	10.221799	10.066783	10.288581	2
59	9.711629	9.933141	9.778487	10.221512	10.066858	10.288371	1
60	9.711839	9.933066	9.778774	10.221226	10.066934	10.288161	0
		Sine.		Tang.		Secant.	M

59 *Degrees.*

31 *Degrees.*

Min.	Sine		Tang.		Secant.		
0	9.711839	9.933066	9.778774	10.221226	10.066934	10.288161	60
1	9.712049	9.932990	9.779060	10.220940	10.067010	10.287951	59
2	9.712260	9.932914	9.779346	10.220654	10.067086	10.287740	58
3	9.712469	9.932838	9.779632	10.220368	10.067162	10.287531	57
4	9.712679	9.932762	9.779918	10.220082	10.067238	10.287321	56
5	9.712889	9.932685	9.780203	10.219797	10.067315	10.287111	55
6	9.713098	9.932609	9.780489	10.219511	10.067391	10.286902	54
7	9.713308	9.932533	9.780775	10.219225	10.067467	10.286692	53
8	9.713517	9.932457	9.781060	10.218940	10.067543	10.286483	52
9	9.713726	9.932380	9.781346	10.218654	10.067620	10.286274	51
10	9.713935	9.932304	9.781631	10.218369	10.067696	10.286065	50
11	9.714144	9.932218	9.781916	10.218084	10.067772	10.285856	49
12	9.714352	9.932151	9.782201	10.217799	10.067849	10.285648	48
13	9.714561	9.932075	9.782486	10.217514	10.067925	10.285439	47
14	9.714769	9.931998	9.782771	10.217229	10.068002	10.285231	46
15	9.714978	9.931921	9.783056	10.216944	10.068079	10.285022	45
16	9.715186	9.931845	9.783341	10.216660	10.068155	10.284814	44
17	9.715394	9.931768	9.783626	10.216374	10.068232	10.284606	43
18	9.715601	9.931691	9.783910	10.216090	10.068300	10.284399	42
19	9.715809	9.931614	9.784195	10.215805	10.068386	10.284191	41
20	9.716017	9.931537	9.784479	10.215521	10.068463	10.283983	40
21	9.716224	9.931460	9.784764	10.215236	10.068536	10.283776	39
22	9.716432	9.931383	9.785048	10.214952	10.068616	10.283568	38
23	9.716639	9.931306	9.785332	10.214668	10.068693	10.283361	37
24	9.716846	9.931229	9.785616	10.214384	10.068771	10.283154	36
25	9.717053	9.931152	9.785900	10.214100	10.068848	10.282947	35
26	9.717259	9.931075	9.786184	10.213816	10.068925	10.282741	34
27	9.717466	9.930998	9.786468	10.213532	10.069002	10.282534	33
28	9.717672	9.930920	9.786752	10.213248	10.069079	10.282328	32
29	9.717879	9.930843	9.787036	10.212964	10.069157	10.282121	31
30	9.718085	9.930766	9.787319	10.212681	10.069234	10.281915	30
31	9.718291	9.930688	9.787603	10.212397	10.069312	10.281709	29
32	9.718497	9.930611	9.787886	10.212114	10.069389	10.281503	28
33	9.718703	9.930533	9.788170	10.211830	10.069467	10.281297	27
34	9.718909	9.930456	9.788453	10.211547	10.069544	10.281091	26
35	9.719114	9.930378	9.788736	10.211264	10.069622	10.280886	25
36	9.719320	9.930300	9.789019	10.210981	10.069700	10.280680	24
37	9.719525	9.930223	9.789302	10.210698	10.069777	10.280475	23
38	9.719730	9.930145	9.789585	10.210415	10.069855	10.280270	22
39	9.719935	9.930067	9.789868	10.210132	10.069933	10.280065	21
40	9.720140	9.929989	9.790151	10.209849	10.070011	10.279860	20
41	9.720345	9.929911	9.790433	10.209566	10.070089	10.279655	19
42	9.720549	9.929833	9.790716	10.209284	10.070167	10.279451	18
43	9.720754	9.929755	9.790999	10.209001	10.070245	10.279246	17
44	9.720958	9.929677	9.791281	10.208719	10.070323	10.279042	16
45	9.721162	9.929599	9.791563	10.208436	10.070401	10.278838	15
46	9.721366	9.929521	9.791846	10.208154	10.070479	10.278634	14
47	9.721570	9.929442	9.792128	10.207872	10.070558	10.278430	13
48	9.721774	9.929364	9.792410	10.207590	10.070636	10.278226	12
49	9.721978	9.929286	9.792692	10.207308	10.070714	10.278022	11
50	9.722181	9.929207	9.792974	10.207026	10.070793	10.277819	10
51	9.722385	9.929129	9.793256	10.206744	10.070871	10.277615	9
52	9.722588	9.929050	9.793538	10.206462	10.070950	10.277412	8
53	9.722791	9.928972	9.793819	10.206180	10.071028	10.277209	7
54	9.722994	9.928893	9.794101	10.205899	10.071107	10.277006	6
55	9.723197	9.928814	9.794383	10.205617	10.071185	10.276803	5
56	9.723400	9.928736	9.794664	10.205336	10.071264	10.276600	4
57	9.723603	9.928657	9.794945	10.205054	10.071343	10.276397	3
58	9.723805	9.928578	9.795227	10.204773	10.071422	10.276195	2
59	9.724007	9.928499	9.795508	10.204492	10.071501	10.275993	1
60	9.724210	9.928420	9.795789	10.204211	10.071579	10.275790	0
		Sine.		Tang.		Secant.	M

58 *Degrees.*

32 Degrees.

Min.	Sine		Tang.		Secant.		
0	9.724210	9.928420	9.795789	10.204211	10.071579	10.275790	60
1	9.724412	9.928341	9.796070	10.203930	10.071658	10.275588	59
2	9.724614	9.928262	9.796351	10.203649	10.071737	10.275386	58
3	9.724816	9.928183	9.796632	10.203368	10.071817	10.275184	57
4	9.725017	9.928104	9.796913	10.203087	10.071896	10.274983	56
5	9.725219	9.928025	9.797194	10.202806	10.071975	10.274781	55
6	9.725420	9.927946	9.797474	10.202525	10.072054	10.274580	54
7	9.725622	9.927867	9.797755	10.202245	10.072133	10.274378	53
8	9.725823	9.927787	9.798036	10.201964	10.072213	10.274177	52
9	9.726024	9.927704	9.798316	10.201684	10.072292	10.273976	51
10	9.726225	9.927628	9.798596	10.201404	10.072371	10.273775	50
11	9.726426	9.927549	9.798877	10.201123	10.072451	10.273574	49
12	9.726626	9.927469	9.799157	10.200843	10.072530	10.273374	48
13	9.726827	9.927390	9.799437	10.200563	10.072610	10.273173	47
14	9.727027	9.927310	9.799717	10.200283	10.072690	10.272973	46
15	9.727228	9.927231	9.799997	10.200003	10.072769	10.272772	45
16	9.727428	9.927151	9.800277	10.199723	10,072849	10.272572	44
17	9.727628	9.927071	9.800557	10.199443	10.072929	10.272372	43
18	9.727828	9.926991	9.800836	10.199163	10.073009	10.272172	42
19	9.728027	9.926911	9.801116	10.198884	10.073089	10.271972	41
20	9.728227	9.926831	9.801396	10.198604	10.073169	10.271773	40
21	9.728427	9.926751	9.801675	10.198325	10.073249	10.271573	39
22	9.728626	9.926671	9.801955	10.198045	10.073329	10.271374	38
23	9.728825	9.926591	9.802234	10.197766	10.073409	10.271175	37
24	9.729024	9.926511	9.802513	10.197487	10.073489	10.270976	36
25	9.729223	9.926431	9.802792	10.197207	10.073569	10.270777	35
26	9.729422	9.926351	9.803072	10.196928	10.073649	10.270578	34
27	9.729621	9.926270	9.803351	10.196649	10.073730	10.270379	33
28	9.729820	9.926190	9.803630	10.196370	10.073810	10.270180	32
29	9.730018	9.926110	9.803908	10.196091	10.073891	10.269982	31
30	9.730216	9.926029	9.804187	10.195813	10.073971	10.269783	30
31	9.730415	9.925949	9.804466	10.195534	10.074051	10.269585	29
32	9.730613	9.925868	9.804745	10.195255	10.074132	10.269387	28
33	9.730811	9.925787	9.805023	10.194977	10.074212	10.269189	27
34	9.731009	9.925707	9.805302	10.194698	10.074293	10.268991	26
35	9.731206	9.925626	9.805580	10.194420	10.074374	10.268794	25
36	9.731404	9.925545	9.805859	10.194141	10.074455	10.268596	24
37	9.731601	9.925465	9.806137	10.193863	10.074535	10.268398	23
38	9.731799	9.925384	9.806415	10.193585	10.074616	10.268201	22
39	9.731996	9.925303	9.806693	10.193307	10.074697	10.268004	21
40	9.732193	9.925222	9.806971	10.193029	10.074778	10.267807	20
41	9.732390	9.925141	9.807249	10.192751	10.074859	10.267601	19
42	9.732587	9.925051	9.807527	10.192473	10.074940	10.267413	18
43	9.732784	9.924979	9.807805	10.192195	10.075021	10.267216	17
44	9.732980	9.924897	9.808083	10.191917	10.075103	10.267020	16
45	9.733177	9.924816	9.808361	10.191639	10.075184	10.266823	15
46	9.733373	9.924735	9.808638	10.191362	10.075265	10.266627	14
47	9.733569	9.924653	9.808916	10.191084	10.075346	10.266431	13
48	9.733765	9.924572	9.809193	10.190807	10.075428	10.266235	12
49	9.733961	9.924491	9.809471	10.190529	10.075509	10.266039	11
50	9.734157	9.924409	9.809748	10.190252	10.075591	10.265843	10
51	9.734353	9.924328	9.810025	10.189975	10.075672	10.265647	9
52	9.734548	9.924246	9.810302	10.189697	10.075754	10.265451	8
53	9.734744	9.924164	9.810580	10.189420	10.075836	10.265256	7
54	9.734939	9.924083	9.810857	10.189143	10.075917	10.265061	6
55	9.735134	9.924011	9.811134	10.188866	10.075909	10.264865	5
56	9.735330	9.923919	9.811410	10.188589	10.076080	10.264670	4
57	9.735525	9.923837	9.811687	10.188313	10.076163	10.264475	3
58	9.735719	9.923755	9.811964	10.188036	10.076245	10.264280	2
59	9.735914	9.923673	9.812241	10.187759	10.076327	10.264086	1
60	9.736109	9.923591	9.812517	10.187483	10.076409	10.264891	0
		Sine		Tang.		Secant.	M

57 Degrees.

33 Degrees.

Min.	Sine		Tang.		Secant.		
0	9.736103	9.923591	9.812517	10.187483	10.076409	10.263891	60
1	9.736303	9.923509	9.812794	10.187206	10.076491	10.263697	59
2	9.736498	9.923427	9.813070	10.186930	10.076573	10.263502	58
3	9.736692	9.923345	9.813347	10.186653	10.076655	10.263308	57
4	9.736886	9.923263	9.813623	10.186377	10.076737	10.263114	56
5	9.737080	9.923180	9.813899	10.186101	10.076819	10.262920	55
6	9.737274	9.923098	9.814175	10.185824	10.076902	10.262726	54
7	9.737467	9.923016	9.814452	10.185548	10.076984	10.262532	53
8	9.737661	9.922933	9.814728	10.185272	10.077067	10.262339	52
9	9.737855	9.922851	9.815004	10.184996	10.077149	10.262145	51
10	9.738048	9.922768	9.815279	10.184720	10.077232	10.261952	50
11	9.738241	9.922686	9.815555	10.184445	10.077314	10.261759	49
12	9.738434	9.922603	9.815831	10.184169	10.077397	10.261566	48
13	9.738627	9.922520	9.816107	10.183893	10.077479	10.261373	47
14	9.738820	9.922438	9.816382	10.183618	10.077562	10.261180	46
15	9.739013	9.922355	9.816658	10.183342	10.077645	10.260987	45
16	9.739205	9.922272	9.816933	10.183066	10.077728	10.260794	44
17	9.739398	9.922189	9.817209	10.182791	10.077811	10.260602	43
18	9.739590	9.922106	9.817484	10.182516	10.077894	10.260410	42
19	9.739783	9.922023	9.817759	10.182240	10.077977	10.260217	41
20	9.739975	9.921940	9.818035	10.181965	10.078060	10.260025	40
21	9.740167	9.921857	9.818310	10.181690	10.078143	10.259833	39
22	9.740359	9.921774	9.818585	10.181415	10.078226	10.259641	38
23	9.740570	9.921691	9.818860	10.181140	10.078309	10.259449	37
24	9.740742	9.921607	9.819135	10.180865	10.078393	10.259258	36
25	9.740934	9.921524	9.819410	10.180590	10.078476	10.259066	35
26	9.741125	9.921441	9.819684	10.180316	10.078559	10.258875	34
27	9.741316	9.921357	9.819959	10.180041	10.078643	10.258684	33
28	9.741507	9.921274	9.810234	10.179766	10.078726	10.258492	32
29	9.741699	9.921190	9.810508	10.179492	10.078810	10.258301	31
30	9.741889	9.921107	9.820783	10.179217	10.078893	10.258110	30
31	9.742080	9.921023	9.821057	10.178943	10.078977	10.257920	29
32	9.742271	9.920939	9.821332	10.178668	10.079061	10.257729	28
33	9.742462	9.920855	9.821606	10.178394	10.079144	10.257538	27
34	9.742652	9.920772	9.821880	10.178120	10.079228	10.257348	26
35	9.742842	9.920688	9.822154	10.177845	10.079312	10.257158	25
36	9.743032	9.920604	9.822429	10.177571	10.079396	10.256967	24
37	9.743222	9.920520	9.822703	10.177297	10.079480	10.256777	23
38	9.743412	9.920436	9.822977	10.177023	10.079564	10.256587	22
39	9.743602	9.920352	9.823250	10.176749	10.079648	10.256398	21
40	9.743792	9.920268	9.823524	10.176476	10.079732	10.256208	20
41	9.743982	9.920184	9.823798	10.176202	10.079816	10.256018	19
42	9.744171	9.920099	9.824072	10.175928	10.079901	10.255829	18
43	9.744361	9.910015	9.824345	10.175654	10.079985	10.255639	17
44	9.744550	9.919931	9.824619	10.175381	10.080069	10.255540	16
45	9.744739	9.919846	9.824893	10.175107	10.080154	10.255261	15
46	9.744928	9.919762	9.825165	10.174834	10.080238	10.255072	14
47	9.745117	9.919677	9.825439	10.174561	10.080322	10.254883	13
48	9.745306	9.919593	9.825713	10.174287	10.080407	10.254694	12
49	9.745494	9.919508	9.825986	10.174014	10.080492	10.254506	11
50	9.745683	9.919424	9.826259	10.173741	10.080576	10.254317	10
51	9.745871	9.919339	9.826532	10.173468	10.080661	10.254129	9
52	9.746059	9.919254	9.826805	10.173195	10.080746	10.253910	8
53	9.746248	9.919169	9.827078	10.172922	10.080831	10.253752	7
54	9.746436	9.919084	9.827351	10.172649	10.080915	10.253564	6
55	9.746624	9.919000	9.827624	10.172376	10.081000	10.253376	5
56	9.746811	9.918915	9.827897	10.172103	10.081085	10.253188	4
57	9.746999	9.918830	9.828170	10.171830	10.081170	10.253001	3
58	9.747187	9.918744	9.828442	10.171558	10.081255	10.252813	2
59	9.747374	9.918659	9.828715	10.171285	10.081341	10.252626	1
60	9.747562	9.918574	9.828987	10.171013	10.081426	10.252438	0
		Sine		Tang.		Secant.	M

56 Degrees.

34 Degrees.

Min.	Sine		Tang.		Secant.		
0	9.747562	9.918574	9.828987	10.171013	10.081426	10.252439	60
1	9.747749	9.918489	9.829260	10.170740	10.081511	10.252251	59
2	9.747936	9.918404	9.829532	10.170468	10.081596	10.252064	58
3	9.748123	9.918318	9.829805	10.170195	10.081682	10.251877	57
4	9.748310	9.918233	9.830077	10.169923	10.081767	10.251690	56
5	9.748497	9.918147	9.830349	10.169651	10.081852	10.251503	55
6	9.748683	9.918062	9.830621	10.169379	10.081938	10.251317	54
7	9.748870	9.917976	9.830893	10.169107	10.082024	10.251130	53
8	9.749056	9.917891	9.831165	10.168835	10.082109	10.250944	52
9	9.749242	9.917805	9.831437	10.168563	10.082195	10.250757	51
10	9.749429	9.917719	9.831709	10.168291	10.082281	10.250571	50
11	9.749615	9.917634	9.831981	10.168019	10.082366	10.250385	49
12	9.749801	9.917548	9.832253	10.167747	10.082452	10.250199	48
13	9.749987	9.917462	9.832525	10.167475	10.082538	10.250013	47
14	9.750172	9.917376	9.832796	10.167204	10.082624	10.249828	46
15	9.750358	9.917290	9.833068	10.166932	10.082710	10.249642	45
16	9.750543	9.917204	9.833339	10.166661	10.082796	10.249457	44
17	9.750729	9.917118	9.833611	10.166389	10.082882	10.249271	43
18	9.750914	9.917032	9.833882	10.166118	20.082968	10.249086	42
19	9.751099	9.916945	9.834154	10.165846	10.083054	10.248901	41
20	9.751284	9.916859	9.834425	10.165575	10.083141	10.248716	40
21	9.751469	9.916773	9.834696	10.165304	10.083227	10.248534	39
22	9.751654	9.916687	9.834967	10.165033	10.083313	10.248346	38
23	9.751838	9.916600	9.835238	10.164762	10.083400	10.248161	37
24	9.752023	9.916514	9.835509	10.164491	10.083486	10.247977	36
25	9.752207	9.916427	9.835780	10.164220	10.083573	10.247792	35
26	9.752392	9.916341	9.836051	10.163949	10.083659	10.247608	34
27	9.752576	9.916254	9.836322	10.163678	10.083746	10.247424	33
28	9.752760	9.916167	9.836593	10.163407	10.083833	10.247240	32
29	9.752944	9.916080	9.836864	10.163136	10.083919	10.247056	31
30	9.753128	9.915994	9.837134	10.162866	10.084006	10.246872	30
31	9.753312	9.915907	9.837405	10.162595	10.084093	10.246688	29
32	9.753495	9.915820	9.837675	10.162324	10.084180	10.246505	28
33	9.753679	9.915733	9.837946	10.162054	10.084267	10.246321	27
34	9.753862	9.915646	9.838216	10.161784	10.084354	10.246138	26
35	9.754046	9.915559	9.838487	10.161513	10.084441	10.245954	25
36	9.754229	9.915472	9.838757	10.161243	10.084528	10.245771	24
37	9.754412	9.915385	9.839027	10.160973	10.084615	10.245588	23
38	9.754595	9.915297	9.839297	10.160702	10.084703	10.245405	22
39	9.754778	9.915210	9.839568	10.160432	10.084790	10.245222	21
40	9.754960	9.915123	9.839838	10.160162	10.084877	10.245040	20
41	9.755143	9.915035	9.840108	10.159892	10.084965	10.244857	19
42	9.755326	9.914948	9.840378	10.159622	10.085052	10.244674	18
43	9.755508	9.914860	9.840647	10.159352	10.085140	10.244492	17
44	9.755690	9.914773	9.840917	10.159085	10.085227	10.244310	16
45	9.755872	9.914685	9.841187	10.158813	10.085315	10.244128	15
46	9.756054	9.914598	9.841457	10.158543	10.085402	10.243946	14
47	9.756236	9.914510	9.841726	10.158273	10.085490	10.243764	13
48	9.756418	9.914422	9.841996	10.158004	10.085578	10.243582	12
49	9.756600	9.914334	9.842266	10.157734	10.085666	10.243400	11
50	9.756781	9.914246	9.842535	10.157465	10.085754	10.243218	10
51	9.756963	9.914158	9.842805	10.157195	10.085842	10.243037	9
52	9.757144	9.914070	9.843074	10.156926	10.085930	10.242856	8
53	9.757326	9.913982	9.843343	10.156657	10.086018	10.242674	7
54	9.757507	9.913895	9.843612	10.156388	10.086106	10.242493	6
55	9.757688	9.913806	9.843882	10.156118	10.086194	10.242312	5
56	9.757869	9.913718	9.844151	10.155849	10.086282	10.242131	4
57	9.758049	9.913630	9.844420	10.155580	10.086370	10.241950	3
58	9.758230	9.913541	9.844689	10.155311	10.086459	10.241770	2
59	9.758411	9.913453	9.844958	10.155042	10.086547	10.241589	1
60	9.758591	9.913364	9.845227	10.154773	10.086635	10.241409	0
		Sine.		Tang.		Secant.	M.

55 Degrees

35 Degrees.

Min	Sine		Tang.		Secant.		
0	9.758591	9.913364	9.845227	10.154773	10.086635	10.241409	60
1	9.758772	9.913276	9.845496	10.154504	10.086724	10.241228	59
2	9.758952	9.913187	9.845764	10.154236	10.086812	10.241048	58
3	9.759132	9.913099	9.846033	10.153967	10.086901	10.240868	57
4	9.759312	9.913010	9.846302	10.153698	10.086990	10.240688	56
5	9.759492	9.912921	9.846570	10.153429	10.087078	10.240508	55
6	9.759672	9.912833	9.846839	10.153161	10.087167	10.240328	54
7	9.759851	9.912744	9.847107	10.152892	10.087256	10.240148	53
8	9.760031	9.912655	9.847376	10.152624	10.087345	10.239969	52
9	9.760211	9.912566	9.847644	10.152356	10.087434	10.239789	51
10	9.760390	9.912477	9.847913	10.152087	10.087523	10.239610	50
11	9.760569	9.912388	9.848181	10.151819	10.087612	10.239431	49
12	9.760749	9.912299	9.848449	10.151551	10.087701	10.239252	48
13	9.760927	9.912210	9.848717	10.151283	10.087790	10.239073	47
14	9.761106	9.912121	9.848985	10.151014	10.087879	10.238894	46
15	9.761285	9.912031	9.849254	10.150746	10.087968	10.238715	45
16	9.761464	9.911942	9.849522	10.150478	10.088058	10.238536	44
17	9.761642	9.911853	9.849790	10.150210	10.088147	10.238358	43
18	9.761821	9.911763	9.850057	10.149942	10.088237	10.238179	42
19	9.761999	9.911674	9.850325	10.149675	10.088326	10.238001	41
20	9.762177	9.911584	9.850593	10.149407	10.088416	10.237822	40
21	9.762356	9.911495	9.850861	10.149139	10.088505	10.237644	39
22	9.762534	9.911405	9.851128	10.148871	10.088595	10.237466	38
23	9.762712	9.911315	9.851396	10.148604	10.088684	10.237288	37
24	9.762889	9.911226	9.851664	10.148336	10.088774	10.237111	36
25	9.763067	9.911136	9.851931	10.148069	10.088864	10.236933	35
26	9.763245	9.911046	9.852199	10.147801	10.088954	10.236755	34
27	9.763422	9.910956	9.852466	10.147534	10.089044	10.236578	33
28	9.763600	9.910866	9.852733	10.147266	10.089134	10.236400	32
29	9.763777	9.910766	9.853001	10.146999	10.089224	10.236223	31
30	9.763954	9.910686	9.853268	10.146732	10.089314	10.236046	30
31	9.764131	9.910596	9.853535	10.146465	10.089404	10.235869	29
32	9.764306	9.910506	9.853802	10.146198	10.089494	10.235692	28
33	9.764485	9.910415	9.854069	10.145931	10.089585	10.235515	27
34	9.764662	9.910325	9.854336	10.145664	10.089675	10.235338	26
35	9.764838	9.910235	9.854603	10.145397	10.089765	10.235162	25
36	9.765015	9.910144	9.854870	10.145130	10.089856	10.234985	24
37	9.765191	9.910054	9.855137	10.144863	10.089946	10.234809	23
38	9.765367	9.909963	9.855404	10.144596	10.090037	10.234633	22
39	9.765544	9.909873	9.855671	10.144329	10.090127	10.234456	21
40	9.765720	9.909782	9.855938	10.144062	10.090218	10.234280	20
41	9.765896	9.909691	9.856204	10.143796	10.090308	10.234104	19
42	9.766071	9.909601	9.856471	10.143529	10.090399	10.233928	18
43	9.766247	9.909510	9.856737	10.143263	10.090490	10.233753	17
44	9.766423	9.909419	9.857004	10.142996	10.090581	10.233577	16
45	9.766599	9.909328	9.857270	10.142730	10.090672	10.233401	15
46	9.766774	9.909237	9.857537	10.142463	10.090763	10.233226	14
47	9.766949	9.909146	9.857803	10.142197	10.090854	10.233051	13
48	9.767124	9.909055	9.858069	10.141931	10.090945	10.232876	12
49	9.767290	9.908964	9.858336	10.141664	10.091036	10.232700	11
50	9.767475	9.908873	9.858602	10.141398	10.091127	10.232525	10
51	9.767649	9.908781	9.858868	10.141132	10.091219	10.232351	9
52	9.767824	9.908691	9.859134	10.140866	10.091310	10.232176	8
53	9.767999	9.908599	9.859400	10.140600	10.091401	10.232001	7
54	9.768173	9.508507	9.859666	10.440334	10.091493	10.231826	6
55	9.768348	9.908416	9.859932	10.140068	10.091584	10.231652	5
56	9.768522	9.908324	9.860198	10.139802	10.091676	10.231478	4
57	9.768697	9.908233	9.860464	10.139536	10.091767	10.231303	3
58	9.768871	9.908141	9.860730	10.139270	10.091859	10.231129	2
59	9.769045	9.908049	9.860995	10.139005	10.091951	10.230955	1
60	9.769219	9.907958	9.861261	10.138739	10.092042	10.230781	0
		Sine		Tang.		Secant.	M

54 Degrees.

36 Degrees.

Sine		Tang.		Secant.		
9.769219	9.907958	9.861261	10.138739	10.092042	10.230781	60
9.769392	9.907866	9.861527	10.138473	10.092134	10.230607	59
9.769566	9.907774	9.861792	10.138208	10.092226	10.230434	58
9.769740	9.907682	9.862058	10.137942	10.092318	10.230260	57
9.769913	9.907550	9.862323	10.137677	10.092410	10.230087	56
9.770087	9.907498	9.862589	10.137411	10.092502	10.229913	55
9.770260	9.907406	9.862854	10.137146	10.092594	10.229740	54
9.770433	9.907314	9.863119	10.136880	10.092686	10.229567	53
9.770606	9.907221	9.863385	10.136615	10.092778	10.229394	52
9.770779	9.907129	9.863650	10.136350	10.092871	10.229220	51
9.770952	9.907037	9.863915	10.136085	10.092963	10.229048	50
9.771125	9.906945	9.864180	10.135820	10.093055	10.228875	49
9.771298	9.906852	9.864445	10.135555	10.093148	10.228702	48
9.771470	9.906760	9.864710	10.135289	10.093240	10.228520	47
9.771642	9.906667	9.864975	10.135024	10.093333	10.228357	46
9.771815	9.906574	9.865240	10.134760	10.093425	10.228185	45
9.771987	9.906482	9.865505	10.134495	10.093518	10.228013	44
9.772159	9.906389	9.865770	10.134230	10.093611	10.227841	43
9.772331	9.906296	9.866035	10.133965	10.093704	10.227669	42
9.772503	9.906204	9.866300	10.133700	10.093796	10.227497	41
9.772675	9.906111	9.866564	10.133436	10.093889	10.227325	40
9.772847	9.906018	9.866829	10.133171	10.093982	10.227153	39
9.773018	9.905925	9.867094	10.132906	10.094075	10.226981	38
9.773190	9.905832	9.867358	10.132642	10.094167	10.226810	37
9.773361	9.905739	9.867623	10.132377	10.094261	10.226639	36
9.773533	9.905645	9.867887	10.132113	10.094355	10.226467	35
9.773704	9.905552	9.868152	10.131848	10.094448	10.226296	34
9.773875	9.905459	9.868416	10.131584	10.094541	10.226125	33
9.774046	9.905366	9.868680	10.131320	10.094634	10.225954	32
9.774217	9.905272	9.868945	10.131055	10.094728	10.225783	31
9.774388	9.905179	9.869209	10.130791	10.094821	10.225612	30
9.774558	9.905085	9.869473	10.130527	10.094915	10.225442	29
9.774729	9.904992	9.869737	10.130263	10.095008	10.225271	28
9.774899	9.904899	9.870001	10.129999	10.095102	10.225101	27
9.775070	9.904804	9.870265	10.129735	10.095196	10.224930	26
9.775240	9.904711	9.870529	10.129471	10.095289	10.224760	25
9.775410	9.904617	9.870793	10.129207	10.095383	10.224590	24
9.775580	9.904523	9.871057	10.128943	10.095477	10.224420	23
9.775750	9.904429	9.871321	10.128679	10.095571	10.224250	22
9.775920	9.904335	9.871585	10.128415	10.095665	10.224080	21
9.776090	9.904241	9.871849	10.128151	10.095759	10.223910	20
9.776259	9.904147	9.872112	10.127888	10.095853	10.223741	19
9.776429	9.904053	9.872376	10.127624	10.095947	10.223571	18
9.776598	9.903959	9.872640	10.127360	10.096041	10.223402	17
9.776768	9.903864	9.872903	10.127097	20.096136	10.223232	16
9.776937	9.903770	9.873167	10.126833	10.096230	10.223063	15
9.777106	9.903676	9.873430	10.126570	10.096324	10.222894	14
9.777275	9.903581	9.873694	10.126306	10.096419	10.222725	13
9.777444	9.903487	9.873957	10.126032	10.096513	10.222556	12
9.777613	9.903392	9.874220	10.125780	10.096608	10.222387	11
9.777781	9.903298	9.874484	10.125516	10.096703	10.222218	10
9.777950	9.903203	9.874747	10.125253	10.096797	10.222050	9
9.778119	9.903108	9.875010	10.124990	10.096892	10.221881	8
9.778287	9.903014	9.875273	10.124727	10.096986	10.221713	7
9.778455	9.902919	9.875536	10.124463	10.097081	10.221545	6
9.778623	9.902824	9.875800	10.124200	10.097176	10.221376	5
9.778792	9.902729	9.876063	10.123937	10.097271	10.221208	4
9.778960	9.902634	9.876326	10.123674	10.097366	10.221040	3
9.779128	9.902539	9.876589	10.123411	10.097461	10.220872	2
9.779295	9.902444	9.876851	10.123148	10.097556	10.220705	1
9.779463	9.902349	9.877114	10.122886	10.097651	10.220537	0
	Sine.		Tang.		Secant.	M

53 Degrees

37 Degrees.

Min	Sine		Tang.		Secant.		
0	9.779463	9.902349	9.877114	10.122886	10.097651	10.220537	60
1	9.779631	9.902253	9.877377	10.122623	10.097747	10.220369	59
2	9.779798	9.902158	9.877640	10.122360	10.097842	10.220202	58
3	9.779965	9.902063	9.877903	10.122097	10.097957	10.220034	57
4	9.780133	9.901967	9.878165	10.121835	10.098033	10.219867	56
5	9.780300	9.901872	9.878428	10.121572	10.098128	10.219700	55
6	9.780467	9.901776	9.878691	10.121309	10.098224	10.219533	54
7	9.780634	9.901681	9.878953	10.121047	10.098319	10.219366	53
8	9.780801	9.901585	9.879216	10.120784	10.098415	10.219199	52
9	9.780968	9.901489	9.879478	10.120522	10.098510	10.219032	51
10	9.781134	9.901394	9.879741	10.120259	10.098606	10.218866	50
11	9.781301	9.901298	9.880003	10.119997	10.098702	10.218699	49
12	9.781467	9.901202	9.880265	10.119735	10.098798	10.218532	48
13	9.781634	9.901106	9.880528	10.119472	10.098894	10.218366	47
14	9.781800	9.901010	9.880790	10.119210	10.098990	10.218190	46
15	9.781966	9.900914	9.881052	10.118948	10.099086	10.218034	45
16	9.782132	9.900818	9.881314	10.118686	10.099182	10.217868	44
17	9.782298	9.900721	9.881576	10.118423	10.099278	10.217702	43
18	9.782464	9.900626	9.881839	10.118161	10.099374	10.217536	42
19	9.782630	9.900529	9.882101	10.117899	10.099471	10.217370	41
20	9.782796	9.900433	9.882363	10.117637	10.099567	10.217204	40
21	9.782961	9.900337	9.882625	10.117375	10.099663	10.217039	39
22	9.783127	9.900240	9.882887	10.117113	10.099760	10.216873	38
23	9.783292	9.900144	9.883148	10.116852	10.099856	10.216708	37
24	9.783457	9.900047	9.883410	10.116590	10.099953	10.216542	36
25	9.783623	9.899951	9.883672	10.116328	10.100049	10.216377	35
26	9.783788	9.899854	9.883934	10.116066	10.100146	10.216212	34
27	9.783953	9.899757	9.884196	10.115804	10.100243	10.216047	33
28	9.784118	9.899660	9.884457	10.115543	10.100340	10.215882	32
29	9.784282	9.899564	9.884719	10.115281	10.100436	10.215718	31
30	9.784447	9.899467	9.884980	10.115019	10.100533	10.215553	30
31	9.784612	9.899370	9.885242	10.114758	10.100630	10.215388	29
32	9.784776	9.899273	9.885503	10.114496	10.100727	10.215224	28
33	9.784941	9.899176	9.885765	10.114235	10.100824	10.215059	27
34	9.785105	9.899078	9.886026	10.113974	10.100922	10.214895	26
35	9.785269	9.898981	9.886288	10.113712	10.101019	10.214731	25
36	9.785433	9.898884	9.886549	10.113450	10.101116	10.214567	24
37	9.785597	9.898787	9.886810	10.113189	10.101213	10.214403	23
38	9.785761	9.898689	9.887072	10.112928	10.101311	10.214239	22
39	9.785925	9.898592	9.887333	10.112667	10.101408	10.214075	21
40	9.786089	9.898494	9.887594	10.112406	10.101506	10.213911	20
41	9.786252	9.898397	9.887855	10.112145	10.101603	10.213748	19
42	9.786416	9.8 8299	9.888116	10.111883	10.101701	10.213584	18
43	9.786579	9.898201	9.888377	10.111622	10.101798	10.213420	17
44	9.786742	9.898104	9.888639	10.111361	10.101896	10.213258	16
45	9.786905	9.898006	9.888900	10.111100	10.101994	10.213094	15
46	9.787069	9.897908	9.889160	10.110839	10.102092	10.212931	14
47	9.787232	9.897810	9.889421	10.110579	10.102190	10.212768	13
48	9.787395	9.897712	9.889682	10.110318	10.102288	10.212605	12
49	9.787557	9.897614	9.889943	10.110057	10.102386	10.212443	11
50	9.787720	9.897516	9.890204	10.109796	10.102484	10.212280	10
51	9.787883	9.897418	9.890465	10.109535	10.102582	10.212117	9
52	9.788045	9.897320	9.890725	10.109275	10.102680	10.211955	8
53	9.788208	9.897222	9.890986	10.109014	10.102778	10.211792	7
54	9.788370	9.897123	9.891247	10.108753	10.102877	10.211630	6
55	9.788532	9.897025	9.891507	10.108493	10.102975	10.211467	5
56	9.788694	9.896926	9.891768	10.108232	10.103073	10.211305	4
57	9.788856	9.896828	9.892028	10.107971	10.103172	10.211143	3
58	9.789018	9.896729	9.892289	10.107711	10.103271	10.210982	2
59	9.789180	9.896631	9.892549	10.107451	10.103369	10.210820	1
60	9.789342	9.896532	0.892810	10.107190	10.103468	10.210658	0
		Sine		Tang.		Secant.	M

52 Degrees.

38 *Degrees.*

Min.	Sine		Tang.		Secant.		
0	9.789342	9.896532	9.892810	10.107190	10.103468	10.210658	60
1	9.789504	9.896433	9.893070	10.106930	10.103567	10.210496	59
2	9.789665	9.896335	9.893331	10.106669	10.103665	10.210335	58
3	9.789827	9.896236	9.893491	10.106409	10.103764	10.210173	57
4	9.789988	9.896137	9.893851	10.106149	10.103863	10.210012	56
5	9.790149	9.896038	9.894111	10.105889	10.103962	10.209851	55
6	9.790310	9.895939	9.894371	10.105628	10.104061	10.209690	54
7	9.790471	9.895840	9.894632	10.105368	10.104160	10.209528	53
8	9.790632	9.895741	9.894892	10.105108	10.104259	10.209367	52
9	9.790793	9.895641	9.895152	10.104848	10.104359	10.209207	51
10	9.790954	9.895542	9.895412	10.104588	10.104458	10.209046	50
11	9.791115	9.895443	9.895672	10.104328	10.104557	10.208885	49
12	9.791275	9.895343	9.895932	10.104068	10.104656	10.208725	48
13	9.791436	9.895244	9.896192	10.103808	10.104756	10.208564	47
14	9.791596	9.895144	9.896452	10.103548	10.104855	10.208404	46
15	9.791757	9.895045	9.896712	10.103288	10.104955	10.208243	45
16	9.791917	9.894945	9.896971	10.103029	10.105055	10.208083	44
17	9.792077	9.894846	9.897231	10.102769	10.105154	10.207923	43
18	9.792237	9.894746	9.897491	10.102509	10.105254	10.207763	42
19	9.792397	9.894646	9.897751	10.102249	10.105354	10.207603	41
20	9.792557	9.894546	9.898010	10.101990	10.105454	10.207443	40
21	9.792716	9.894446	9.898270	10.101730	10.105554	10.207284	39
22	9.792876	9.894346	9.898530	10.101470	10.105654	10.207124	38
23	9.793035	9.894246	9.898789	10.101211	10.105754	10.206964	37
24	9.793195	9.894146	9.899049	10.100951	10.105854	10.206805	36
25	9.793354	9.894056	9.899308	10.100692	10.105954	10.206646	35
26	9.793513	9.893946	9.899568	10.100432	10.106054	10.206486	34
27	9.793673	9.893846	9.899827	10.100173	10.106154	10.206327	33
28	9.793832	9.893745	9.900086	10.099913	10.106255	10.206168	32
29	9.793991	9.893645	9.900346	10.099654	10.106355	10.206009	31
30	9.794150	9.893544	9.900605	10.099395	10.106456	10.205850	30
31	9.794308	9.893444	9.900864	10.099135	10.106556	10.205692	29
32	9.794467	9.893343	9.901124	10.098876	10.106657	10.205533	28
33	9.794626	9.893243	9.901383	10.098617	10.106757	10.205374	27
34	9.794784	9.893142	9.901642	10.098358	10.106858	10.205216	26
35	9.794942	9.893041	9.901901	10.098099	10.106959	10.205057	25
36	9.795101	9.892940	9.902160	10.097840	10.107060	10.204899	24
37	9.795259	9.892839	9.902419	10.097580	10.107160	10.204741	23
38	9.795417	9.892738	9.902679	10.097321	10.107261	10.204583	22
39	9.795575	9.892637	9.902938	10.097062	10.107362	10.204425	21
40	9.795733	9.892536	9.903197	10.096803	10.107463	10.204267	20
41	9.795891	9.892435	9.903455	10.096544	10.107565	10.204109	19
42	9.796049	9.892335	9.903714	10.096286	10.107666	10.203951	18
43	9.796206	9.892233	9.903973	10.096028	10.107767	10.203794	17
44	9.796364	9.892132	9.904232	10.095768	10.107868	10.203636	16
45	9.796521	9.892030	9.904491	10.095509	10.107970	10.203479	15
46	9.796679	9.891929	9.904750	10.095250	10.108071	10.203321	14
47	9.796836	9.891827	9.905008	10.094922	10,108173	10.203164	13
48	9.796993	9.891726	9.905267	10.094733	10.108274	10 203007	12
49	9.797150	9.891624	9.905526	10.094474	10.108376	10.202850	11
50	9.797307	9.891523	9.905784	10.094215	10.108477	10.202692	10
51	9.797464	9.891421	9.906043	10.093957	10.108580	10.202736	9
52	9.797621	9.891319	9.906302	10.093698	10.108681	10.202379	8
53	9.797777	9.891217	9.906560	10.093440	10.108483	10.202222	7
54	9.797934	9.891115	9.906819	10.093181	10.108885	10.202066	6
55	9.798091	9.891013	9.907077	10.092923	10.108987	10.201909	5
56	9.798247	9.890911	9.907336	10.092664	10.109089	10.201753	4
57	9.798403	9.890809	9.907594	10.092406	10.109191	10.201597	3
58	9.798560	9.890707	9.907852	10.092147	10.109293	10.201440	2
59	9.798719	9.890605	9.908111	10.091889	10.109395	10.201284	1
60	9.798872	9.890503	9.908369	10.091631	10.109497	10.201128	0
		Sine		Tang		Secant.	M

51 *Degrees.*

39 *Degrees.*

Min.	Sine		Tang.		Secant.		
0	9.798872	9.890503	9.908369	10.091631	10.109497	10.201128	60
1	9.799028	9.890400	9.908627	10.091372	10.109600	10.200972	59
2	9.799184	9.890298	9.908886	10.091114	10.109702	10.200816	58
3	9.799339	9.890195	9.909144	10.090856	10.109805	10.200661	57
4	9.799495	9.890093	9.909402	10.090598	10.109907	10.200505	56
5	9.799651	9.889990	9.909660	10.090340	10.110010	10.200349	55
6	9.799806	9.889888	9.909918	10.090081	10.110112	10.200194	54
7	9.799962	9.889785	9.910177	10.089823	10.110215	10.200038	53
8	9.800117	9.889682	9.910435	10.089565	10.110318	10.199883	52
9	9.800272	9.889579	9.910693	10.089307	10.110421	10.199728	51
10	9.800427	9.889476	9.910951	10.089049	10.110523	10.199573	50
11	9.800582	9.889374	9.911209	10.088791	10.110626	10.199418	49
12	9.800737	9.889271	9.911467	10.088533	10.110729	10.199262	48
13	9.800892	9.889167	9.911724	10.088275	10.110832	10.199108	47
14	9.801047	9.889064	9.911982	10.088018	10.110936	10.198953	46
15	9.801201	9.888961	9.912240	10.087760	10.111039	10.198798	45
16	9.801356	9.888858	9.912498	10.087502	10.111142	10.198644	44
17	9.801511	9.888755	9.912756	10.087244	10.111245	10.198489	43
18	9.801665	9.888651	9.913014	10.086986	10.111349	10.198335	42
19	9.801819	9.888548	9.913271	10.086729	10.111452	10.198181	41
20	9.801973	9.888444	9.913529	10.086471	10.111556	10.198026	[illegible]
21	9.802128	9.888341	9.913787	10.086213	10.111659	10.197872	[illegible]
22	9.802282	9.888237	9.914044	10.085956	10.111763	10.197718	[illegible]
23	9.802435	9.888133	9.914302	10.085698	10.111866	10.197564	[illegible]
24	9.802589	9.888030	9.914560	10.085440	10.111970	10.197411	[illegible]
25	9.802743	9.887926	9.914817	10.085183	10.112074	10.197257	[illegible]
26	9.802897	9.887822	9.915075	10.084925	10.112178	10.197103	[illegible]
27	9.803050	9.887718	9.915332	10.084668	10.112282	10.196950	[illegible]
28	9.803204	9.887614	9.915590	10.084410	10.112386	10.196796	[illegible]
29	9.803357	9.887510	9.915847	10.084153	10.112490	10.196643	[illegible]
30	9.803510	9.887406	9.916104	10.083895	10.112594	10.196489	[illegible]
31	9.803664	9.887302	9.916362	10.083638	10.112698	10.196336	[illegible]
32	9.803817	9.887198	9.916619	10.083381	10.112802	10.196183	[illegible]
33	9.803970	9.887093	9.916876	10.083123	10.112907	10.196030	[illegible]
34	9.804123	9.886989	9.917134	10.082866	10.113011	10.195877	[illegible]
35	9.804276	9.886885	9.917391	10.082609	10.113115	10.195724	[illegible]
36	9.804428	9.886780	9.917648	10.082352	10.113220	10.195572	[illegible]
37	9.804581	9.886676	9.917905	10.082095	10.113324	10.195419	[illegible]
38	9.804734	9.886571	9.918163	10.081837	10.113429	10.195266	[illegible]
39	9.804886	9.886466	9.918420	10.081580	10.113534	10.195114	[illegible]
40	9.805038	9.886362	9.918677	10.081323	10.113638	10.194961	[illegible]
41	9.805191	9.886257	9.918934	10.081066	10.113743	10.194809	[illegible]
42	9.805343	9.886152	9.919191	10.080809	10.113848	10.194657	[illegible]
43	9.805495	9.886047	9.919448	10.080552	10.113954	10.194505	[illegible]
44	9.805647	9.885942	9.919705	10.080296	10.114058	10.194353	[illegible]
45	9.805799	9.885837	9.919962	10.080038	10.114163	10.194201	[illegible]
46	9.805951	9.885732	9.920219	10.079781	10.114268	10.194049	[illegible]
47	9.806103	9.885627	9.920476	10.079524	10.114373	10.193897	[illegible]
48	9.806254	9.885521	9.920733	10.079267	10.114478	10.193746	[illegible]
49	9.806406	9.885416	9.920990	10.079010	10.114584	10.193594	[illegible]
50	9.806557	9.885311	9.921247	10.078753	10.114689	10.193442	[illegible]
51	9.806709	9.885205	9.921503	10.078497	10.114794	10.193291	[illegible]
52	9.806860	9.885100	9.921760	10.078240	10.114900	10.193140	[illegible]
53	9.807011	9.884994	9.922017	10.077983	10.115005	10.192989	[illegible]
54	9.807163	9.884889	9.922274	10.077726	10.115111	10.192837	[illegible]
55	9.807314	9.884783	9.922530	10.077470	10.115217	10.192686	[illegible]
56	9.807465	9.884677	9.922787	10 077213	10.115322	10.192535	[illegible]
57	9.807615	9.884572	9.923044	10.076956	10.115428	[illegible]	[illegible]
58	9.807766	9.884466	9.923300	10.076700	10.115534	10.192234	[illegible]
59	9.807917	9.884360	9.923557	10.076443	10.115640	10.192083	[illegible]
60	9.808067	9.884254	9.923813	10.076186	10.115746	10.191932	[illegible]
		Sine		Tang.		Secant.	

50 *Degrees.*

40 Degrees.

Min.	Sine.		Tang.		Secant.		
0	9.808067	9.884254	9.923813	10.076186	10.115746	10.191932	60
1	9.808218	9.884148	9.924070	10.075930	10.115852	10.191782	59
2	9.808368	9.884042	9.924327	10.075673	10.115958	10.191632	58
3	9.808519	9.883936	9.924583	10.075417	10.116064	10.191481	57
4	9.808669	9.883829	9.924840	10.075160	10.116170	10.191331	56
5	9.808819	9.883723	9.925096	10.074904	10.116277	10.191180	55
6	9.808969	9.883617	9.925352	10.074648	10.116383	10.191030	54
7	9.809119	9.883510	9.925609	10.074391	10.116490	10.190881	53
8	9.809269	9.883404	9.925865	10.074135	10.116596	10.190731	52
9	9.809419	9.883297	9.926121	10.073878	10.116703	10.190581	51
10	9.809569	9.883191	9.926378	10.073622	10.116809	10.190431	50
11	9.809718	9.883084	9.926634	10.073366	10.116916	10.190282	49
12	9.809868	9.882977	9.926890	10.073110	10.117023	10.190132	48
13	9.810017	9.882871	9.927147	10.072853	10.117129	10.189983	47
14	9.810167	9.882764	9.927403	10.072597	10.117236	10.189833	46
15	9.810316	9.882657	9.927659	10.072341	10.117343	10.189684	45
16	9.810465	9.882550	9.927915	10.072085	10.117450	10.189535	44
17	9.810614	9.882443	9.928171	10.071829	10.117557	10.189386	43
18	9.810763	9.882336	9.928427	10.071573	10.117664	10.189237	42
19	9.810912	9.882228	9.928683	10.071316	10.117771	10.189088	41
20	9.811061	9.882121	9.928940	10.071060	10.117879	10.188939	40
21	9.811210	9.882014	9.929196	10.070804	10.117986	10.188790	39
22	9.811358	9.881907	9.929452	10.070548	10.118093	10.188642	38
23	9.811507	9.881799	9.929708	10.070292	10.118201	10.188493	37
24	9.811655	9.881692	9.929964	10.070036	10.118308	10.188345	36
25	9.811804	9.881584	9.930219	10.069780	10.118416	10.188196	35
26	9.811952	9.881477	9.930475	10.069524	10.118523	10.188048	34
27	9.812100	9.881369	9.930731	10.069269	10.118631	10.187900	33
28	9.812248	9.881261	9.930987	10.069013	10.118739	10.187752	32
29	9.812395	9.881153	9.931243	10.068757	10.118847	10.187603	31
30	9.812544	9.881045	9.931499	10.068501	10.118954	10.187456	30
31	9.812692	9.880938	9.931755	10.068245	10.119062	10.187308	29
32	9.812840	9.880830	9.932010	10.067989	10.119170	10.187160	28
33	9.812988	9.880721	9.932266	10.067734	10.119278	10.187012	27
34	9.813135	9.880613	9.932522	10.067478	10.119387	10.186865	26
35	9.813283	9.880505	9.932778	10.067222	10.119495	10.186717	25
36	9.813430	9.880397	9.933033	10.066967	10.119603	10.186570	24
37	9.813578	9.880289	9.933289	10.066711	10.119711	10.186422	23
38	9.813725	9.880180	9.933545	10.066455	10.119820	10.186275	22
39	9.813872	9.880072	9.933800	10.066200	10.119928	10.186128	21
40	9.814019	9.879963	9.934056	10.065944	10.120037	10.185981	20
41	9.814166	9.879855	9.934311	10.065689	10.120145	10.185834	19
42	9.814313	9.879746	9.934567	10.065433	10.120254	10.185687	18
43	9.814460	9.879637	9.934822	10.065177	10.120362	10.185540	17
44	9.814607	9.879529	9.935078	10.064922	10.120471	10.185393	16
45	9.814753	9.879420	9.935333	10.064666	10.120580	10.185246	15
46	9.814900	9.879311	9.935589	10.064411	10.120689	10.185100	14
47	9.815046	9.879202	9.935844	10.064156	10.120798	10.184954	13
48	9.815193	9.879093	9.936100	10.063900	10.120907	10.184807	12
49	9.815339	9.878984	9.936355	10.063645	10.121016	10.184661	11
50	9.815485	9.878875	9.936610	10.063389	10.121125	10.184515	10
51	9.815631	9.878766	9.936866	10.063134	10.121234	10.184368	9
52	9.815778	9.878656	9.937121	10.062879	10.121314	10.184222	8
53	9.815923	9.878547	9.937376	10.062623	10.121453	10.184076	7
54	9.816069	9.878438	9.937632	10.062368	10.121562	10.183931	6
55	9.816215	9.878328	9.937887	10.062113	10.121672	10.183785	5
56	9.816361	9.878219	9.938142	10.061858	10.121781	10.183639	4
57	9.816507	9.878109	9.938397	10.061602	10.121891	10.183493	3
58	9.816652	9.877999	9.938653	10.061347	10.122001	10.183348	2
59	9.816797	9.877890	9.938908	10.061092	10.122110	10.183202	1
60	9.816943	9.877780	9.939163	10.060837	10.122220	10.183057	0
		Sine.		Tang.		Secant.	M

49 Degrees.

41 Degrees.

Min.	Sine		Tang.		Secant.		
0	9.816943	9.877780	9.939163	10.060837	10.122220	10.183057	60
1	9.817088	9.877670	9.939418	10.060582	10.122330	10.182919	59
2	9.817233	9.877560	9.939673	10.060327	10.122440	10.182767	58
3	9.817378	9.877450	9.939928	10.060072	10.122550	10.182621	57
4	9.817523	9.877340	9.940183	10.059816	10.122660	10.182476	56
5	9.817668	9.877230	9.940438	10.059561	10.122770	10.182331	55
6	9.817813	9.877120	9.940694	10.059306	10.122880	10.182187	54
7	9.817958	9.877010	9.940949	10.059051	10.122990	10.182042	53
8	9.818103	9.876899	9.941204	10.058796	10.123101	10.181897	52
9	9.818247	9.876789	9.941458	10.058541	10.123211	10.181752	51
10	9.818392	9.876678	9.941713	10.058286	10.123321	10.181608	50
11	9.818536	9.876568	9.941968	10.058032	10.123432	10.181464	49
12	9.818681	9.876457	9.942223	10.057777	10.123543	10.181319	48
13	9.818825	9.876345	9.942478	10.057522	10.123653	10.181175	47
14	9.818969	9.876236	9.942733	10.057267	10.123764	10.181031	46
15	9.819113	9.876125	9.942988	10.057012	10.123875	10.180887	45
16	9.819258	9.876014	9.943243	10.056757	10.123985	10.180743	44
17	9.819401	9.875904	9.943498	10.056502	10.124096	10.180599	43
18	9.819545	9.875793	9.943752	10.056248	10.124207	10.180455	42
19	9.819689	9.875682	9.944007	10.055993	10.124318	10.180311	41
20	9.819832	9.875571	9.944262	10.055738	10.124429	10.180167	40
21	9.819976	9.875459	9.944517	10.055483	10.124541	10.180024	39
22	9.820120	9.875348	9.944771	10.055229	10.124652	10.179880	38
23	9.820263	9.875237	9.945026	10.054974	10.124763	10.179737	37
24	9.820406	9.875126	9.945281	10.054719	10.124874	10.179594	36
25	9.820550	9.875014	9.945535	10.054465	10.124986	10.179450	35
26	9.820693	9.874903	9.945790	10.054210	10.125097	10.179307	34
27	9.820836	9.874791	9.946045	10.053955	10.125209	10.179164	33
28	9.820979	9.874670	9.946299	10.053700	10.125320	10.179021	32
29	9.821122	9.874568	9.946554	10.053446	10.125432	10.178878	31
30	9.821265	9.874456	9.946805	10.053192	10.125554	10.178735	30
31	9.821407	9.874344	9.947063	10.052937	10.125656	10.178593	29
32	9.821550	9.874232	9.947317	10.052682	10.125767	10.178450	28
33	9.821693	9.874120	9.947572	10.052428	10.125879	10.178307	27
34	9.821835	9.874008	9.947826	10.052173	10.125991	10.178165	26
35	9.821977	9.873896	9.948081	10.051919	10.126103	10.178022	25
36	9.822120	9.873784	9.948335	10.051664	10.126216	10.177880	24
37	9.822262	9.873672	9.948590	10.051410	10.126328	10.177738	23
38	9.822404	9.873560	9.948844	10.051156	10.126440	10.177596	22
39	9.822546	9.873448	9.949099	10.050901	10.126552	10.177454	21
40	9.822688	9.873335	9.949353	10.050647	10.126665	10.177312	20
41	9.822830	9.873223	9.949607	10.050392	10.126777	10.177170	19
42	9.822972	9.873110	9.949862	10.050138	10.126090	10.177028	18
43	9.823114	9.872998	9.950116	10.049884	10.127002	10.176886	17
44	9.823255	9.872885	9.950370	10.049629	10.127115	10.176744	16
45	9.823397	9.872772	9.950625	10.049375	10.127228	10.176603	15
46	9.823539	9.872659	9.950879	10.040121	10.127341	10.176461	14
47	9.823680	9.872547	9.951133	10.048867	10.127453	10.176320	13
48	9.823821	9.872431	9.951388	10.048612	10.127566	10.176179	12
49	9.823963	9.872321	9.951642	10.048358	10.127679	10.176037	11
50	9.824104	9.872208	9.951856	10.048104	10.127792	10.175896	10
51	9.824245	9.872004	9.952150	10.047850	10.127905	10.175755	9
52	9.824386	9.871081	9.952401	10.047595	10.128019	10.175614	8
53	9.824527	9.871868	9.952659	10.047341	10.128132	10.175473	7
54	9.824668	9.871755	9.952913	10.047087	10.128245	10.175332	6
55	9.824808	9.871641	9.953167	10.046833	10.128359	10.175192	5
56	9.824949	9.871528	9.953421	10.046579	10.128472	10.175051	4
57	9.825090	9.871414	9.953675	10.046325	10.128586	10.174910	3
58	9.825230	9.871301	9.953929	10.046071	10.128699	10.174770	2
59	9.825370	9.871187	9.954183	10.045817	10.128813	10.174629	1
60	9.825511	9.871073	9.954437	10.045563	10.128926	10.174489	0
		Sine.		Tang.		Secant.	M

48 Degrees.

42 *Degrees.*

Min.	Sine.		Tang.		Secant.		
0	9.825511	9.871073	9.954437	10.045563	10.128926	10.174489	60
1	9.825651	9.870960	9.954691	10.045308	10.129040	10.174349	59
2	9.825791	9.870846	9.954945	10.045054	10.129154	10.174209	58
3	9.825931	9.870732	9.955199	10.044800	10.129268	10.174069	57
4	9.826071	9.870618	9.955453	10.044546	10.129382	10.173928	56
5	9.826211	9.870504	9.955707	10.044292	10.129496	10.173789	55
6	9.826351	9.870390	9.955961	10.044038	10.129610	10.173649	54
7	9.826491	9.870276	9.956215	10.043785	10.129724	10.173509	53
8	9.826631	9.870161	9.956469	10.043531	10.129839	10.173370	52
9	9.826770	9.870047	9.956723	10.043277	10.129953	10.173230	51
10	9.826910	9.869933	9.956977	10.043023	10.130067	10.173090	50
11	9.827049	9.869818	9.957231	10.042769	10.130182	10.172951	49
12	9.827189	9.869704	9.957485	10.042515	10.130296	10.172811	48
13	9.827328	9.869589	9.957739	10.042261	10.130411	10.172672	47
14	9.827467	9.869474	9.957993	10.042007	10.130526	10.172533	46
15	9.827606	9.869360	9.958246	10.041753	10.130640	10.172394	45
16	9.827745	9.869245	9.958500	10.041500	10.130755	10.172255	44
17	9.827884	9.869130	9.958754	10.041246	10.130870	10.172116	43
18	9.828023	9.869015	9.959008	10.040992	10.130985	10.171977	42
19	9.828162	9.868900	9.959262	10.040738	10.131100	10.171838	41
20	9.828301	9.868785	9.959515	10.040484	10.131215	10.171699	40
21	9.828439	9.868670	9.959770	10.040231	10.131330	10.171561	39
22	9.828578	9.868555	9.960023	10.039977	10.131445	10.171422	38
23	9.828716	9.868440	9.960277	10.039723	10.131560	10.171284	37
24	9.828855	9.868324	9.960530	10.039470	10.131676	10.171145	36
25	9.828993	9.868209	9.960784	10.039216	10.131791	10.171007	35
26	9.829131	9.868093	9.961038	10.038962	10.131907	10.170869	34
27	9.829269	9.867978	9.961291	10.038708	10.132022	10.170731	33
28	9.829407	9.867862	9.961545	10.038455	10.132138	10.170592	32
29	9.829545	9.867747	9.961799	10.038201	10.132253	10.170455	31
30	9.829683	9.867631	9.962052	10.037947	10.132369	10.170317	30
31	9.829821	9.867515	9.962306	10.037694	10.132485	10.170179	29
32	9.829959	9.867399	9.962560	10.037440	10.132601	10.170041	28
33	9.830097	9.867283	9.962813	10.037187	10.132717	10.169903	27
34	9.830234	9.867167	9.963067	10.036933	10.132833	10.169766	26
35	9.830372	9.867051	9.963320	10.036680	10.132949	10.169628	25
36	9.830509	9.866935	9.963574	10.036426	10.133065	10.169491	24
37	9.830646	9.866819	9.963827	10.036172	10.133181	10.169354	23
38	9.830784	9.866703	9.964081	10.035919	10.133297	10.169216	22
39	9.830921	9.866587	9.964335	10.035665	10.133414	10.169079	21
40	9.831058	9.866470	9.964588	10.035412	10.133530	10.168942	20
41	9.831195	9.866353	9.964842	10.035158	10.133647	10.168805	19
42	9.831332	9.866237	9.965095	10.034905	10.133763	10.168668	18
43	9.831469	9.866120	9.965349	10.034651	10.133880	10.168531	17
44	9.831606	9.866004	9.965602	10.034399	10.133997	10.168394	16
45	9.831742	9.865887	9.965855	10.034144	10.134113	10.168257	15
46	9.831879	9.865770	9.966109	10.033891	10.134230	10.168121	14
47	9.832015	9.865653	9.966362	10.033638	10.134347	10.167984	13
48	9.832152	9.865536	9.966616	10.033384	10.134464	10.167848	12
49	9.832288	9.865419	9.966869	10.033131	10.134581	10.167712	11
50	9.832425	9.865302	9.967122	10.032877	10.134698	10.167575	10
51	9.832561	9.865185	9.967376	10.032624	10.134815	10.167439	9
52	9.832697	9.865068	9.967630	10.032371	10.134932	10.167303	8
53	9.832833	9.864950	9.967883	10.032118	10.135050	10.167167	7
54	9.832970	9.864833	9.968136	10.031864	10.135167	10.167031	6
55	9.833105	9.864716	9.968389	10.031611	10.135284	10.166895	5
56	9.833241	9.864598	9.968643	10.031357	10.135402	10.166759	4
57	9.833377	9.864481	9.968897	10.031104	10.135519	10.166623	3
58	9.833512	9.864363	9.969150	10.030851	10.135637	10.166488	2
59	9.833648	9.864245	9.969403	10.030597	10.135755	10.166352	1
60	9.833783	9.864127	9.969656	10.030344	10.135872	10.166217	0
		Sine.		Tang.		Secant.	M

47 *Degrees.*

43 *Degrees.*

Min.	Sine		Tang.		Secant.		
0	9.833783	9.864127	9.969656	10.030344	10.135872	10.166217	60
1	9.833919	9.864010	9.969909	10.030091	10.135990	10.166081	59
2	9.834054	9.863892	9.970162	10.029838	10.136108	10.165946	58
3	9.834189	9.863774	9.970416	10.029584	10.136226	10.165811	57
4	9.834325	9.863656	9.970669	10.029331	10.136344	10.165675	56
5	9.834460	9.863538	9.970922	10.029078	10.136462	10.165540	55
6	9.834595	9.863419	9.971175	10.028825	10.136581	10.165405	54
7	9.834730	9.863301	9.971429	10.028571	10.136699	10.165270	53
8	9.834865	9.863183	9.971682	10.028318	10.136817	10.165135	52
9	9.834999	9.863064	9.971935	10.028065	10.136936	10.165001	51
10	9.835134	9.862946	9.972188	10.027812	10.137054	10.164866	50
11	9.835269	9.862827	9.972441	10.027559	10.137173	10.164731	49
12	9.835403	9.862709	9.972694	10.027305	10.137291	10.164597	48
13	9.835538	9.862590	9.972948	10.027052	10.137410	10.164462	47
14	9.835672	9.862471	9.973201	10.026799	10.137529	10.164328	46
15	9.835807	9.862353	9.973454	10.026546	10.137647	10.164193	45
16	9.835941	9.862234	9.973707	10.026293	10.137766	10.164059	44
17	9.836075	9.862115	9.973960	10.026040	10.137885	10.163925	43
18	9.836209	9.861996	9.974213	10.025787	10.138004	10.163791	42
19	9.836343	9.861877	9.974466	10.025534	10.138123	10.163657	41
20	9.836477	9.861758	9.974719	10.025280	10.138242	10.163523	40
21	9.836611	9.861638	9.974973	10.025027	10.138362	10.163389	39
22	9.836745	9.861519	9.975226	10.024774	10.138481	10.163255	38
23	9.836878	9.861400	9.975479	10.024521	10.138600	10.163122	37
24	9.837012	9.861280	9.975732	10.024268	10.138720	10.162988	36
25	9.837146	9.861161	9.975985	10.024015	10.138839	10.162854	35
26	9.837379	9.861041	9.976238	10.023762	10.138959	10.162721	34
27	9.837412	9.860921	9.976491	10.023509	10.139078	10.162587	33
28	9.837546	9.860802	9.976744	10.023256	10.139198	10.162454	32
29	9.837679	9.860682	9.976997	10.023003	10.139318	10.162321	31
30	9.837812	9.860562	9.977250	10.022750	10.139438	10.162188	30
31	9.837945	9.860442	9.977503	10.022497	10.139558	10.162055	29
32	9.838078	9.860322	9.977756	10.022244	10.139678	10.161922	28
33	9.838211	9.860202	9.978009	10.021991	10.139798	10.161789	27
34	9.838344	9.860082	9.978262	10.021738	10.139918	10.161656	26
35	9.838477	9.859962	9.978515	10.021485	10.140038	10.161523	25
36	9.838607	9.859842	9.978768	10.021232	10.140158	10.161390	24
37	9.838742	9.859721	9.979021	10.020979	10.140279	10.161258	23
38	9.838875	9.859601	9.979274	10.020726	10.140399	10.161125	22
39	9.839007	9.859480	9.979527	10.020473	10.140520	10.160993	21
40	9.839140	9.859360	9.979780	10.020220	10.140640	10.160860	20
41	9.839272	9.859239	9.980033	10.019967	10.140761	10.160728	19
42	9.839404	9.859119	9.980286	10.019714	10.140881	10.160596	18
43	9.839536	9.858998	9.980538	10.019461	10.141002	10.160464	17
44	9.839668	9.858877	9.980791	10.019209	10.141123	10.160332	16
45	9.839800	9.858756	9.981044	10.018956	10.141244	10.160200	15
46	9.839932	9.858635	9.981297	10.018703	10.141365	10.160068	14
47	9.840064	9.858514	9.981550	10.018450	10.141486	10.159936	13
48	9.840196	9.858393	9.981803	10.018197	10.141607	10.159804	12
49	9.840328	9.858272	9.982056	10.017944	10.141728	10.159672	11
50	9.840460	9.858150	9.982309	10.017691	10.141849	10.159541	10
51	9.840591	9.858020	9.982562	10.017438	10.141971	10.159409	9
52	9.840722	9.857908	9.982814	10.017185	10.142092	10.159278	8
53	9.840854	9.857786	9.983067	10.016933	10.142214	10.159146	7
54	9.840985	9.857665	9.983320	10.016680	10.142335	10.159015	6
55	9.841116	9.857543	9.983573	10.016427	10.142457	10.158884	5
56	9.841247	9.857421	9.983826	10.016174	10.142579	10.158753	4
57	9.841378	9.857300	9.984079	10.015921	10.142700	10.158621	3
58	9.841509	9.857178	9.984331	10.015668	10.142822	10.158490	2
59	9.841640	9.857056	9.984584	10.015416	10.142944	10.158360	1
60	9.841771	9.856934	9.984837	10.015163	10.143066	10.158229	0
		Sine.		Tang.		Secant.	M

46 *Degrees*

44 *Degrees.*

Min.	Sine		Tang.		Secant.		
0	9.841771	9.856934	9.984837	10.015163	10.143066	10.158229	60
1	9.841902	9.856812	9.985090	10.014910	10.143188	10.158098	59
2	9.842033	9.856690	9.985343	10.014657	10.143310	10.157967	58
3	9.842163	9.856568	9.985596	10.014404	10.143432	10.157837	57
4	9.842294	9.856445	9.985848	10.014152	10.143554	10.157706	56
5	9.842424	9.856323	9.986101	10.013899	10.143677	10.157576	55
6	9.842555	9.856201	9.986354	10.013646	10.143799	10.157445	54
7	9.842685	9.856078	9.986607	10.013393	10.143922	10.157315	53
8	9.842815	9.855956	9.986860	10.013140	10.144044	10.157185	52
9	9.842946	9.855833	9.987112	10.012888	10.144167	10.157054	51
10	9.843076	9.855711	9.987365	10.012635	10.144289	10.156924	50
11	9.843206	9.855588	9.987618	10.012382	10.144412	10.156794	49
12	9.843336	9.855465	9.987871	10.012129	10.144535	10.156664	48
13	9.843465	9.855342	9.988123	10.011877	10.144658	10.156534	47
14	9.843595	9.855219	9.988376	10.011624	10.144781	10.156405	46
15	9.843725	9.855096	9.988629	10.011371	10.144904	10.156275	45
16	9.843855	9.854973	9.988882	10.011118	10.145027	10.156145	44
17	9.843984	9.854850	9.989134	10.010866	10.145150	10.156016	43
18	9.844114	9.854727	9.989387	10.010613	10.145273	10.155886	42
19	9.844243	9.854603	9.989640	10.010360	10.145397	10.155757	41
20	9.844372	9.854480	9.989893	10.010107	10.145520	10.155627	40
21	9.844502	9.854356	9.990145	10.009855	10.145644	10.155498	39
22	9.844631	9.854233	9.990398	10.009602	10.145767	10.155369	38
23	9.844760	9.854109	9.990651	10.009349	10.145891	10.155240	37
24	9.844889	9.853986	9.990903	10.009096	10.146014	10.155111	36
25	9.845018	9.853862	9.991156	10.008844	10.146138	10.154982	35
26	9.845147	9.853738	9.991409	10.008591	10.146262	10.154853	34
27	9.845276	9.853614	9.691662	10.008338	10.146386	10.154724	33
28	9.845404	9.853490	9.991914	10.008086	10.146510	10.154595	32
29	9.845533	9.853366	9.992167	10.007833	10.146634	10.154467	31
30	9.845662	9.853242	9.992420	10.007580	10.146758	10.154338	30
31	9.845790	9.853118	9.992672	10.007328	10.146882	10.154210	29
32	9.845919	9.852994	9.992925	10.007075	10.147006	10.154081	28
33	9.846047	9.852869	9.993178	10.006822	10.147131	10.153952	27
34	9.846175	9.852745	9.993430	10.006569	10.147255	10.153825	26
35	9.846304	9.852620	9.993683	10.006317	10.147380	10.153696	25
36	9.846432	9.852496	9.993936	10.006064	10.147504	10.153568	24
37	9.846560	9.852371	9.994189	10.005811	10.147629	10.153440	23
38	9.846688	9.852247	9.994441	10.005559	10.147753	10.153312	22
39	9.846816	9.852122	9.994694	10.005306	10.147878	10.153184	21
40	9.846944	9.851997	9.994947	10.005053	10.148003	10.153056	20
41	9.847074	9.851872	9.995199	10.004801	10.148128	10.152229	19
42	9.847199	9.851747	9.995452	10.004548	10.148253	10.152801	18
43	9.847327	9.851622	9.995705	10.004295	10.148378	10.152673	17
44	9.847454	9.851497	9.995957	10.004043	10.148503	10.152546	16
45	9.847582	9.851372	9.996210	10.003790	10.148628	10.152418	15
46	9.847709	9.851246	9.996463	10.003537	10.148753	10.152291	14
47	9.847836	9.851121	9.996715	10.003285	10.148879	10.152163	13
48	9.847964	9.850996	9.996968	10.003032	10.149004	10.152036	12
49	9.848091	9.850870	9.997221	10.002779	10.149130	10.151909	11
50	9.848213	9.850745	9.997473	10.002527	10.149255	10.151782	10
51	9.848345	9.850619	9.997726	10.002274	10.149381	10.151655	9
52	9.848472	9.850493	9.997979	10.002021	10.149507	10.151528	8
53	9.848599	9.850367	9.998231	10.001769	10.149632	10.151401	7
54	9.848726	9.850242	9.998484	10.001516	10.149758	10.151274	6
55	9.848852	9.850116	9.998737	10.001263	10.149884	10.151148	5
56	9.848979	9.849990	9.998989	10.001011	10.150010	10.151021	4
57	9.849106	9.849864	9.999242	10.000758	10.150136	10.150894	3
58	9.849232	9.849737	9.999494	10.000503	10.150262	10.150768	2
59	9.849359	9.849611	9.999747	10.000253	10.150389	10.150641	1
60	9.849485	9.849485	1.000000	10.000000	10.150515	10.150515	0
		Sine.		Tang.		Secant.	M

45 *Degrees.*

A Table of Angles, which every Rhomb (or point of the Compass) maketh with the Meridian.

North.	*South.*	*Point.*		*D.*	*M.*	*North.*	*South.*
			¼	02	49		
			½	05	37		
			¾	08	26		
N. *by* E.	S. *by* East.	1		11	15	N. *by* W.	S. *by* W.
		1	¼	14	04		
		1	½	16	52		
		1	¾	19	41		
N. N. E.	S. S. E.	2		22	30	N. N. W.	S. S. W.
		2	¼	25	19		
		2	½	28	07		
		2	¾	30	56		
N. E. *by* N.	S. E. *by* S.	3		33	45	N. W. *by* N.	S. W. *by* S
		3	¼	36	34		
		3	1/9	39	22		
		3	¾	42	11		
No. East.	So. East.	4		45	00	N. West.	So. West.
		4	¼	47	49		
		4	½	50	38		
		4	¾	53	26		
N. E. *by* E.	S. E. *by* E.	5		56	15	N. W. *by* W.	S. W. *by* W.
		5	¼	59	04		
		5	½	61	53		
		5	¾	64	41		
E. N. E.	E. S. E.	6		67	30	W. N. W.	W. S. W.
		6	¼	70	19		
		6	½	73	08		
		6	¾	75	56		
E. *by* N.	E. *by* S.	7		78	45	W. *by* N.	W. *by* S.
		7	¼	81	34		
		7	½	84	23		
		7	¾	87	11		
East.	East.	8		90	00	West.	[illegible]

F I N I S.

THE ELEMENTS OF NAVAL ARCHITECTURE: OR, A PRACTICAL TREATISE ON SHIP-BUILDING.

LATELY PUBLISHED AT PARIS.

By M. DUHAMEL du MONCEAU,

Inspector General of the Marine to his most Christian Majesty, Member of the Royal Academy of Sciences at Paris, and Fellow of the Royal Society at London.

CAREFULLY ABRIDGED

By MUNGO MURRAY.

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M,DCC,LIV.

CONTENTS.

A

PRACTICAL TREATISE

ON

SHIP-BUILDING

CHAP. I.

General Proportions for Building.

NAVAL Architecture may be divided into three principal parts.

I. To give the ſhip ſuch a figure or exterior form, as may ſuit the ſervice ſhe is deſigned for.

II. To find the true form of all the pieces of timber that ſhall be neceſſary to compoſe ſuch a ſolid.

III. To make proper accommodations for guns, ammunition, proviſions, and apartments for all the officers, and likewiſe for the cargo.

We ſhall at preſent only treat of the firſt of theſe, namely the exterior figure, and conſider it firſt, as it regards the bottom, that is, the part which lies under water, and may be called the quick-work; or ſecondly, the part which is above water, and may be called the dead-work.

In order to give a proper figure to the bottom, all the qualities which are neceſſary to make a ſhip anſwer the ſervice for which ſhe is deſign'd, ſhould be conſidered. A ſhip of war ſhould carry her lower tier of guns four or five feet out of the water. A ſhip for the merchants ſervice ſhould ſtow the cargo well, and both of them ſhould be made to go well, carry a good ſail, ſteer well, and lie too eaſily in the ſea.

Some

Some eminent geometricians have endeavoured to find the form of a solid which may best answer all these qualities, and meet with the least resistance in dividing the fluid through which it is to pass; but have not been able to reduce their theory to practice by reason of the different positions a ship is obliged to be in when under sail. The ship-builders despairing to establish this point by mathematical rules, have applied themselves wholly to their own observations and experience, which may indeed supply the deficiencies of art, but though they may thereby discover that a ship has several bad qualities, it will not be easy to determine where the fault lies; for it may be owing to the rigging; and though the fault be not there, yet they cannot be certain in what particular part of the body it is. If their observations be assisted by principles drawn from theory, it will conduce very much to attain their end.

As there have been several ships built which have seemed to answer all the services for which they have been designed, some builders have made it their principal study to copy ships which have gained the applause of the seamen. This method they very improperly call the principal rule which should be observed in building. Now, as the bodies of ships are very different from one another, so there are, by this means, as many different methods used; some chusing one, and some another for a standard. But it must be observed, that even though it were possible to find such a body as should give intire satisfaction, and have all the good qualities that should be necessary to answer the services proposed, yet this could by no means be established as a standard by which other ships of different dimensions may be built. For admitting we have a first rate of 100 guns, which by experience has been found to be a very good ship in all respects, yet we should find ourselves very much deceived, if we should build a ship of 20 guns by making all the parts have the same proportion to one another that they have in that of 100 guns.

The first thing to be done in order to lay down the draught of a ship is to determine the length, which should be either on the lower gun deck, or at the load-water line; for there must be great care taken that there is a sufficient space betwixt the ports. This will oblige us first to fix the number and dimensions of the ports, the distance of the aftermost port from the transom, and of the foremost from the stem, and the distance betwixt the ports. This article may be determined by the following tables:

A Table

A Table of the Number of Ports on each Side of a Ship, according to the Number of Guns, and the Weight of the Shot.

A Ship of 112 Guns.

Decks	Ports	Shot
1	15	48 or 36
2	16	24
3	15	12
Quarter	5	8
Forecaſtle	3	8
Poop	2	4

A Ship of 102 Guns.

Decks	Ports	Shot
1	14	36
2	15	18
3	14	12
Quarter, Forecaſtle, Poop	13	6

A Ship of 74 Guns.

Decks	Ports	Shot
1	13	36
	14	
2	14	18
	15	
Quarter	8	8
Forecaſtle	8	
Poop	2	4

A Ship of 64 Guns.

Decks	Ports	Shot
1	12	24
	13	
2	13	12
	14	
Quarter	7	6
Forecaſtle	5	

A Ship of 58 Guns.

Decks	Ports	Shot
1	12	18
2	13	12
Quarter, Forecaſtle	4	4

The Tiger.

Decks	Ports	Shot
1	11	18
2	12	8
Quarter	3	6
Forecaſtle	2	4

A Ship of 50 Guns.

Decks	Ports	Shot
1	11	18
2	12	12
Quarter	2	4

A Frigate of 46 Guns.

Decks	Ports	Shot
1	11	12
2	12	8

A Frigate of 32 Guns

Decks	Ports	Shot
1	10	6
Quarter, Forecaſtle	6	4

A Frigate of 32 Guns.

Decks	Ports	Shot
1	4	8
2	10	6
Quarter, Forecaſtle	2	4

A Frigate of 32 Guns.

Decks	Ports	Shot
1	10	8
2	6	6

A Frigate of 28 Guns.

Decks	Ports	Shot
1	3	8
2	10	4
Quarter	1	4

A Frigate of 24 Guns.

Decks	Ports	Shot
1	10	6
Quarter	2	4

A Frigate of 22 Guus.

Decks	Ports	Shot
1	9	6
Quarter	2	4

Frigates of 20 Guns have 10 Ports on each ſide on one Deck. Shot 6 lb.

Veſſels of 16 guns have 8 ports on one deck, the guns to carry 4 lb. ſhot.
Veſſels of 12 guns have 6 ports on one deck, the guns to carry 4 lb. ſhot.

A Table of the Dimensions of the Ports and Height of their Sells, according to the Weight of the Shot.

Shot	Hei. and brea. of ports.							Height of the ports				Sells			
lb.	f.	in.	f.	in.		f.	in.	1st Deck		2d Deck		3d Deck		Quarter-Deck and Forecastle	
								f.	in.	f.	in.				
48	2	9	3	0	or	3	2								
36	2	8	3	0	or	3	1	2	2						
24	2	5	2	8	or	2	10	2	1	2	0				
18	2	4	2	7	or	2	8	1	11	1	9				
12	2	2 2	2	4	or	2	6	1	10	1	8	1	7		
8	1	9	1	11	or	2	3	1	8	1	6			1	5
6	1	6	1	8	or	1	11	1	7	1	6			1	5
4	1	4	1	6	or	1	9	1	5					1	3

A Table of the Number, Dimensions and Distances betwixt the Ports on the lower Deck; also the Distance betwixt the foremost Port and the Stem, betwixt the Aftmost and the Sternpost.

Ships Names.	N° of Ports	Breadth of Ports		Dist. betw. Ports		Foremost from Stem		Aftmost from Post		Length on L. Deck	
Amiable	13	2	8	7	6	13	4	9	0	147	
Invincible	13	2	8	7	4	12	4	9	0	144	
Achilles	12	2	8	7	8	18	2	10	6	145	
Toulouse	12	2	8	7	6	17	4	9	2	141	
Ardent, 64 guns	12	2	8	7	6	17	0	9	2	140	8
Fleuron, 64 guns	12	2	8	7	8	18	10	10	6	145	8
Dauphin Royal 74 guns	13	2	10	7	7	18	2	10	0	156	

Note, An inch *French* measure is equal to $1\frac{1}{18}$ inch *English*, and is divided into twelve parts called lines, which are divided into twelve parts, called points.

The next thing to be done is to establish the breadth by the midship beam; the builders are pretty much divided in proportioning this to the length. Most of them conform to dimensions taken from ships of the same burthen, and designed for the same service.

After these two dimensions are determined, the depth of the hold must be fixed, which in most ships is half the breadth; but the form of the body should be considered; for a flat floor will require less hold than a sharp one. The distances likewise between the decks must be determined. The following table may be very useful towards ascertaining the three aforesaid dimensions,

A Table

A Table of the Length, Breadth, and Depth in Hold of the following Ships.

Ships Names	Guns	Length at load-water line		Breadth		Depth in the Hold	
		feet	in.	f.	in.	f.	in.
Monarque	74	165		43		20	6
Intrepide	74	165		43		20	6
Alcide	64	149		40	6	19	4
Renommée	30	120		31	8	15	7
Palme	12	85		22	6	10	5
Soleil Royal	80	182		48		23	
Formidable	80	178		44	10	21	10
Tonnant	80	168		46		23	
Sceptre	74	165		43		20	6
Superbe	74	153	6	42	8	21	
Esperance	74	154		42		21	
Magnifique	74	165		43		20	6
Northumberland	68	149		40		20	
Lis	64	149		40		19	
Hercule	64	149		40	6	19	4
Protee	64	150		40	6	19	4
Illustre	64	150		40	8	20	
Opinatre	64	150		40	4	19	5
Dragon	64	149		40		19	
Leopard	64	146		39	6	18	6
St Laurent	60	145		39	4	18	8
Amphion	50	145		39		18	
Amazon	44	118					
Brillant	50	135		35			
Arc-en-Ciel	50	135		37		17	9
Tigre	52	131		37		17	
Alcion	50	132		35	4	18	
Aquilon	46	127		34		17	
Junon	46	136		36	6	16	6
Favorite	36	127		33		14	
Anglesea	32	121	8	33	6	16	
Serenne	30	118		31	8	15	9
Emeraude	28	118		31	8	16	
Galatée	24	110		29		14	6
Mutine	24	110		29		14	6
Cumberland	24	102		26		13	
Marshal Saxe	22	100		27		14	
Anemone	12	84		22		9	

Ships Names	Guns	Length	Breadth		Depth	
		feet	f.	in.	f.	in.
Amarante	12	84	22		9	
Elizabeth	64	143	38	4	18	
Brave	80	172	44		21	
Floriſſant	74	165	45		22	6
Couronne	74	167	44		22	7
Hardi	64	149	40	6	20	9
Aigle	50	144	39		19	6
Hermione	26	126	33	8	13	8
Juſte	70	151	42		21	
Triponne	26	114	31	8		
Panthere	20	108	28	6		
Badine	6	66	18	4		

We may then proceed to fix the length of the keel, which will oblige us to determine the rake of the ſtem and poſt, for which the builders have given us no invariable rule, they being very much divided in their opinions; for where ſome have given a rake of 18 or 20 feet, others have given none at all. The height of the ſtem and wing-tranſom muſt alſo be determined, which may be regulated by the decks.

The difference betwixt the draught of water abaft and that afore, ſhould likewiſe be conſidered; for though ſome imagine that when a ſhip is loaded her keel ſhould be parallel to the ſurface of the water, yet in many caſes it will be found neceſſary that the keel abaft ſhould be deeper in the water than it is afore. This will give the rudder more power, and thereby contribute to make a ſhip ſteer well; but this difference of the draught of water is intirely arbitrary; for in large ſhips ſome have given five, whereas others have given but three, or even two feet of difference. Though I could not procure the true difference of the draught of water of many ſhips of war, yet I am aſſured that the following are pretty exact.

The Difference of the Draught of Water in the following Ships.

	feet	in.		feet	in.
Northumberland	1	2	Panthere	1	4
Auguſte	1	6	Couronne	2	1
Aloze	1	0	Triponne	2	
Hermione	2	0	Renommée	1	4
Amazone	1	6	Tigre	3	2
Badine	0	10	Intrepide	2	3
Palme	1	4	Alcide	2	0

The

The length of the wing tranſom muſt alſo be determined; ſome make it $\frac{2}{3}$ of the main breadth; but this is likewiſe arbitrary, the broader a ſhip is abaſt, the more room there will be for accommodations for the officers; but this will be diſadvantageous to her ſailing upon a wind.

The following Examples will be ſufficient to fix the Length of the Wing tranſom for any Ship,

For a ſhip of 110 guns, $\frac{2}{3}$ of the main breadth, and 3 lines more to every foot.
102 guns, $\frac{2}{3}$ of the main breadth, and 8 inches more.
82 guns, $\frac{2}{3}$ of ditto.
74 guns, 7 inches, 9 lines for every foot in breadth.
62 guns, 7 inches, 8 lines for ditto.
56 guns, 7 inches, 7 lines, 3 points for ditto.
50 guns, 7 inches, 6 lines, 6 points for ditto.
46 guns, 7 inches, 6 lines for ditto.
32 guns, 7 inches, $5\frac{1}{2}$ lines for ditto.
For a frigate of 22 guns, 7 inches 4 lines.
12 guns, 7 inches.

Some, without regarding theſe proportions, make the wing tranſoms of the firſt and ſecond rates two thirds of the breadth, and for all the reſt one foot leſs.

After theſe dimenſions are determined, the timbers may be conſidered which form the ſides of the ſhip. A frame of timbers is compoſed of one floor timber, two or three futtocks and a top timber on each ſide: All theſe being united together, and ſecured by croſs-bars, form a circular incloſure, that which incloſes the greateſt ſpace is called the midſhip frame: The curve of this frame is inverted at the lower part, ſo that the floor timber will be ſomewhat hollow in the middle, whereby the ends will form a very obtuſe angle; but this angle decreaſes the farther the frames are removed from the midſhips, in ſuch a manner, that the foremoſt and aftermoſt will become very ſharp, and form a very acute angle. Theſe floor timbers are called crutches.

The builders ſeem to agree nearly as to the length of the midſhip floor timber, making it generally half the length of the main beam; but they differ very much about the riſing of it, ſome chuſing a flat and others a ſharp floor. And if we conſider the advantages and diſadvantages that attend the one and the other, we ſhall not be much ſurprized to find them ſo much divided upon this article; for it is certain, the more riſing a ſhip has, ſhe will hold the better wind, but then this will occaſion her to draw more water, which will be ſometimes attended with very great inconveniencies.

A Table

A Table of the riſing of the Midſhip Floor Timbers.

Guns	f.	in.	lines		Guns	f.	in.	lines	
110	0	0	10	to every foot in length.	56	0	1	4	to every foot in length.
102	0	0	10½		32	0	1	4	
86	0	1	0		28	0	1	4	
74	0	1	0		22	0	1	6	
62	0	1	0½		16	0	1	6	

Note, *What we have here rendered the riſing of the floor timbers, the author calls the* Aculement, *and makes a diſtinction betwixt it and the riſing, which we ſhall ſee when we come to form the frames.*

They differ as much in determining the ſtation of the midſhip frame, ſome placing it before, others at the middle of the ſhip; others again have two floor timbers of equal length, and riſing, one of which is placed exactly in the middle, or the breadth of the timber before the middle, and the other at a proper diſtance before it. Thoſe who place it before alledge that if a ſhip is full forward, after ſhe has once opened a column of water, ſhe will afterwards meet with no reſiſtance, and the water will eaſily unite abaft, and by that means force the ſhip a-head, and have more power on the rudder the farther it is from the centre of gravity; and beſides this comes neareſt the form of fiſhes, which ſhould ſeem to be the moſt advantageous for dividing fluids.

Thoſe who would have it placed in midſhips ſay, that by that means the water-lines forward will be eaſier, and of conſequence properer for dividing fluids; and that there will be ſpace enough betwixt it and the rudder for forming very fair water-lines, ſo that the water will eaſily unite at the rudder; and beſides it will be eaſier by this means to balance the fore body and after body; and in general the building will by this means be very much facilitated: ſo that, in my opinion, it will be propereſt to place it very near the middle, though it is the general practice to place it before it.

After the riſing of the midſhip floor timber is determined, we may then proceed to fix the height of the riſing line of the floor abaft on the poſt, and afore upon the ſtem.

Now, as all ſhips are narrower abaft and afore, than in midſhips, the other floor timbers will of conſequence be ſhorter and have a greater riſing, which will be ſtill increaſing till it ends on the poſt and ſtem. There are ſeveral different methods uſed by the builders to ſettle the height of this line. Some imagine, that by narrowing the floor abaft, which will occaſion the riſing line to be high upon the poſt, the ſhip will

will thereby ſteer better, and beſides, the water which is opened by the midſhip frame will then have a greater preſſure upon the after part of the ſhip, and thereby contribute to her ſailing: Yet theſe arguments are of very little weight; for if we only conſider the ſteerage, it is certain, that the higher the riſing line is carried abaft, and the narrower a ſhip is, the water will have the eaſier paſſage, and more power upon the rudder. But then we ſhall thereby run the riſk of falling into two great inconveniencies; for by this means we take away the buttock, which is the only thing we have to ſupport all the weight of the after part of the ſhip; neither ſhall we be able to give a proper balance betwixt the fore and after part, and when the fore and after parts are not duly balanced, it will occaſion a ſhip to pitch very hard, and be in danger of being frequently pooped by the ſea when it runs high. To prevent theſe inconveniencies it will be proper to give all ſhips, eſpecially the large ſort, a full buttock. As to the height of the riſing line afore, it ſhould be determined by the form of the water lines; but before this can be done, the timbers muſt be formed.

Note, What we have rendered the riſing line of the floor, our author calls les façons, *which, he ſays, is the increaſe of the* acculement, *the extreme points of which upon the perpendicular of the ſtem and poſt, are now to be determined.*

The height of the lower deck is the next thing to be conſidered: It is determined in midſhips by the depth of the hold, and ſome builders make it no higher at the ſtem; but they raiſe it abaft more than it is in midſhips, as much as the load-water mark abaft exceeds that afore. As to the height betwixt decks, it is altogether arbitrary, and muſt be determined by the rate of the ſhip, and the ſervice that ſhe is deſigned for.

We come now to conſider the upper works, or all that is above water, called the dead-work: And here the ſhip muſt be narrower, ſo that all the weight that lies above the load-water line will thereby be brought nearer the middle of the ſhip; by which means ſhe will ſtrain leſs by working the guns, and the main ſail will be eaſier trimmed when the ſhrouds do not ſpread ſo much. But though theſe advantages are gained by narrowing a ſhip above water, great care muſt be taken not to narrow her too much, for there muſt be ſufficient room upon the upper deck for the guns to recoil. The ſecurity of the maſts ſhould likewiſe be conſidered, which requires ſufficient breadth to ſpread the ſhrouds, though this may be aſſiſted by enlarging the breadth of the channels.

CHAP. II.

Of the Scantlings and Dimensions of the principal Pieces of Timber in a Ship.

ALTHOUGH it is not my intention, as I obſerved in the beginning of the laſt chapter, to treat of all the pieces that compoſe the ſhip, yet I think it neceſſary to ſay ſomething of the principal pieces. I ſhall therefore in the following plate lay down each piece by itſelf, by which means we ſhall ſee the length of the ſcarphs, and in what manner they are to be joined together.

Explanation of Plate I.

I.

A. The keel in four pieces, to be well bolted together, and clinched.

II.

I. The fore foot, one end of which is ſcarphed to the fore end of the keel, of which it is a part, and the other end makes a part of the ſtem, to which it is ſcarphed.

III.

u u. Two pieces of dead wood, one afore and the other abaft, fayed upon the keel.

IV.

C C. The ſtem in two pieces, to be ſcarphed together.

V.

E E. The apron in two pieces, to be ſcarphed together, and fayed on the inſide of the ſtem, to ſupport the ſcarph of the ſtem; for which purpoſe the ſcarph of the apron muſt be clear from that of the ſtem.

VI.

ó. The ſtemſon in two pieces, to ſupport the ſcarph of the apron.
o. The falſe poſt, which is fayed to the fore part of the poſt.

VII.

B. The ſtern poſt: It is tenanted into the keel, to which it is faſtened with a knee.

D. The

VIII.

D. The back of the poſt, which is likewiſe tenanted into the keel and well bolted to the poſt; the deſign of it is to give ſufficient breadth to the poſt, which ſeldom can be got broad enough in one piece.

IX.

F. The knee which faſteneth the poſt to the keel.

X.

N. The wing tranſom. It is fayed acroſs the ſtern poſt, and bolted to the head of it: The faſhion pieces are faſtened to the ends of it; underneath this and parallel to it is the deck tranſom.

XI.

O O. Two tranſoms faſtened to the ſtern poſt and faſhion pieces, in the ſame manner as the wing tranſom.

XII.

P. The tranſom knee, which faſteneth it to the ſhip's ſide.

XIII.

Q. The faſhion piece, of which there is one on each ſide: Their heels are faſtened to the ſtern poſt at the height of the floor ribbands, and their heads are faſtened to the wing tranſom.

XIV.

T. A floor timber. It is laid acroſs the keel, to which it it faſtened by a bolt through the middle.

XV.

K. The lower futtock.

XVI.

T T T T T. 2d, 3d, 4th futtocks and top timbers. Theſe ſhew the proper length and ſcarph of the timbers in midſhips frame.

XVII.

U U. Riders. Theſe are fayed in the inſide of the ſhip, and conſiſt of floor and futtock riders.

XVIII.

Z. The keelſon. This is made of two or three large pieces of timber ſcarphed together in the ſame manner as the keel. It is placed over the middle of the floor timbers, and ſcored about an inch and a half down upon each of them.

XIX.

R, S. Breaſt-hooks. Theſe are fayed in the inſide to the ſtem, and to the bow on each ſide of it, to which they are faſtened with proper bolts. There are generally four or five in the form of R in the hold, one in the form of S into which the lower deck planks are rabbited; there is one right under the hawſe holes, and another under the ſecond deck.

XX.

X, Y, Z. are thick planks which are fayed in the inſide, and ſtretch fore and aft to ſtrengthen the ſcarphings of the timbers.

XXI.

Z. are thick planks in the inſide, called clamps, which ſupport the ends of the beams.

XXII.

15, 15, 15, 15, 15, are the wales. They are planks broader and thicker than the reſt, which are faſtened to the outſide of the ſhip in the wake of the decks. We ſhall have occaſion in another place to ſhow how they are laid down in a draught. As to the plank below the wale to the keel, and above it to the top of the ſide, we refer to the ſection of one half of the midſhip frame, as laid down in the plate.

XXIII.

d, d, d, d, d, d, d, are knees. Theſe are crooked pieces of timber conſiſting of two arms which form an angle, either within or without a ſquare, or exactly ſquare; their uſe is to faſten any two pieces together, as the beams to the ſhip ſides.

XXIV.

19. The rudder. This is joined to the ſtern poſt by the rudder irons, upon which it turns round in the googings which are faſtened upon the ſtern poſt for that purpoſe. There is a mortiſe cut out of the head of

it,

it, into which a long bar is fitted, called the tiller, by which the rudder is turned from one ſide to the other.

XXV.

23. The cat heads. Theſe are two large pieces of ſquare timber, one on each ſide of the bowſprit. They project out before the bow, in order to keep the anchor clear of the ſhip, which is hove up by a rope called the cat fall, that paſſes through ſhivers in the outer end of the cat-head: Their inner ends are faſtened upon the forecaſtle.

XXVI.

m, *m*, *i*, *i*, *i*, are the ſeveral pieces which compoſe the knee of the head; the lower part *m* is fayed to the ſtem, the heel of it is ſcarphed to the head of the forefoot; it is faſten'd to the bows by two knees called cheeks, in the form of *ſ*, and to the ſtem by a knee call'd a ſtandard, in the form of K.

XXVII.

Beams &c, *a*, X Y, are large pieces of timber which ſupport the planks of each deck.

Having thus explained all the pieces in the plate, we ſhall in the following table give their ſcantlings.

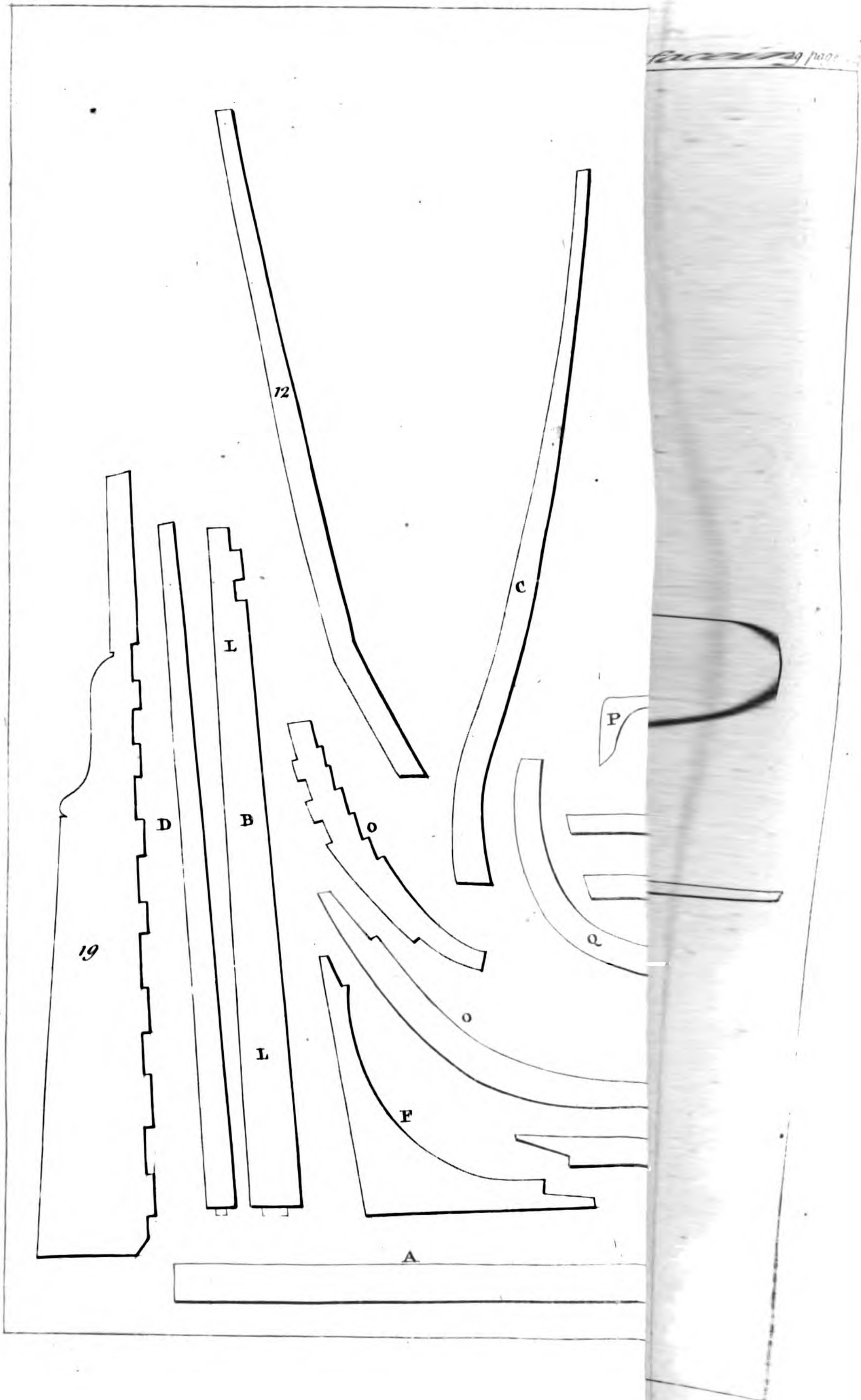
The

SCANTLINGS of the principal Pieces of Timber in a SHIP.

BREADTH of the SHIPS / LENGTH of the SHIPS	48 f. 0 in. / 176 f. 0 in.			46 f. 0 in. / 169 f. 11 in.			44 f. 0 in. / 163 f. 9 in.			42 f. 0 in. / 157 f. 6 in.			40 f. 0 in. / 151 f. 0 in.			38 f. 0 in. / 144 f. 7 in.			36 f. 0 in. / 138 f. 0 in.			34 f. 0 in. / 131 f. 3 in.			32 f. 0 in. / 124 f. 5 in.			30 f. 0 in. / 117 f. 6 in.			27 f. 0 in. / 106 f. 10 in.			24 f. 0 in. / 96 f. 0 in.		
NAMES of the PIECES.	f.	in.	l.	f.	in.	l.	f.	in.	l.	f.	in.	l.	f.	in.	l.	f.	in.	l.	f.	in.	l.	f.	in.	l.	f.	in.	l.	f.	in.	l.	f.	in.	l.	f.	in.	l.
BEAMS. First deck square	1	4	0	1	3	6	1	3	0	1	2	6	1	2	0	1	1	6	1	1	0	1	0	4	0	11	10	0	11	2	0	11	7	0	10	0
BEAMS. Second deck square	1	1	6	1	0	11	1	0	5	0	11	10	0	11	3	0	10	9	0	10	2	0	9	7	0	9	1	0	8	6	0	0	0	0	0	0
BEAMS. Quarter deck and forecastle, Moulded	0	10	3	0	9	9	0	9	3	0	8	9	0	8	3	0	7	9	0	7	3	0	6	9	0	9	3	0	5	9	0	5	3	0	4	9
BEAMS. Quarter deck and forecastle, Sided	0	11	3	0	10	10	0	10	6	0	10	1	0	9	8	0	9	4	0	8	11	0	8	6	0	8	2	0	7	9	0	7	5	0	7	0
BEAMS. Poop, Moulded	0	7	6	0	7	1	0	6	9	0	6	5	0	6	1	0	5	8	0	5	4	0	5	0												
BEAMS. Poop, Sided	9	3	9	0	9	2	0	8	9	0	8	6	0	8	3	0	8	0	0	7	9	0	7	6												
BREAST HOOK sided on the lower deck	1	3	6	1	3	0	1	2	7	1	2	2	1	1	8	1	1	3	1	0	9	1	0	4	0	11	10	0	11	5	0	10	10	0	10	6
CLAMPS, as broad as can be had, and half as thick as the timbers to which they are fastened																																				
KEEL, Deep	1	7	0	1	6	5	1	5	11	1	5	4	1	4	10	1	4	3	1	3	9	1	3	2	1	2	8	1	2	1	1	1	6	1	1	0
KEEL, Broad	1	5	0	1	4	4	1	3	8	1	3	0	1	2	4	1	1	8	1	1	0	1	0	5	0	11	9	0	11	2	0	10	7	0	10	0
KNEES. First deck, Sided	1	3	0	1	2	4	1	1	8	1	0	11	1	0	3	0	11	7	0	10	11	0	10	3	0	9	6	0	8	10	0	8	2	0	7	6
KNEES. First deck, Moulded	1	6	0	1	5	3	1	4	6	1	3	9	1	3	0	1	2	3	1	1	6	1	0	9	1	0	0	0	11	3	0	10	6	0	9	9
KNEES. Second deck, Sided	1	1	0	1	0	3	0	11	6	0	10	9	0	10	0	0	9	3	0	8	6	0	7	9	0	7	0	0	6	3						
KNEES. Second deck, Moulded	1	3	0	1	2	3	1	1	5	1	1	8	0	11	9	0	11	1	0	10	4	0	9	7	0	8	9	0	8	4						
KNEES. Quarter deck and forecastle, Sided	0	9	3	0	8	10	0	8	5	0	7	11	0	7	6	0	7	1	0	6	8	0	6	3	0	5	9	0	5	4	0	4	11	0	4	6
KNEES. Quarter deck and forecastle, Moulded	0	11	6	0	11	0	0	10	7	0	10	2	0	9	8	0	9	3	0	8	0	0	8	4	0	7	10	0	7	5	0	6	11	0	6	6
KNEES. Poop, Sided	0	6	0	0	5	8	0	5	5	0	5	2	0	4	10	0	4	7	0	4	3	0	4	0												
KNEES. Poop, Moulded	0	8	3	0	7	11	0	7	7	0	7	3	0	6	11	0	6	8	0	6	4	0	6	0												
KNEES. Transom, Sided	1	3	0	1	2	6	1	2	1	1	1	8	1	1	2	1	0	9	1	0	3	0	11	10	0	11	4	0	10	11	0	10	5	0	10	0
KNEES. Transom, Moulded	1	6	0	1	5	6	1	5	0	1	4	6	1	4	[illegible]	1	3	6	1	3	0	1	2	6	1	2	0	1	1	6	1	1	0	1	0	6
Planchere thick	0	5	0	0	4	9	0	4	6	0	4	4	0	4	1	0	3	10	0	3	8	0	3	5	0	3	2	0	2	11	0	2	9	0	2	6
STEM, Thwartships	1	5	0	1	4	4	1	3	9	1	3	1	1	2	5	1	1	10	1	1	2	1	0	6	0	11	11	0	11	3	0	10	8	0	10	0
STEM, Fore and aft.	1	9	0	1	8	5	1	7	10	1	7	3	1	6	8	1	6	0	1	5	5	1	4	10	1	4	3	1	3	8	1	3	1	1	2	6
Ditto, false or apron thick	1	2	0	1	1	5	1	0	11	1	0	4	0	11	10	0	11	3	0	10	9	0	10	2	0	9	8	0	9	1	0	8	6	0	8	0
STERN-POST, Thick	1	5	0	1	4	4	1	3	9	1	3	1	1	2	5	1	1	10	1	1	2	1	0	6	0	11	11	0	11	3	0	10	8	0	10	0
STERN-POST, Fore and aft, Below	1	9	0	1	8	5	1	7	10	1	7	3	1	6	8	1	6	0	1	5	5	1	4	10	1	4	3	1	3	8	1	3	1	1	2	6
STERN-POST, Fore and aft, Above	1	5	0	1	4	6	1	4	0	1	3	7	1	3	1	1	2	7	1	2	2	1	1	8	1	1	2	1	0	8	1	0	3	0	11	9
TIMBERS. Floor, Broad on the keel	1	3	0	1	2	4	1	1	8	1	0	11	1	0	3	0	11	7	0	11	10	0	10	3	0	9	6	0	8	10	5	8	2	0	7	6
TIMBERS. Floor, Deep on the keel	1	11	0	1	9	11	1	8	11	1	7	10	1	6	10	1	5	10	1	4	9	1	3	8	1	2	8	1	1	7	1	0	6	0	11	6
TIMBERS. Floor, Out and in at the head	1	3	0	1	2	4	1	1	8	1	0	11	1	0	3	0	11	7	0	10	11	0	10	3	0	9	6	0	8	10	0	8	2	0	7	6
TIMBERS. Futtocks out and in at the lower deck	1	1	[illegible]	1	0	5	0	11	10	0	11	3	0	10	8	0	10	8	0	9	5	0	8	10	0	8	3	0	7	8	0	7	1	0	6	6
TIMBERS. Top timber, Broad	1	1	0	1	0	6	0	11	11	0	11	5	0	10	11	0	10	5	0	9	4	0	8	10	0	8	3	0	7	9	0	7	6	0	7	3
TIMBERS. Top timber, Out and in at the lower deck ports	1	0	6	0	11	11	0	11	4	0	10	9	0	10	3	0	9	1	0	9	1	0	8	6	0	7	11	0	7	5	0	6	10	0	6	3
TIMBERS. Top timber, Out and in at the head	0	10	0	0	9	6	0	9	0	0	8	7	0	8	1	0	7	7	0	7	1	0	6	8	0	6	2	0	5	8	0	5	2	0	4	9
TIMBERS. Fashion-piece, Sided	1	2	6	1	1	10	1	1	2	1	0	6	0	11	10	0	11	2	0	10	6	0	9	11	0	9	3	0	8	7	0	7	11	0	7	3
TIMBERS. Fashion-piece, Out and in, Below	1	3	0	1	2	3	1	1	8	1	0	1	1	0	3	0	11	7	0	10	11	0	10	3	0	9	7	0	8	10	0	8	2	0	7	6
TIMBERS. Fashion-piece, Out and in, Aloft	1	1	0	1	0	5	0	11	10	0	11	3	0	10	0	0	10	8	0	9	5	0	8	10	0	8	3	0	7	8	0	7	1	0	6	6
TRANSOMS. Wing square	1	6	0	1	5	5	1	4	10	1	4	3	1	3	8	1	3	0	1	2	5	1	1	10	1	1	3	1	0	8	1	0	1	0	11	6
TRANSOMS. Deck, Moulded	1	2	0	1	1	5	1	0	10	1	0	3	0	11	7	0	11	0	0	10	5	0	9	10	0	9	3	0	8	8	0	8	1	0	7	5
TRANSOMS. Deck, Sided	1	5	6	1	4	10	1	4	2	1	3	5	1	2	9	1	2	1	1	1	5	1	1	9	1	0	0	0	11	5	0	10	8	0	10	0
TRANSOMS. Second and third, Moulded	1	2	0	1	1	4	1	0	8	1	0	0	0	11	4	0	10	8	0	10	0	0	9	4	0	8	8	0	8	0						
TRANSOMS. Second and third, Sided	1	5	0	1	4	5	1	3	11	1	3	4	1	2	9	1	2	3	1	1	8	1	1	1	1	0	7	1	0	0						
WALES. 1st and 2d, thick	0	9	0	0	8	8	0	8	3	0	7	1	0	7	6	0	7	2	0	6	10	0	6	5	0	6	1	0	5	9	0	5	4	0	5	0
WALES. 1st and 2d, broad	1	3	0	1	2	8	2	2	3	1	1	11	1	1	6	1	1	2	1	0	10	1	0	5	1	0	1	0	11	9	0	11	4	0	11	0
WALES. 3d and 4th, thick	0	7	0	0	6	8	0	6	4	0	6	0	0	5	8	0	5	4	0	5	9	0	4	8	0	4	4	0	4	0						
WALES. 3d and 4th, broad	1	1	0	1	0	8	1	0	4	1	0	0	0	11	8	0	11	4	0	11	0	0	10	8	0	10	4	0	10	0						
WALES. 5th and 6th, thick	0	6	0	0	5	8	0	5	6	0	5	6	0	4	10	0	4	7	0	4	3	0	4	0												
WALES. 5th and 6th, broad	1	0	0	0	11	6	0	11	0	0	10	7	0	10	1	0	9	7	0	9	2	0	8	8												

Note, The Scantlings of the Knees moulded is $\frac{2}{3}$ from the Throat.

Pl.I.



CHAP. III.

A Method to lay down a 70 Gun Ship upon the Plane of Elevation.

THE Dimenſions we have given of the principal parts of a ſhip of each claſs collected from the practice of different builders, which many have ſo great a regard to, as not to vary from them in the minuteſt article, we think only to be ſo far obſerved, as they ſhall produce ſuch a form as the ſervice the ſhip is deſigned for, ſhall require, agreeable to mathematical principles.

We ſhall now illuſtrate what has been ſaid on that head by drawing a ſhip from theſe dimenſions. But it will be firſt neceſſary to obſerve, that the builders make uſe of three different planes for one ſhip; 1ſt, the plane of elevation, in which the whole length is laid down according to a ſide view; 2d, The plane of the projection, which ſome call a vertical plane of the timbers, becauſe it gives us an end view of the form of all the timbers, before the plank is put on. 3d, The horizontal plane, upon which are deſcribed all the curves that are formed by ſections of the body parallel to the horizon, which muſt be conſidered as well as thoſe vertical ſections which form the curves of the timbers. We may likewiſe form the curves of the ribbands upon this plane, which will be of great uſe in proving whether the form we give the timbers will produce a fair ſide.

It is indifferent with which of theſe we begin, though that of the elevation ſeems moſt commodious. But firſt of all it will be very proper to draw out a liſt of all the dimenſions of the veſſel we are to build, ſo that we may have a view of the whole deſign.

This ſhip then is to have two tier of guns, ſo there muſt be two decks quite fore and aft, likewiſe a quarter deck as far as the main maſt, a forecaſtle 33 feet long, and a poop to the mizen maſt.

There are to be 13 ports of a ſide on the lower deck, the guns to carry 24 lb. ſhot; 14 ports on each ſide upon the upper deck, the guns to carry 18 lb ſhot; on the quarter deck 4 guns, and on the forecaſtle 2 guns of 8 lb. ſhot on each ſide, and 2 of 4 lb. on each ſide on the poop.

Ports

	feet	in.	l.
Ports on the lower deck fore and aft	2	10	
Diſtance betwixt the ports	7	9	
Aftermoſt port from the poſt	9	3	
Foremoſt from the ſtem	17	2	
Height of the ſells, including the lower deck planks	2	5	
Ports up and down on the lower deck	2	7	
Diſtance from the upper ſide of the lower deck beam to the upper ſide of the upper deck beams	6	11	
Riſing of the ſecond deck abaft		11	
Second deck ports up and down	2	4	
Second deck ports fore and aft	2	6	
Height of the ſells from the deck line	1	11	6
Diſtance betwixt the ſecond deck and quarter deck from plank to plank	6	6	
Quarter deck ports up and down	1	10	
Quarter deck ports fore and aft	2		
Height of the ſells	1	4	
Diſtance betwixt the quarter deck and poop	6	2	
Ports on the poop fore and aft	1	10	
Height of the ſells	1	0	
Length from rabbit to rabbit on the gun deck	156	3	
Extreme breadth	42		
Depth in the hold below the plank	21	0	0
Riſing of the lower deck abaft, not including the difference of the draught of water	2	11	6
Height of the ſtem	31	9	3
Height of the poſt	31	7	9
Rake of the ſtem	15	7	2
Rake of the poſt	3	1	5
Length by the keel	139	6	10
Depth of the keel	1	7	3
Length of the wing tranſom	27		
Length of the midſhip floor timber	21		
Riſing of ditto	1	9	
Difference of the draught of water abaft more than afore	3	2	0
Height of the riſing line of the floor abaft	13	6	0
Height of the riſing line of the floor afore	5	7	5

I would adviſe young beginners in the art of drawing to conform exactly to theſe dimenſions, which we have here given for an example, and

and observe all the particular directions which we shall give in laying down a ship of 70 guns; for they must begin by making themselves acquainted with the terms, and thereby gain a general idea of the whole design. After finishing this draught, they may then proceed to another of a different rate, and as we have given the principal dimensions of several good ships, they may chuse such a one as will best answer their design.

Plate II. Fig I.] 1st. Provide a scale of equal parts properly divided into feet and inches, adapted to the intended length of the draught, and draw the line A B, which make 156 feet 3 inches for the length of the gun deck, from the rabbit of the stem to that of the post.

	f.	in.	l.
To find the length on the gun deck, multiply 13, the number of ports, by 2 f. 10 in. the dimensions of each port fore and aft, the product is	36	10	0
Again multiply 7 f. 9 in. the distance betwixt the ports, by 12, the number of spaces, the product is	93	0	0
Aftmost port before the post	9	3	0
Foremost abaft the stem	17	2	0
Length on the gun deck	156	3	0

2dly, Draw the line CD equal and parallel to AB, let 21 feet, the half of the main breadth, be the distance betwixt them, and erect the perpendiculars CF and D Z.

3dly. Set off 3 feet 2 inches, the difference of the draught of water, from B to G, and draw the line A G, which will give the position of the lower side of the keel. From A set off 1 foot 7 inches 3 lines, the depth of the keel, as in the table of scantlings, to K, and draw the line K I parallel to A G, which will be the upper side of the keel.

4thly, Set off 1 foot 3 inches 3 lines, the breadth of the stem from G to M, and draw the dotted line M N parallel to G Z. From G set off 15 feet 7 inches 2 lines, the rake of the stem, to O.

5thly, Set up 31 feet 9 inches 3 lines, the height of the stem from G to P. With the radius I P describe the arch P O, which will be the fore side of the stem, and from the same center describe another arch within the former, which will give the inside of the stem, and another arch for the rabbit may be described four inches before the inside of the stem.

6thly, Set up 25 feet 1 inch from K to L for the height of the gun deck abaft, and 21 feet 6 inches from I to *e*, for the height afore.

7thly, Set up 2 feet 5 inches, the height of the port-sells from L to *d*, which will give the upper side of the wing transom; from which set up

2 feet 7 inches, the height of the ports; alſo 1 foot for the depth, and 6 inches 9 lines for the round of the helm port tranſom, to the point F, which will be the height of the poſt; ſo K F will be 31 feet 7 inches 9 lines. From K ſet off 3 feet 1 inch 5 lines to *f*, for the rake of the poſt, and draw the line F*f* for the aft ſide of the poſt. From *f* to *b* ſet off the depth of the keel, and draw the line *b d* for the fore ſide of the poſt, making F *d* $\frac{1}{4}$ of *f b*, ſo ſhall *f* O be the whole length of the keel.

The builders are very much divided about aſſigning a proper place for the midſhip frame, for which the following method may be uſed:

Divide the line CD into two equal parts, then take 5 feet 6 inches 10 lines, that is $\frac{1}{28}$ part of 156 feet 3 inches, the length of the gun deck: Set off this before the middle of the line C D, which will give the point F the ſtation of the midſhip frame. Set up 21 feet from F to Z, which will give the height of the gun deck at the midſhip frame. From the point Z ſet off 2 feet 7 inches 6 lines (the $\frac{1}{8}$ of the height of the gun deck at the midſhip frame,) through which point draw a line VT, parallel to CD, which will be the load-water line. Through the point F draw the line G*g*, parallel and equal to the load-water line, which will ſhew how much water the ſhip will draw abaft more than afore.

One of the frames is placed pretty near the cheſs tree, which is called the loof frame; to find its place, from the point of D ſet off $\frac{1}{4}$ of the line D C, and there draw a dotted line perpendicular to A B. Again divide the line F G into nine equal parts, and draw eight lines perpendicular to A B, which will ſtation eight frames in the fore-body beſides that of the loof.

There is in the after-body a frame to balance that of the loof in the fore-body; theſe two are of equal breadth in ſome points, and this will occaſion the center of gravity of that part contained betwixt theſe two frames to be near the plane of the midſhip frame, which will keep the fore part and after part upon a balance. It muſt be as far abaft the middle of the line CD as that of the loof is before it.

The frames in the after-body are the ſame diſtance from one another as they are in the fore, which will occaſion one more abaft than before; ſo there are nine abaft, beſides that of the balance.

We ſhall in the next place lay down the deck lines, and firſt for the lower deck draw a fair curve through the points L Z *e*, and parallel to it draw another curve for the port-ſells, which are 2 feet 5 inches above the deck line.

The

The aftermoſt port is 9 feet 3 inches before the poſt, which ſet off to *u*, and the ports are 2 feet 10 inches fore and aft; which ſet off from *u* to *x*, the diſtance betwixt the ports is 7 feet 9 inches, which ſet off from *x* to Y for the aft ſide of the ſecond port; from Y again ſet off 2 feet 10 inches, which will give the foreſide of the next. Proceed in the ſame manner till all the ports are ſpaced; ſo ſhall the foremoſt port be 17 feet 2 inches abaft the rabbit of the ſtem. The height of the ports is 2 feet 7 inches; which ſet up from *u*, draw a curve parallel to the deck line, which will give the upper part of all the ports; after which theſe two lines may be wiped off the draught, which muſt be therefore drawn with a black lead pencil, and only the ports inked in.

Draw a line for the upper deck, which is 6 feet 11 inches above the lower from the midſhip frame forward, and 6 inches more abaft. We may then draw a line for the port-ſells, and one for their height parallel to the deck line, and ſpace the ports ſo that they may be exactly over the middle of the diſtance betwixt the lower deck ports.

Before we can ſet off the height of the Quarter deck we muſt find the true place of the main maſt. The general rule is to take 4 lines for every foot the gun deck is in length, and ſet it off abaft the middle, which will give the fore ſide of the maſt: now the length 156 feet 3 inches $\times$ 4 lines $= 625 =$ 4 feet 2 inches 1 line, which ſet off abaft the middle of the line C D, and there erect a line perpendicular to the water-line, which will be the fore ſide of the maſt, and parallel to it draw a line for the middle, and one for the aft ſide of the maſt, the diameter of which is 35 inches. Set off 6 feet 6 inches on the aft ſide of the main maſt, for the height of the quarter deck afore, and 6 feet 10 inches for the height abaft; and draw a line nearly parallel to that of the upper deck, which will be the line for the quarter deck. We may then ſpace the ports, ſo that they may be exactly over thoſe of the lower deck. The forecaſtle is 6 feet 6 inches high, at which diſtance draw a line parallel to the upper deck line, which will give the line for the forecaſtle deck. As to the length of this deck, it ends forward at the beak head, and is carried aft diſcretionally, obſerving to leave room for the capſtan bars. In ſpacing the ports upon the forecaſtle, care muſt be taken that none be oppoſite to the fore maſt. Now to find the center of this maſt, take 15 feet 7 inches 2 lines, the tenth part of the whole length, which ſet off from the rabbit of the ſtem upon the lower deck abaft, from which point ſet off 32 inches and 1 line, being the diameter of the maſt, through the middle of this draw a perpendicular line, as in the plate. The boltſprit generally makes an angle of 34 or 35 degrees with the load-water line.

The poop is pretty near parallel to the quarter deck; the diſtance betwixt them forward is 6 feet, an abaft 6 feet 3 inches. It ends about 18 inches before the mizen maſt, the aft ſide of which is $\frac{2}{7}$ of the main breadth before the rabbit of the poſt upon the gun deck.

The counter is generally an arch paſſing from the upper ſide of the wing-tranſom to the lower ſide of the beam of the ſecond deck. The rake of the lower counter is $\frac{3}{4}$ of an inch for every foot of the main breadth. The rake of the ſecond counter is $\frac{2}{3}$ of the lower; its height above the deck is 3 feet 5 inches. The hollow of the counters is altogether arbitrary, inſomuch that ſome give none to the lower. The upright of the ſtern rakes 2 inches in a foot, as in the plate.

The beauty of a ſhip depends much upon giving the wales a proper hanging; for by them the ſheer and drift rails are regulated, being all nearly parallel to one another, though they generally riſe a little more abaft on account of the accommodations for the officers. It is this which makes a ſhip look airy and graceful in the water. There is no certain rule for laying them down; this is left entirely to the fancy and taſte of the artiſt; but in placing the wales great care muſt be taken that they be wounded as little as poſſible by the ports; the foremoſt port on the gun deck muſt be 1½ or 2 inches above, and the third port from abaft juſt touch the upper ſide of the upper ſtrake of the main wales. The lower edge of the lower ſtrake may glance with the edge of the water when loaded. There are two ſtrakes of wales, and one ſtrake between them of 15 inches broad each. The range of the deck ſhould be conſidered in placing the wales, ſo that the ſcuppers may be in the ſtrake betwixt the wales. The like caution muſt be uſed for the channel wales as may be ſeen in the plate, where they are all laid down, together with the ſheer and drift rails; the rails, cheeks, and knee of the head are likewiſe laid down in the plate, and being for an ornament to the ſhip, are left to the fancy and taſte of the builder. Though the knee may help a ſhip to hold a good wind, the fore part of it is generally one twelfth part of the length before the ſtem.

CHAP.

CHAP. IV.

To lay down the Frames upon the Plane of Projection.

HAVING thus explained all that is neceſſary to be delineated upon the plane of elevation, the next thing to be determined is the different breadths of the ſhip at any aſſigned points of the length, whereby we ſhall gain the forms of all the planes that are made by ſections, perpendicular to the load-water line. The timbers that compoſe the body of a ſhip are ſuppoſed to have their planes in that poſition, and may be all delineated upon the plane of the projection; but as both ſides of a ſhip are exactly the ſame, it will ſuffice to lay down the half of each, thoſe of the fore-body on the right, and thoſe of the after-body on the left hand. And whereas theſe planes diminiſh afore and aft, the planes of all the frames may be all delineated upon the plane of the mid-ſhip one, which may be called the maſter frame. The firſt thing then neceſſary to be known is how to form this frame.

The mid-ſhip frame is that which is at the broadeſt part of the ſhip. The builders differ about the form of this frame, but there are ſeveral preliminary operations which are neceſſary to be obſerved in all the different methods uſed in forming it.

Preliminary Operations for forming the Midſhip Frame.

Plate III, *Fig.* I. and II.] 1ſt. Draw the line A B to repreſent the upper ſide of the keel; it muſt be at leaſt as long as the ſhip is broad. This line our author calls the line of *aculement*, becauſe upon it the aculement of the midſhip floor timber terminates.

2dly, Draw the line C D parallel and equal to A B, ſo that A C and B D may be equal to the riſing of the midſhip floor timber. This line may be called the riſing line, becauſe it limits the height of the ends of the midſhip floor timber above the keel.

3dly, Draw the line G H for the height of the lower deck parallel to the former, and below this draw a line to repreſent the load-water line, taking its diſtance below the deck line from the plane of elevation at the midſhip frame. Draw alſo the lines I K and L M, the one for the ſecond deck, and the other for the ſheer rail or top of the ſide in midſhips. The height of both are to be taken from the plane of elevation.

4thly,

4thly, Draw the line N O perpendicular to A B; this is called the middle line, and repreſents the middle line of the ſtem and poſt, dividing the whole ſhip into two equal parts; and parallel to N O draw the lines A L and B M, to limit the breadth; alſo a line for half the thickneſs of the ſtem, and one for half the thickneſs of the poſt. Draw the lines *z x* parallel to N O, dividing the lines O A and O B into two equal parts. Draw alſo the diagonal G B. Theſe lines being drawn, we may proceed to form the midſhip frame by ſome of the following methods.

METHOD I.

To form a Midſhip Frame that ſhall be neither too ſharp nor too flat.

Plate III. *Fig.* I.] 1ſt, Divide the line *a x*, which marks the head of the floor timber into three equal parts; ſet off one from *a* to *b*.

2d, Divide the line *d* B, the diſtance betwixt the load-water line and the upper ſide of the keel, into ſeven equal parts; ſet off one of theſe from *d* to *e*, and from *e* to *m*, and draw the diagonal *a* V, which divide into two equal parts in the point *n*. *Note, the diagonal* a V *is wiped out after finding the point n.*

3d. Deſcribe an arch of a circle to paſs through the points *b* and *e*, make the radius the whole length and half the length of the line B *e*, ſo the center A may be found by deſcribing an arch with that radius from *e*, and one from *b* to interſect one another in A, we ſhall only make uſe of that part of this arch betwixt *l* and *m*. Now, to find the other arches *m d*, *l a*, *a n*, *n* V, it muſt be obſerved, that in order to reconcile two arches, ſo as to make a fair curve, a ſtrait line muſt paſt through the centers of both, and through the points where they unite or touch one another; draw therefore the lines A *m* and A *l*, ſo ſhall *k* be the center of the arch *m d*, and *o* the center of the arch *l a*. Again through the center *o* draw the line *a o*, produce it to P, which will be the center of the arch *a n*. Laſtly, from P thro' *n* draw the line P *s*, *s* will be the center of the inverted arch *n* V. *Note, the center s will be without the Plate.*

4th. To form the top timber ſet back the tenth part of the half breadth from K to S, upon the line of the ſecond deck; deſcribe an arch of a circle thro' the points *d* and K, taking $\frac{2}{7}$ of the whole breadth for the radius: Again, from the point M, upon the line L M, ſet back the fifth part of the whole breadth to *g*. Deſcribe an arch of a circle through the points S and I, taking the diagonal G B for the radius. As this arch is inverted in reſpect of the arch *d* S, the center will be without the figure. This compleats the form of half the midſhip frame, and by the ſame operations we may find the other half.

It muſt be obſerved that there is no regard had to the round of the beam in ſetting off the deck line or depth of the hold. M E-

METHOD II.

To describe a Midship Frame of a circular Floor.

Plate III. *Fig.* II.] From the center G, the point where the middle line intersects the deck line, making the half breadth the radius describe the arch *b*, G, *c*, O: Let *d* be the head of the floor timber, and *d x* the rising. Assume the point *f*, according to what round you propose to give to the second futtock, and describe the arch *d f*; the center may be found as directed in the preceding method. Divide the arch *c* O into three equal parts; set off one from *c* to *g*, and from the center *b*; describe the arch *d g*: there remains only the inverted arch *g* Y to be described; the center may be found as before directed.

METHOD III.

To draw a Midship Frame which shall be very full.

Plate III. *Fig.* III.] 1st. Draw the rising and deck lines as before; let *d x* be the rising.

2d. Make *d b* the side of the square *d b a c* equal to C *b* the ½ of the breadth.

3d. Inscribe the two quadrants *c e b*, and *c f b* into the square.

4th. Divide the side *c a* into a certain number of equal parts in the points O, N, M, L *a*; draw the lines *i* L, *b* M, &c. perpendicular to *a c*.

5. Divide the line C G, the depth of the hold after deducting the rising, into the same number of equal parts in the points E, F, I, K, and make the lines E*p*, F*q*, I *r*, K *s*, in the frame, equal to the lines O *t*, N *n*, M *e*, L *m* in the square, describe a curve through the points G, *p*, *q*, *r*, *s*, *b*, and the remaining part of the frame may be described by the preceding methods.

METHOD IV.

To describe a Midship Frame for a very sharp Ship.

Plate III. *Fig.* IV.] Let the length of the floor timber be half the breadth as before, and the rising one fifth or one sixth of the whole length of the floor timber; lay this off from *x* to E, and describe a parabola through the points G, P, Q, E, of which the point G is the vertex, and G C the axis. This method is extracted from M. *Bouguer*. The parabola may be formed by the following method: 1. Through the point E draw the line T *x* perpendicular to G C, and the line *d* E perpendicular to A G, and produce

produce the line C G to D. 2dly, Upon the line G D find the center of a semicircle that shall pass through the points T, *d*, and D, so shall G D be the parameter of the parabola, by which we may find any number of points through which the curve must pass: for instance, suppose it were required to find a point in the perpendicular X P, through which the curve must pass; upon the line G D find the center of a semicircle which shall pass through the points D and X; this will intersect the line A G in *b*, make *b* P equal and parallel to G X, so shall P be the point required; in like manner the points *a* Q *f* may be found. The remainder of the curve from E to *y* will be composed of two arches, the one to reconcile with the parabola in the point E, and the other inverted to pass through the point *y*; the center of which may be found by any of the preceding methods. In order to find the center of that which joins with the parabola make T R equal to half the parameter G D, and draw the line E R, upon which find a point S for the center of the arch.

We might shew a great many more methods of describing this midship frame. It is very true that great care ought to be had in forming this frame, because upon it chiefly depends the form of all the other timbers; I say chiefly, but not altogether; for two ships may be similar as to their midship frames, and yet very different afore and abaft; and though the artists should make themselves acquainted with all the different ways of forming this frame, I should recommend that method to them which is the simplest and which gives them the most liberty to vary the form of it, according to every one's particular taste or fancy; and it is very possible there may be several other methods as easy and plain as those we have described. This frame being once formed, we may form all the rest upon the same plane. We shall in the next place shew the different methods used by the builders for that purpose.

The ancient builders not being acquainted with the methods of laying down their designs in a draught, found out a mechanic way of doing this only by help of the midship frame, which they might have formed by some of the preceding methods or any other contrivance of their own; and though this method is defective in several points, yet as it is an ingenious contrivance, we shall give it a place here.

METHOD I.

Of forming the Timbers by a Mould made to the Midship Frame; a rising Staff and overcast Staff.

Plate III. *Fig.* VII.] 1st. Having formed the midship frame and set off its scantlings, make a mould to fit both outside and inside, which may be called the bend mould

2d.

2d. Draw the line Z *x* to limit the head of the floor timber at *d*; let *d u* be the rising, and draw the line *a u*; let *t* be the height of the rising line abaft, and draw the line *d t* to represent the floor heads, or floor ribband. Set off *d x* from *d* to H; and from *e*, the head of the first futtock, to 6, and divide each into six equal parts, being the number of frames from midships to the balance frame.

3d. Divide the line *a u* into five equal parts, and set off two of them from *a* to S; divide the line *a* S into the same proportion, that the part A 6 of the base A C of the right angled triangle (*fig.* 5.) is divided into, and transfer these divisions to the bend mould, and let them be numbered 0, 1, 2, 3, 4, 5, 6, which points will give the narrowing of the floor, as we shall shew, after constructing the triangle. We shall only remark, that the line *a* S, which is $\frac{2}{5}$ of *a u*, is nearly the difference betwixt half the length of the midship floor timber, and half the length of the floor timber at the balance frame. But as this appears to be too much, we may take $\frac{2}{5}$ as in the figure, or any other quantity which shall be thought most convenient.

To construct the Triangle, Fig. 5.

Upon the line A C, drawn at pleasure, set off any distance from A to 1, and double that distance from 1 to 2, treble from 2 to 3, and so on in the same progression till we have as many divisions on the line A C as we propose to have frames abaft the midship. Erect a perpendicular at A, which may be produced at pleasure, and from any point B draw lines to all the divisions of the base A C. Observe that though in the triangle we have drawn a line for every frame to the fashion piece, we shall only make use of six, there being so many to the balance frame. The triangle being thus constructed, apply the line *a* S to it, in such a manner, that it may be parallel to A C, and be contained betwixt the lines B A and B 6, the lines drawn from the point B to the points 1, 2, *&c.* will divide it into the required proportion.

To construct the Rising Staff, Fig. 5.

This staff K may be of the same breadth with the keel, and a little longer than *a t*, the height of the rising of the floor. In order to graduate that staff, set off *x u*, the rising of the midship floor from K to *o*, and make *o* L equal to *a t*; apply the line *o* L to the triangle, so that it may be parallel to the base, and contained betwixt the lines A B and B C the

 lines

lines from the point B to the ſeveral points in the baſe will divide it into the required proportion, which will give the riſing of the floor.

Note, Our author calls x u *the acculement, and* u d *the riſing; the line* u a *will paſs through the point where the inverted arch joins the floor ſweep.*

To conſtruct the over caſt Staff, Fig. 5.

That we may have a clear underſtanding of what is meant by *over-caſt*, it will be proper to obſerve, that in forming the frames by the bend mould, when it is ſet to the narrowing of the floor, the head of the mould will come too far in at the deck; the mould muſt therefore be moved round upon the point which repreſents the floor ribband, till the head goes out to the proper breadth; this will occaſion the lower part of the mould to riſe a certain quantity, which is called the over-caſt. In order to graduate this ſtaff we muſt determine the difference betwixt the main breadth at the midſhip frame, and at the balance frame, which ſuppoſe D F, let this be placed parallel to the baſe, and contained betwixt the line B A and B 6; ſo ſhall the lines B 6, B 5, *&c.* divide it into the required proportion.

Theſe are the inſtruments that are neceſſary for forming the after frames, thoſe for the fore part are conſtructed in the ſame manner, only the graduations for theſe are but half the graduations of the former, for which reaſon there muſt be another bend mould graduated for the fore body.

Now, in order to form the frames by theſe inſtruments, place the bend mould upon the riſing ſtaff in ſuch a manner that the middle line of the ſtaff produced may paſs through the narrowing of the floor upon the bend mould, expreſſed by the diviſion correſponding to the frame to be formed; ſuppoſe frame 6, (*Fig.* 7.) the lower or ſtrait part of it expreſſed by the dotted line in the figure being applied to the riſing ſtaff, till the middle line B *a* paſs through the diviſion 6 on the bend mould: mark by the edge of the riſing ſtaff the point 6, which expreſſes the riſing of the floor at that frame. Set up the over caſt (expreſſed by the ſpace contained betwixt the points 5 and 6 upon the over caſt ſtaff) from the lower part of the bend mould to the point 6 upon the line B *a*; then keeping the point *d* immoveable, turn the bend mould upon this point till the lower part riſe to the overcaſt at the point 6 upon the line B *a*, and when in this poſition we may deſcribe the curve to the floor head, and then invert the bend mould, and placing the point 6 (betwixt *d* and H) to the point ſet off before to expreſs the riſing, turn the mould till

till the ſtrait part touch the curve before deſcribed, and then draw the lower part, which compleats the frame.

This is the method that is uſed when they mould the timbers, and it may likewiſe be uſed to lay them down upon a draught; for if the line *a u* of the bend mould (*Fig.* 8.) be laid upon the line A V, we may, when in that poſition, deſcribe the midſhip frame from the point *d* to the point X. In like manner we may deſcribe all the reſt of the frames, by giving each its proper over-caſt and riſing; as for inſtance, if it were required to deſcribe frame 6, take the riſing K 6 upon the riſing ſtaff, and ſet it off from the point B to the point *a* upon the line B G, and draw a line through the point *a* parallel to A V, upon which laying the bend mould in ſuch a manner that the point 6 which expreſſes the narrowing of the floor, ſhall be upon the point *a*; then will the point *d* be upon the point R S: ſet up the proper over-caſt from *a* to 6, and keeping the point *d* immoveable, puſh up the bend-mould which at firſt was placed at the point *a*, till it be raiſed to the point 6, which will throw out the point X to the proper breadth at the deck. But becauſe the deck is higher at timber 6 than at the midſhip frame. Take the diſtance betwixt *e* and 6, at the head of the futtock on the bend mould, and ſet it up from *x* to 6, and then inverting the bend mould, ſo that the point 6 betwixt *d* and H be at the point X, and the ſtrait part of the mould touch the curve before deſcribed: we may then deſcribe the lower part to the point X, which compleats the whole frame. The timbers for the fore body may be deſcribed by the ſame proceſs as thoſe of the after body, only making uſe of the bend mould, riſing and overcaſt ſtaff graduated for that purpoſe; but as we obſerved before, we cannot lay down any timbers by this method but thoſe betwixt the midſhip and ballance frame.

The builders finding how very advantageous it would be for them to form all the timbers upon the plane of the projection, becauſe they could then at one view ſee how they would compare one with another, have tried ſeveral expedients to perform this, of which I might inſtance ten or twelve, but ſhall content myſelf with explaining three, which may be ſufficient for thoſe purpoſes, in order to which I ſhall firſt ſhew another method of forming the midſhip frame, different from thoſe we have ſhewn before.

Plate III. *Fig.* 6. 1ſt. Draw the riſing deck and load-water lines, and ſet off the length of the floor timber as before.

2d. Take one fourth of the length of the floor timber, and ſet it off from O to *d*, upon which erect the perpendicular *d c*, and divide it into two equal parts in the point *e*.

3d.

3d. Describe an arch through the point *a*, the head of the floor timber, and the point *e*, taking for the radius the distance from the upper edge of the keel to the port-sells, or a little more or less, according to what round you propose to the floor head. This determines the rising of the floor timber, and with the radius O *l*, half the length of the floor timber, describe the arch *e* Y, which determines the *acukment* of the floor timber.

4th. At the point *l*, the middle of the line A O, erect the perpendicular *l m*; and at the point *n*, the middle of the line A *l*, erect the perpendicular *n o*; erect also the perpendicular *p q* at the middle of the line A *n*; and another *r s*, at the middle of the line A *p*; and lastly, another *t u*, at the middle of the line A *r*.

5th. Take the distance *l n*, which set off on the line *n o* from *n* to *z*; and on the line *p q*, from *p* to *g*; then taking the distance from *a* to *g* set that off from *p* to *y*; again take the distance *p y*, which set off from *r* to *b*, and the distance *b a* from *r* to F; and lastly take the distance *r* F, which set off from *t* to E, and then the distance E *a* from *t* to *x*, a curve passing through the point *a*, *z*, *y*, F, *x*, T, will form the midship frame under water. We may then set off half the thickness of the post and stem on each side of the middle line, and form the rest of the timbers; those for the fore body on the right, and for the after body to the left of the middle line.

Plate II. 1st. To lay down the post upon the plane of projection, take the difference of the draught of water abaft more than in midships, as marked on the plane of elevation (*Fig.* 2.) set off this from F to *d* (*Fig.* 3.) and draw the line *d e* parallel to A B; take also K F, the height from the plane of elevation, which set off from *e* to *r*, so shall the point *r* be the head of the post.

2d, to lay down the wing transom, take its height from the plane of elevation, which set up on the plane of projection to *f*, and draw the line *g f* perpendicular to the middle line, so *g f* represents the upper side of the wing transom without regarding the round up or the round aft. Take also the height of the rising line upon the post from the plane of elevation, which set off from *e* to G.

3d. To form the fashion piece; take upon the plane of the projection *n* G the height of the load-water line, above the rising line upon the post, which set off from *n* to *o* upon the water line; take also G P, the distance betwixt the rising line and lower deck, which set off from P to *q* upon the deck line, and describe a circle through the points *f*, *q*, *o*. There is a problem in geometry to find the center of this arch. *Note*, the point *q* may be taken further out or in, as you design a lank or full fashion piece.

Lastly,

Laſtly, deſcribe the arch *o* G; the radius of this arch may be the main half breadth; ſo ſhall *f*, *q*, *o*, G be the form of the faſhion piece, which may be varied according to the fancy of the artiſt, by altering the centers.

Having thus formed the midſhip and after frames, we ſhall in the next place ſhew how to ſpace the ribband lines, which are repreſented by the diagonals in the figure, but it will be proper to remark, that the ribbands are thin narrow planks which are made ſo, that they may eaſily be bent to the timbers. That which is nailed to the poſt at the height of the riſing line, and to the midſhip frame, at the end of the riſing of the floor timber, is called the floor ribband. That which anſwers to the wing tranſom and to the height of the lower deck, on the midſhip frame, is called the breadth ribband; all the reſt betwixt theſe two are called intermediates.

From the Point H draw the line H G for the floor ribband, and from the point T draw the curve T, E, *q*, *p* for the breadth ribband, and draw the two intermediates betwixt them, ſo that by them the curve of the midſhip frame and faſhion piece may be divided into three equal parts.

Now, it is very plain, that if the ribbands had a proper form, and nail'd at the proper heights and poſitions, they would compoſe a kind of a model, by which the circular form of every timber might eaſily be diſcovered; but as we have only the extreme points of each given, we cannot from thence form ſuch a curve as ſhall be neceſſary. We muſt therefore find a method to form ſome intermediate timbers betwixt the midſhip and after one, and thereby form the ribbands ſo that they ſhall make fair curves. There are ſome preliminary operations which are neceſſary towards performing this.

1ſt. *To conſtruct an equilateral Triangle for the Progreſſion of the Frames in the After-Body.*

Plate IV. *Fig.* 1. From the point M ſet off any diſtance to 1, upon any ſtrait line, and from 1 to 2 treble that diſtance, from 2 to 3 five times that diſtance, from 3 to 4 ſeven times that diſtance, and proceed in that progreſſion, increaſing the ſpaces betwixt the figures by equal differences, *viz.* double the diſtance betwixt M and 1, till we have as many diviſions leſs one as there are frames betwixt the midſhip and poſt, including that of the midſhip and poſt; and becauſe there are nine frames the line muſt conſiſt of ten diviſions, from the point M to the point E. Let them be numbered 1, 2, 3, &c. make M E the baſe of an equilateral triangle S M E, and draw the lines S 1, S 2, &c. obſerving to produce them all till the diſtance betwixt the lines S E and S 9, upon a line parallel to the baſe, be at leaſt equal to the diſtance betwixt the frames in the plane

of

of elevation. The line S M repreſents the midſhip frame, and the line S E, the poſt, and the nine intermediates, repreſent the nine frames betwixt the midſhip and poſt.

In order to give us a clear underſtanding of the uſe of this triangle, it will be neceſſary to remark, that the midſhip frame being that which incloſeth the greateſt ſpace, and the aftermoſt that which incloſeth the leaſt, it will follow, that the intermediate frames will partake of the form of each; but moſtly of that to which they are neareſt; yet they will ſtill retain a little of the form of each. Hence, when the intermediate frames are all formed, their curves will divide all the diagonals; drawn in the plane of projection, into as many parts as there are frames; and all the methods the builders have invented ſerve only to divide them into ſuch a proportion as ſhall produce the faireſt curves.

Now, if the proportion pitched upon for that purpoſe, be as 1, 3, 5, 7, 9; *&c.* then they muſt all be divided into the ſame proportions as the baſe of the triangle is divided into; and this may be performed very readily, only by taking the length of each diagonal from the plane of the projection, and applying it to the triangle in ſuch a manner that it ſhall become the baſe of an equilateral triangle; as for inſtance, to divide the firſt intermediate diagonal; take the length of it in the plane of projection, (*Plate* II. *Fig.* 3) and ſet it off from the point S to *m* and *k* on the ſides of the triangle S M and S E; and draw the line *m k*, which being parallel to the baſe of the triangle, will be divided into the ſame proportion. In like manner all the reſt of the diagonals may be divided; but as the builders are not agreed as to the preciſe form of a ſhip's bottom, ſome chuſe to divide the baſe of the triangle into another proportion; others again in applying the diagonals to the triangle give them different inclinations to the line M S. It would be very proper to try ſeveral of theſe methods, by which means we might diſcover which would be moſt convenient; and after all the diagonals are divided into as many points as there are frames, curves paſſing through theſe points will determine the form of all the frames from the midſhip to the poſt. It only remains to ſhew how to end each frame upon the poſt. It was before obſerved that the keel is not parallel to the ſurface of the water, ſo that it will be very eaſy to conceive that the height of each frame taken from the upper ſide of the keel, upon a perpendicular to the ſurface of the water, will always increaſe, the nearer the frame is to the ſtern poſt. Now *g* K is what the keel is deeper abaft than at the midſhip frame; and to find how much any frame abaft exceeds that of the midſhip, ſuppoſe the firſt; take the diſtance betwixt the line *g* G and K I at that frame, from the plane of of elevation, (*Plate* II.) which ſet off from F towards *d*, (*Fig.* 3.) and at that point draw a line parallel to *d e*, which will be the firſt frame upon the keel. In like manner we may draw lines parallel to *d e*, for all the reſt, as in the figure, which will determine their heights from the upper ſide of the keel to the ſurface of the water.

It muſt be obſerved, that the diagonals in the plane of the projection, which end on the faſhion piece, muſt likewiſe end on the faſhion piece on the plane of elevation; we muſt therefore draw the faſhion piece on the plane of elevation. Thus, take the diſtance of the point G, in the plane of the projection, from the upper ſide of the keel, which ſet off upon the ſtern poſt in the plane of elevation to the point *b*; through *n*, the rabbit of the wing tranſom, draw the ſtrait line *b n*, which will repreſent the faſhion piece on the plane of elevation. Now as only the loweſt diagonal ends upon the poſt, in the plane of projection, which in the plane of elevation ends at *b*, ſo the other diagonals that end upon the faſhion piece, muſt likewiſe end on the faſhion piece in the plane of elevation. Their height muſt therefore be transferred from the plane of the projection to that of the elevation; ſo the ſecond diagonal will end at the point P, upon the faſhion piece in the plane of elevation. In like manner all the reſt may be transferred to the plane of elevation; and as the line that repreſents the faſhion piece upon the plane of elevation rakes aft, this will occaſion the line P S, which is perpendicular to the line that repreſents frame 9, to exceed the line *b* M. In the triangle, the line S M repreſents the midſhip frame and the line S E the poſt; that is, if the point where the ribband ends on the poſt, be equally diſtant from frame 9, that frame 9 is from frame 8. Now as M *b* is longer than M L, we muſt draw the line S D without the triangle, which is to be uſed inſtead of the line S E, when we come to apply the diagonal H G to the triangle; for the point H muſt touch the line S M, and the point G the line S E. To find the point D, take M L from the plane of elevation, and apply it to the triangle, ſo that B C ſhall be equal to it; and parallel to M E; it muſt alſo be contained betwixt the line S 9 and S E. Then take *b* M and ſet off from B, which will give the point D. In like manner the line S F muſt be uſed when we divide the diagonal M K; and to find the point F ſet off P S in the plane of elevation, from B to F in the triangle; and draw the line S F. In the ſame manner there muſt be lines drawn for every diagonal without the line S E; ſo the line S E is not uſed in dividing the diagonals. Let it be further obſerved, that in applying each diagonal to the triangle, it muſt not only be contained betwixt the line S M and the line correſponding to the diagonal, which is to be divided, but it muſt likewiſe form a certain angle with the line M S,

Plate IV. *Fig.* I.]

that

that is, with that part of it which is intercepted betwixt the diagonal and the point S. These which appear to me to be properest for that purpose are as follows: The first diagonal to make an angle of 60 degrees; the second $62\frac{1}{2}$, the third 68, the fourth 86, the fifth 65, the sixth 60 degrees; but the artists vary these angles according to the form they design to give to the timbers; nay, some draw them always parallel to the base of the triangle.

Our author then proceeds to the forebody, and forms a triangle, the base of which he divides in the same manner as that already described, by which he divides each diagonal. He likewise shews how to space the diagonals upon the stem; but as the artists leave us so much undetermined as to the angles that each diagonal is to make with the line S M, *when they are applied to the triangle, it will be very difficult to apply this method to practice. So we presume it will be needless to say any more on that head, judging what has been already said sufficient to give our readers an idea of the principles on which the method is grounded; we shall proceed therefore to the next method he proposes.*

To form the Timbers by a Quarter of a Circle, Plate IV. Fig. 2. 3.

1st. Form the midship frame, the fashion piece, the foremost timber, also the two balance frames, by some of the preceding methods. *Note, Those who make use of the following method of forming the rest of the timbers are supposed to be previously acquainted with the manner of forming the midship frame,* &c.

2d. Space all the diagonals for the ribbands as directed in the preceding method.

3d. From the center A with any radius describe a quarter of a circle, and divide it into so many equal parts, that there may be a point for each timber to be formed, and draw the radii A 1, A 2, *&c.* to A 9, so we shall have one for each frame.

4th. Take *a b* the first diagonal, which set off from the point A upon the line A C, to 1.

5th, Take *a c*, the distance upon the lower ribband, betwixt the post and balance frame, which in the plane of projection is the 6th frame, set off this distance upon a perpendicular erected upon the line A B, to intersect the radius A 6, in such a manner that the perpendicular G 1 shall be equal to *a c*.

6th. Produce the line C A to F, and upon this line find a point, which shall be the center of a circle whose circumference shall pass through the point 1, before marked upon the line A C, and the point 1, now marked upon the radius A 6; describe the arch through these two points to the point 1 on the line A B.

7th,

7th. Let fall perpendiculars to the line A B from the points where the arch 1, 1, 1 interfects the feveral radii. Transfer thefe perpendiculars to the line *a b*, which will divide the lower diagonal into the points through which the curves of the frames muft pafs. *Note, the perpendiculars are not drawn to avoid confufion.*

After the fame manner all the other diagonals are graduated, firft by taking the whole length of each diagonal, and fetting them up on the line A C, from the point A to the points 5, 4, 3, 1, 2, and fecondly, by taking the feveral diftances upon each diagonal intercepted betwixt the after frame and the balance frame, and applying them feverally to the radius A 6, in fuch a manner that they fhall be contained betwixt the radius A 6 and the line A B, upon the perpendiculars let fall from the points 5, 4, 3, 1, 2. And thirdly by defcribing arches through the points in the line A C, to pafs through the points of the fame number upon the radius A 6, whofe centers are in the line A F; the arches to be produced to interfect the line A B in the points 5, 4, 3, 1, 2, will interfect all the radii; the perpendiculars let fall from the interfections of the radii with the arch correfponding to each diagonal, will divide that diagonal into the points through which the curves of the frames muft pafs.

The diagonals for forming the frames in the fore body are divided into the points through which the curves muft pafs by the fame operations, only obferving that frame 4 is the balance frame for the fore body.

CHAP. V.

Of the Projections on the horizontal Planes, and of the Water and Ribband Lines on the Plane of Elevation, and that of the Projection.

WATER Lines are defcribed upon a fhip's bottom by the furface of the water into which fhe fwims; that which determines how much is under water when fhe is loaded is called the load-water line. Now it is plain, that if a fhip is lightened, fhe will rife higher out of the water; and if fhe be lightened fo as to rife equally afore and abaft, the furface of the water will then form another water-line parallel to the load-water line. Again, if the fhip is lightened more fhe will ftill rife higher, and if the fame difference ftill continues betwixt the draught of water abaft and afore, we fhall have another water line parallel to the two former;

ſo that by this means we may deſcribe as many water lines as we pleaſe, all parallel to one another.

In order to form an idea how theſe lines are repreſented on the different planes, let us ſuppoſe a ſhip upon the ſtocks upon a level ground, and her keel in the ſame poſition, with reſpect to the horizon, that it is to be in the water when loaded; we may then deſcribe ſeveral black lines upon the ſhip's bottom, which may be whitened for that purpoſe, all parallel to the horizon: Theſe will all be water lines.

Now, if a ſpectator be removed at any conſiderable diſtance from the ſhip upon a line in the ſame direction with the keel, all theſe black lines which were drawn upon the ſhip's bottom, parallel to the horizon, and which are actually curves, will appear to him all ſtrait lines, becauſe he ſees them all upon a plane formed by a ſection paſſing through the midſhip frame perpendicular to the keel. Hence the water lines will be repreſented by ſtrait lines upon the plane of the projection.

Again, if a ſpectator is removed at any conſiderable diſtance from the ſhip upon a line perpendicular to the keel, ſo as to ſee the whole length of the ſhip at one view, the water lines will then appear to him ſtrait lines, becauſe he ſees them upon a plane erected perpendicular to the horizon upon the middle line of the keel. Hence the water lines will be repreſented by ſtrait lines upon the plane of elevation.

But if the ſpectator be ſuppoſed to be placed underneath the middle of the ſhip at any conſiderable depth, in a line perpendicular to the level ground, he will then, viewing the ſhip's bottom upwards, diſcover the curvings of all the water lines. Theſe curves are all projected upon a plane, which we muſt imagine to be formed by a ſection of the ſhip through the load-water line, and we are now to ſhew how theſe are formed:

To form the Water Lines upon the Horizontal Plane.

Let the water lines to be formed be repreſented in the plane of the projection by ſtrait lines all parallel to one another. Theſe will be repreſented by the ſtrait lines in the plane of elevation. Suppoſe *q r*, *s t* *b x*, and T V, all parallel to one another, and the ſame diſtance from the load-water line T V that the lines which repreſent them in the plane of the projection are from it. In order to form theſe upon the horizontal plane,

1ſt, Take half the thickneſs of the poſt from the plane of the projection, and lay it off on the horizontal plane from A to E, and through the point

point E draw the line E*s* parallel to A B, five or six feet long; lay off the same distance from B to F, and thro' the point F draw a line F R parallel to A B, five or six feet long.

2d. From the points where the water lines intersect the stern post upon the plane of elevation, let fall perpendiculars. In like manner let fall perpendiculars from the points where the water lines intersect the stem,

3d. Take upon the water lines, in the plane of the projection, the several distances intercepted betwixt the middle line and the curve of the midship frame, and lay them off from the line A B in the horizontal plane, upon the perpendicular that represents the midship frame. Take also from the plane of projection the several distances intercepted betwixt the middle line and the curvings of the other frames, and lay them off in the horizontal plane from the line A B upon the perpendiculars corresponding to their respective frames, both in the fore body and after body, and curves passing through all these points will give the true form of all the water lines. They end forward at the points where the perpendiculars intersect the line F R. The water lines abaft which end upon the post in the plane of elevation, will end where the perpendiculars intersect the line E*s* upon the horizontal plane. But the 3d and 4th water lines cannot end upon the post, by reason of the fashion pieces; and in order to find the points where these shall end, we must proceed in the following manner.

To find the point where the load-water line ends, let fall a perpendicular from the point *k*, where it intersects the fashion piece on the plane of elevation, to N. Take from the plane of the projection upon the line that represents the load-water lines, the distance betwixt the fashion piece and the mid-line; lay this off upon the horizontal plane from the line A B to the point N, which will end the load-water line upon the horizontal plane, from whence it may be drawn to *g*; so *g* N will be the flat of the Tuck; and to find the point *g* draw a line parallel to *k* N thro' the point where the line T V cuts the rabbit of the post, which will give the point *g*. We may after the same manner find the ends of the other water lines that do not go the stern post for a square tuck.

To form the Ribbands upon the Horizontal Plane.

We observed before, that the ribbands were thin planks nailed to all the frames from the post to the stem; and that when they are carried round, so as to make fair curves, the form of all the filling timbers may be by them determined. These filling timbers are to be placed betwixt

 the

the frames, which were methodically laid down in the draught. We ſhall here further obſerve, that theſe ribbands will round two ways, one in a vertical, and one in an horizontal ſenſe, occaſioned by the nature of the form of the ſhip's body; for they will, in carrying them about, naturally fly higher abaſt and before than they are in midſhips, which gives them a vertical curve, and the narrowing of the ſhip's breadth from the midſhips both ways gives them the horizontal curve; thence they will be repreſented by different lines on all the planes.

They are repreſented upon the plane of the projection by ſtreight lines, all but the breadth ribband, which is uſually repreſented by a curve; but upon the plane of elevation, and that of the horizon they will be repreſented by curves. The reaſon of theſe different appearances, ariſes from the different ſituations in which they are ſuppoſed to be viewed, as was obſerved in reſpect of the water lines.

Now in order to comprehend the relation betwixt theſe horizontal curves, and the lines that repreſent them upon the plane of the projection; it will be ſufficient to remark, that theſe horizontal curves reſult from the different lengths of the perpendiculars that are ſuppoſed to be drawn in the plane of the projection, from the points where the lines that repreſent the ribbands interſect the frames, to the middle line. Hence, if the lengths of theſe perpendiculars are transferred to the lines correſponding to each frame in the horizontal plane, we ſhall have the points thro' which the curve that forms the ribband muſt paſs.

But if theſe ribbands are to be repreſented upon a plane placed in an oblique poſition to the horizon, that is to ſay, a plane that has the ſame inclination to another plane erected perpendicularly upon the middle line of the keel, that the line that repreſents that ribband, has to the middle line in the plane of the projection; in that caſe, they will have a quite different form from what they have upon the plane of the horizon.

Now, to conceive the relation betwixt theſe and the lines that repreſent them upon the plane of the projection, it will be ſufficient to remark, that if the ſeveral diſtances taken upon each diagonal intercepted betwixt the middle line and the points where theſe diagonals interſect the curves of the timbers in the plane of the projection; I ſay, if theſe be transferred to the lines that repreſent thoſe timbers, we ſhall have the points thro' which the curves that form the ribbands muſt paſs.

Again, if theſe ribbands are to be repreſented upon the plane of elevation, they will have a different form from any of the former; to find which we need only take the perpendicular diſtances from the points where the diagonals interſect the curves of the timbers in the plane of of the projection to the line that repreſents the upper ſide of the keel, and transfer them to the plane of elevation, ſetting them up from the upper ſide of the keel upon the line correſponding to the timber, from which they were taken upon the plane of the projection. This will give us the points thro' which their curves muſt paſs.

Having thus given a general deſcription of theſe curves, we ſhall now proceed to deſcribe them upon the different planes.

To deſcribe the Floor Ribband upon the Plane of Elevation.

1ſt. Take the perpendicular diſtance betwixt the point *a*, where the diagonal interſects frame 9, and the lower water line in the plane of the projection.

2d. Set up this diſtance from the point S, where the lower water line interſects frame 9 in the plane of elevation, and we ſhall have a point G, thro' which the curve muſt paſs.

Now, it is plain, that we may, by repeating the ſame operations, have a point in each frame, thro' which the curve of the ribband muſt paſs upon the plane of elevation. After the ſame manner are all the other ribbands formed.

To deſcribe Ribbands upon the Horizontal Plane.

The breadth ribband is formed by transferring the lengths of all the perpendiculars that are ſuppoſed to be drawn from the points where the curve that repreſents this ribband interſects the timbers, to the middle line in the plane of the projection: This curve, in the plane of the projection, is drawn from the breadth in midſhips to the extremity of the wing tranſom.

1ſt. Lay off the length of the wing tranſom upon the perpendicular *n* L.

2d. Take the length of the perpendicular drawn, from the point where the curve that repreſents the breadth interſects frame 9, to the mid-line in the plane of the projection; lay off this from the line AB upon the perpendicular repreſenting frame 9 in the horizontal plane, to the point S, which will be one of the points thro' which the curve of the ribband muſt paſs. We may proceed in the ſame manner to find points upon all the perpendiculars, both afore and abaft, ſo ſhall the curve L, S, Q, 1, be the form of the breadth ribband. But to compleat this ribband, the round aft of the wing tranſom muſt be ſet off.

To form the Oblique or Cant Ribbands.

We obſerved before, that theſe ribbands could not be formed either upon the horizontal plane or that of elevation, upon which account they were ſeldom drawn, becauſe each muſt be drawn upon a ſeparate plane. However, thoſe who incline to draw them may uſe the following method:

Let it then be required to form the firſt ribband repreſented in the plane of the projection, by the diagonal H G.

1ſt. Produce line H G to the point *p* in the middle line upon the plane of the projection.

2d. Take the height of the point *p* above the line that repreſents the upper ſide of the keel in midſhips, in the plane of the projection; ſet up this from the ſame line in the plane of elevation, on a perpendicular, upon the poſt, from which point let fall a perpendicular to the point F in the line C D, and produce all the perpendiculars that repreſent the frames to the line C D; ſo F, Q will be the axis of the ribband from the poſt to the midſhips.

3d. Take upon the plane of the projection in the line H P, the diſtance *p* G, which ſet off upon the perpendicular from the point F to *f*.

4th. Take the diſtance on the diagonal from the point *p* in the middle line to its interſection with the frame 9. Set off this from the line C D upon the perpendicular correſponding to frame 9; this will give us a point thro' which the curve muſt paſs. Do the ſame for all the other frames to the midſhip.

In like manner the curve for the fore part of the ribband is formed from the interſections of the diagonal 4, 5, with the curves of the frames in the plane of the projection; but it is evident, this is a different plane from that of the line H*p*; therefore we muſt have a different axis for the curve of the fore part of the ribband. In order to which, take from the plane of the projection the diagonal 4, 5; ſet off this from the point O to Z, and draw the line Z X parallel to C D. We muſt likewiſe take the height of the point 4 in the plane of the projection, and ſet it up on the ſtem; from which point letting fall a perpendicular to the line Z X, we ſhall limit the fore end of the ribband. The points thro' which the curve muſt paſs will be found in the ſame manner as thoſe for the after-body.

The builders make uſe of the cant ribbands to find the bevellings of the timbers; For we muſt repreſent each frame as one intire piece of circular timber, and being all faſtened to the keel they form the ſide of the ſhip. They are ſquare upon the upper ſide of the keel; but becauſe both

both the outside and inside of the ship's sides, length-ways, form curves, it is plain, that the sections of any of the frames, the midship only excepted, will produce a surface in the form of a lozenge or rhombus; the angles which are formed by these sections are what are called the bevellings of the timbers.

The ship-wrights take these angles mechanically by an instrument call'd a bevel; thus they draw, upon the plane of the ribband, a line parallel to that which represents the frame, and distant from it the whole breadth of the timber; and applying the stock of the bevel to the line that represents the frame, and the tongue to the ribband, they have the quantity of the angle which forms the bevelling of the timber at that place.

It is plain the angle *b*, *a*, *c*, which points to the midship frame, will be obtuse, whereas the angle *b*, *a*, *d*, which points to the post, will be acute.

Now, as every timber has two planes, that which points to the midships will have what they call a standing bevelling, and that which points either to the post or stem will be under bevelling.

We shall shew in another place how the modern builders, by putting the frames in an oblique position to the keel afore and abaft, lessen the bevellings.

CHAP. VI.

Another Method of laying down the Horizontal Plane, and the Plane of Projection.

THOSE who are well versed in the art of drawing have taken a method quite different from any of those we have described, which shall be the subject of this chapter.

After forming the plane of elevation, and drawing all the perpendiculars for the frames, as before, the following method must be observed:

I.

To lay down the Breadth Ribband on the Horizontal Plane.

The extremities of it on the stem and post, and the point thro' which it is to pass on the midship frame are found as directed in the preceeding chapter. It remains now to find the points in the balance frames, thro' which it is to pass.

To find the point in the fore balance frame take $\frac{113}{100}$ parts of half the main

main breadth, which ſet off on the line that repreſents that frame in the horizontal plane from K to L.

To find the point in the balance frame abaft, take $\frac{184}{200}$ parts of the half of the main breadth from M to N. It will be neceſſary to have another point in the fore-body, thro' which the curve muſt paſs; for which purpoſe uſe the following method:

Divide the ſpace contained betwixt the line that repreſents the balance frame afore and the rabbit of the ſtem, into two equal parts, and draw the line O P, on which ſet off the 160th part of the main breadth, which will give the point P, thro' which the curve is to paſs. It muſt be obſerved, that the proportions for finding theſe points may be varied according to the form we propoſe to give to the ribband. After the points H, N, Q, L, P, I are thus ſet off, we may deſcribe the curve either by moulds or penning battens.

II.

To lay down the Floor Ribband on the Horizontal Plane.

1ſt. The height of this ribband muſt be determined both upon the poſt and ſtem, from which points letting fall perpendiculars, we ſhall have the extremities of it on the horizontal plane, obſerving to allow for the rabbit.

2d. Take half the length of the midſhip floor timber, and ſet off on the line that repreſents the midſhip frame on the horizontal plane from *a* to S, which will be the point thro' which the curve muſt paſs.

3d. Take $\frac{143}{200}$ of the line *a s*, the breadth at the midſhip frame, and ſet it off on the balance timber afore, from K to T, and ſet off $\frac{149}{200}$ of the ſame line, upon the balance timber abaft to V, and draw the curve thro' the points G, V, S, T, R.

III.

To lay down the after Balance Timber upon the Plane of Projection.

1ſt. Produce the line which repreſents it on the horizontal plane to the ſheer rail, on the plane of elevation, and take the diſtance upon this line betwixt the upper ſide of the keel, and the lower edge of the ſecond wale, which here repreſents the breadth ribband; ſet up this from A to C on the plane of the projection, from which point draw the line C D, perpendicular to the middle line. (*Plate* II. *Fig.* 1, 2, 3.)

2d. Take the line M N in the horizontal plane, and ſet off from D to E, which will give one point, through which the curve of the timber muſt paſs.

3d. Take

3d, Take the height of the floor ribband, in the plane of elevation, and ſet it up on the plane of the projection to G; from the point H, at the end of the floor timber, draw the line H G which will repreſent the floor ribband on the plane of the projection. (*Plate* II. *fig.* 1. 2. 3.)

4th, Take the diſtance M V, in the horizontal plane, with a pair of compaſſes, and move the compaſſes with one foot, in the middle line, and the other in a line perpendicular to it, till it interſect the diagonal in the point L, thro' which the curve of the frame muſt paſs. To thoſe who are acquainted with drawing, the three points E, L, F, will be ſufficient to form the timber; they who incline to have another point may divide the line A C into two equal parts by a perpendicular M K drawn to the middle line, from which ſetting off $\frac{138}{200}$ of the line M K we ſhall have another point thro' which the curve muſt paſs.

IV.

To lay down the ninth Frame abaft on the Plane of the Projection.

Take the height of the breadth ribband at this frame, in the plane of elevation, and ſet it up on the plane of the projection from F to O, and draw the line O P perpendicular to the middle line. (*Plate* II. *fig.* 1.2.3.)

2d, Take the diſtance X S in the horizontal plane, and ſet off from O to P, which will be the point thro' which the curve muſt paſs.

3d, Take the diſtance X Z in the horizontal plane, which ſet off from the middle line, to interſect the diagonal that repreſents the floor ribband, in the plane of projection in Q, obſerving to keep the compaſſes as before directed.

4th, Divide the line K O in two equal parts, and draw the line R S perpendicular to the middle line, on which ſet off $\frac{154}{200}$ of the line P O, from R to S, and draw the curve thro' the points P, S, Q, F, which will be the form of the ninth frame.

V.

To lay down the intermediate Ribbands abaft on the Plane of the Projection.

1ſt, Take the diſtance betwixt the upper ſide of the keel and the breadth, upon the line that repreſents the midſhip frame, in the plane of elevation, and ſet it up from A to T, and from B to T, in the plane of the projection, ſo ſhall the line T T give the height of the breadth ribband in midſhips.

2d, Divide the curve H M T into as many equal parts as there are to be intermediate ribbands; divide alſo the curve of the ninth frame Q S P into the ſame number, and, thro' theſe diviſions, draw the diagonals which will repreſent the ribbands as in the plate.

VI.

To lay down the firſt intermediate Ribband upon the Horizontal Plane.

1ſt, Take the neareſt diſtance of the point V (which is the extremity of the diagonal in the plane of the projection) to the middle line O F, ſet off this on the line which repreſents the midſhip frame in the horizontal plane, which will give the point thro' which the curve muſt paſs at that place. After the ſame manner we may find the points in the lines that repreſent the balance and ninth frames in the horizontal plane.

2d, Take F Z, the height of the ribband upon the rabbit of the poſt, in the plane of the projection, and ſet it up on a perpendicular, from N to the point *k* on the line that repreſents the rabbit of the poſt in the plane of elevation; take the neareſt diſtance of the point *k* to the perpendicular of the poſt, which ſet off from E to *e*, aud this will be the end of the ribband: ſo a curve paſſing thro' the points *e*, *d*, *c*, *b*, will be the form of the ribband.

VII.

To lay down the Wing Tranſom upon the Plane of the Projection, and on the Horizontal Plane.

1ſt, Take the height of the upper ſide of the wing tranſom (including the round up) in the plane of elevation, and ſet it up in the plane of the projection to the point *e*.

2d, Take the height in the plane of elevation, without regarding the round up, and ſet off from F to *f*, and draw the line *f g* perpendicular to the middle line, on which ſet off the length of the tranſom from *f* to *g*, this is equal to the line G H in the horizontal plane. The curve *g e* repreſents the upper ſide of the wing tranſom.

The round aft of the tranſom is repreſented upon the horizontal plane by the curve L *k e*; H L is the ſquare end of it.

VIII.

To lay down all the Frames in the after Body.

All theſe are laid down in the ſame manner as the ninth and balance frames before deſcribed, that is, by taking the half breadth of the ribbands at each frame in the horizontal plane, and ſetting them off from the middle line in the plane of the projection to interſect the diagonal correſponding to the ribband, as directed in forming the balance frame, by this means we ſhall divide each into as many points as there are frames: the curves drawn thro' theſe points will give the form of all the frames in the after body.

IX.

To lay down the Position of the Fashion Piece on the Horizontal Plane.

Let fall the perpendicular GH, from the end of the wing transom, and draw the line H *l*, which will represent the plane of the fashion piece upon the horizontal plane, observing to make the angle G H *l*, about 25 degrees.

X.

To form the Fashion Piece in the same Manner it is to be, when put into its proper place in the Ship.

The fashion piece laid down in the plane of the projection, regards that frame as it would appear when viewed from abaft; but as the fashion pieces on each side are not in one plane, as all the rest of the frames are, we shall be much deceived, if we imagine that the fashion piece laid down in the plain of projection, will give the true form of that which is to be put in the ship. We must therefore lay it down upon another plane, and, to avoid confusion, we shall separate it from the plane of projection.

Note, The fashion piece, mention'd by our author, described in the plane of the projection, is that betwixt the ninth frame, aud the curve f q o G, *which represents the fashion piece of a square tuck; it is formed in the same manner as the rest of the frames, by transferring the lines* n m, p o, *&c. in the horizontal plane, to the plane of projection, to intersect the diagonals corresponding to these ribbands in the points* i, l, *&c*:

1st, Draw the line *f* G, to represent the middle line of the plane of projection. *(Fig. 4.)*

2d, Draw the line *f g* perpendicular to G *f*, to represent the wing transom.

3d, From *l*, the point where the fashion piece intersects the floor ribband in the plane of the projection, take the nearest distance to the line *f g*, which represents the wing transom, and set off this distance in *Fig.* 4. from *f* to *h*, and draw the line *h l* parallel to *f g*.

4th, From the point *l*, where the fashion piece intersects the first intermediate diagonal, in the plane of projection take the nearest distance to the line *f g*, set it off from *f* to *k*, in *Fig.* 4. and draw the line *k m*, parallel to *f g*.

5th, In like manner, the points where the fashion piece intersects the second and third diagonals in the plane of projection, are to be transferr'd to the points *q* and *n*, *Fig.* 4. and the lines *p q*, *n o* drawn parallel to *f g*.

 6th,

6th, To find the points through which the curve muſt paſs: Take the line *l* H, which repreſents the poſition of the faſhion piece upon the horizontal plane; lay this off from *f* to *g*: Again, take the diſtance *l y* in the horizontal plane, which lay off from *q* to *p*, in like manner ſet off the diſtance *l z*, from *n* to *o*; and the diſtance *l p* from *k* to *m*, and laſtly, the diſtance *l n*, from *b* to *l*; ſo a curve drawn through the points *g*, *p*, *o*, *m*, *l*, will give the true form of the faſhion piece.

XI.

To lay down the Faſhion Piece upon the Plane of Elevation.

1ſt, Take the ſeveral heights above the keel, of the points where the faſhion piece interſects the diagonals in the plane of projection, and transferr them to the lines *o*, *p*, *q z*, *y*, in the plane of elevation, drawn parallel to the keel, and the ſame height above it, that their coreſponding points are in the plane of projection.

2d, Take the neareſt diſtance of the point *n*, in the plane of elevation, to the line C F, the perpendicular from the head of the poſt, ſet off this from the ſame line in the plane of elevation upon the line *p*; which will be the point through which the curve muſt paſt.

3d, In like manner the points *z*, *y*, muſt be transferr'd from the horizontal plane, to the plane of elevation in the points *z*, *x*, a curve paſſing through theſe points will be the projection of the faſhion piece on the plane of elevation.

We ſhall hear remark, that ſome builders to avoid giving a great bevelling to the timbers, and likewiſe that they may not require ſuch compaſs timber, do change the direction of all the frames in the fore-body before that of the loof; that is, the lines that repreſent them in the horizontal plane make an acute angle with the line that repreſents the keel; theſe are called cant timbers, and may be formed in the ſame manner as the faſhion piece, which we have now deſcribed. Tho' ſeveral builders form all the frames perpendicular to the keel, to have the floor timbers in one piece, which will be much ſtronger than when in two pieces, and this will inevitably be the caſe when the timbers are canted.

We might here ſhew how to lay down the top timbers, but as that part under water is the moſt material, we ſhall proceed to form the timbers afore.

XII.

To lay down the Frames for the Fore-body.

The balance and the eighth frame muſt firſt be formed in the ſame manner as the balance and ninth frame abaft: In order to which the curve that

that represents the breadth ribband must be laid down in the plane of the projection afore The diagonal, which represents the floor ribband must likewise be laid down in the plane of the projection, for which purpose we must take the height of the ribband above the keel upon the rabbit of the stem, and set it upon the line that represents the rabbit of the post in the plane of the projection, to the point 4; from which draw a line to the floor head, so 4 5 will represent the floor ribband.

XIII.

To space the Diagonals that represent the Ribbands afore, in the Plane of the Projection.

1st. As the points of their intersection at the midship frame are the same afore that they are abaft, we need only transfer them from abaft to the fore body.

2d. Take the height of the breadth ribband upon the stem in the plane of elevation, and set it up from F to 17 in the plane of projection.

3d. Divide the distance betwixt 4 and 17 into four equal parts, which will give the points in the plane of projection, where the intermediate diagonals end on the stem.

After the diagonals are drawn in the plane of the projection, the ribbands may be laid down in the horizontal plane, and from thence all the other frames may be laid down in the plane of projection, in the very same manner that the horizontal ribbands and the frames for the after-body were laid down.

CHAP.

CHAP. VII.

General Remarks on Ship Building.

ALL the rules we have hitherto laid down, collected from the principal dimensions of ships built by the most eminent masters, should only be so far regarded as they may assist the artist in forming the body in such a manner as to produce effects answerable to the service for which the vessel is designed.

In order to qualify a builder for such an undertaking, it is necessary he should understand the nature of fluids, and of such bodies as will float in the water; when he has made himself acquainted with these, I would recommend him to Mr *Bouguer*'s treatise on ship-building.

The principal Qualities belonging to Ships.

1st. To be able to carry a good sail, not only because in forming the body, the water lines are all supposed to be described when a ship is upright in the water, but likewise for doubling a cape, or getting off a lee shore, which will be impossible to be done when a ship lies over in the water, this will likewise render her lower tier, if not all her guns useless.

2d. A ship should steer well, and feel the least motion of the helm.

3d. A ship should carry her lower tier of guns four feet and a half, or five feet out of the water, otherwise a great ship that cannot open her ports upon a wind, but in smooth water, may be taken by a small one, that can make use of her guns, or she must bare away before the wind, to have the use of her guns; on which account it will be proper to raise the ports higher before than in midships, because the fore part of the ship is often pressed into the water by carrying sail.

4th. A ship should be duly poised, so as not to dive or pitch hard, but go smooth and easy through the water, rising to the sea when it runs high, and the ship under her courses, or lying to under a mainsail, otherwise she will be in danger of carrying away her masts.

5th. A ship should sail well before the wind, large, but chiefly close hawled, keep a good wind, not fall off to the leeward.

Now the great difficulty consists in uniting so many different qualities in one ship, which seems indeed to be impossible; the whole art therefore consists in forming the body in such a manner, that none of these qualities shall be entirely destroyed, and in giving the preference to that which is most required in the particular service for which the vessel is built; in order to which

which it will be neceſſary to know, at leaſt nearly, what form will give a veſſel one of theſe qualities, conſidered abſtractly from the reſt.

To make a Ship carry a good Sail.

A flat floor timber, and ſomewhat long, or the lower futtock pretty round, a ſtreight upper futtock, the top timber to throw the breadth out aloft; at any rate to carry her main breadth as high as the lower deck: now, if the rigging be well adapted to ſuch a body, and the upper works lightened as much as poſſible ſo that they all concurr to lower the center of gravity, there will be no room to doubt of her carrying a good ſail.

To make a Ship Steer well, and quickly Anſwer the Helm.

If the faſhion pieces be well formed, and the tuck carried pretty high; the midſhip frame carried pretty forward; a conſiderable difference of the draught of water abaft more than afore, a great rake forward and none abaft, a ſnug quarer deck and forecaſtle, all theſe will make a ſhip ſteer well; but to make her feel the leaſt motion of her helm, it will be neceſſary to regard her maſts. There is one thing not to be forgot, that a ſhip which goes well will certainly ſteer well.

To make a Ship carry her Guns well out of the Water.

It is plain that a long floor timber, and not of a great riſing, a very full midſhip frame, and low tuck with light upper works will make a ſhip carry her guns high.

To make a Ship go ſmoothly through the Water without pitching hard.

A long keel, a long floor not to riſe to high afore and abaft, the area or ſpace contained in the fore body, duly proportioned to that of the after body, according to the reſpective weights they are to carry; all theſe are neceſſary to make a ſhip go ſmoothly through the water.

To make a Ship keep a good Wind.

A good length by the keel, not too broad, but pretty deep in the hold, which will occaſion her to have a ſhort floor timber, and great riſing.

As ſuch a ſhip will meet with great reſiſtance in the water going over the broad ſide, and little when going a head, ſhe will not fall much to the leeward.

Now ſome builders imagine that it is not poſſible to make a ſhip carry her guns well; carry a good ſail; and to be a prime ſailer, becauſe it

would

would require a very full bottom to gain the first two qualities, whereas a ſharp ſhip will beſt anſwer for the latter; but when it is conſidered that a full ſhip will carry a great deal more ſail than a ſharp one, a good artiſt may ſo form the body as to have all theſe three good qualities, and likewiſe ſteer well, for which purpoſe I would recommend ſomewhat in length more than has been formerly practiſed.

After what has been ſaid upon this head, I believe it will not be thought impoſſible to unite all theſe different qualities in one ſhip, ſo that all of them may be diſcerned in ſome degree of eminence, but when it happens otherwiſe, the fault muſt be owing to the builder, who has not applyed himſelf to ſtudy the fundamental rules and principles of his art.

Excepting ſome antient builders, who were happily born with a natural genius, and our moderns, who being inſtructed in the principles of the mathematics, have truly laboured very hard to make a progreſs in the art of ſhipbuilding, one may, without violating the truth, affirm that the greateſt part ſatisfy themſelves with copying ſuch ſhips as they eſteem good ſailors, and it is theſe ſervile mechancick methods, which to the great reproach of the art, are but too common, that have produced all theſe pretended rules of proportion, all theſe methods of deſcribing the midſhip frame, and forming the reſt of the timbers, which every builder endeavours if poſſible to conceal and keep wholly in his own family.

How low and mean is this? it is as if a great architect ſhould endeavour to conceal the proportions of the different orders of architecture, whereas they are publiſhed every where, and ſo well known that many can raiſe a very beautifull porch or triumphal arch; but tho' the methods of deſcribing the midſhip frame and forming the reſt of the timbers be known to moſt apprentices, yet we have but few good maſter builders: This requires more than thoſe mechanick rules, they ſhould at leaſt have ſuch a knowledge of the mathematics, phyſicks, mechanicks, of the nature of ſolids and fluids, as to be able to diſcover what figure would procure ſome good quality without hazarding or putting a bad one in its place.

Let us ſuppoſe one to have a collection of draughts of a vaſt number of ſhips, and whoſe good and bad qualities have been remarked with all poſſible exactneſs, ſuch a valuable treaſure would be of great ſervice to a perſon who could calculate preciſely by the draughts where the fault lay, and how it might be rectified. For inſtance, ſuppoſe a ſhip ſails well, but carries her guns too low, a builder who is not acquainted with theſe principles would raiſe her deck, in conſequence of which ſhe would not ſail well; whereas one that could exactly calculate how much the reſiſtance of the fluid is diminiſhed upon the prow, would take great care

or

to add no more to any of the other parts than he could find by an exact calculation might be done without augmenting the resistance in the fluids.

M. *Bouguer* has published several useful problems for making these calculations, to which we refer the reader, and only explain what regards the height of the gun deck, and the resistance of the fluid, in one example of a 70 gun ship.

CHAP. VIII.

To know by the Draught how high a Ship will carry her Guns out of the Water.

THIS is only to know if, when a ship is loaded with all her ammunitions and provisions on board, and ready to sail, her seat in the water will then agree exactly with the load water line in the draught.

It may be demonstrated by several experiments, that any floating body of whatsoever figure will just sink so far in the water as to displace a bulk of water of equal weight with itself.

Hence it will be necessary, first to find a method of calculating the exact weight of a ship ready equipt for sea, and, secondly, to know the exact weight of the water the ship displaces, when loaded to the water line in the draught.

In order to the first, the exact weight of all the timber, iron, lead, masts, sails, rigging, and in short of all the materials, men, provisions, and every thing else on board the ship must be known.

It must be confessed that this is a very laborious task, yet the zeal of our modern builders has surmounted all these difficulties, and got the exact weight of a ship of each class with all its furniture, and six months provisions on board. It will be sufficient for our purpose to give the particulars, of the two followings, one of 30 and another of 50 guns, both ready equipt for sea, with six months provisions on board.

An Estimate of the Weight of the RENOMEE *Frigate of Thirty Guns, with Six Months Provisions.*

WEIGHT of the HULL.

	Cubic feeet	Under water Pounds.	Above water Pounds.	Total in Tuns. Tons	Pounds
Oak timber { under w. at 72 *lb.* per f.	5640	406080		299	800
Oak timber { above w. at 66 *lb.*	2920		192720		
Fir at 50 *lb.* per foot { under water	600	30000		29	000
Fir at 50 *lb.* per foot { above water	560		28000		
Carved work ———			2200	1	200
Iron knees and ſtandards		4200	7010	5	1210
Bolts, rudder irons, chain plates, nails		11650	6558	9	208
Lead for the hauſe holes & ſcuppers		250	430	0	680
Locks ———			170	0	170
Oakum ———		1200	1830	1	1030
Pitch and Tar ———			650	0	650
Paint ——— ———			440	0	440
In the Cook room ———			8000	4	000
Total		453380	248008	350	1388

WEIGHT of the FURNITURE.

	Pounds.	Pounds.	Tons	Pds
Maſts compleat ſet and ſpare	3000	37000	20	00
Blocks ——— ———	1000	5444	2	444
Pumps ——— ———	1734	670	1	404
Cables and Hawſers	24444		12	444
Sails and their Caſes	4222	3778	4	000
Anchors and their Stocks	2611	6944	4	1555
Cordage for the rigging		17282	8	1282
The maſter's Stores	3333		1	1333
Boats ——— ———		6666	3	666
	40344	75784	58	128

WEIGHT of the PROVISIONS, &c.

	Under water Pounds.	Above water Pounds.	Total in Tuns. Tons	Pounds
Proviſions for 6 months for 200 men with all their equipage	245420		122	1420
Water for two months and a half	100000		50	000
Caſks ———	32800		16	800
The Captain's table	15000	5000	10	000
Total	393220	5000	199	220

Weight of the Officers Stores.

	Under water Pounds.	Above water Pounds.	Total in Tons: Tons	Pds
The Carpenter's Stores	3000	1000	2	00
The Caulker's Stores	1000		0	1000
The Surgeon's Effects	2400		1	400
The Pilot's Effects	740	360	0	1100
The Chaplain's Effects		100	0	100
	7140	1460	4	600

Weight of the Guns and Ammunition.

	Under water Pounds.	Above water Pounds.	Total in Tons: Tons	Pds
Iron Guns ———		60300	30	300
Carriages fitted ———		14000	7	00
Balls round and crofs bar	11570	2430	7	00
Balls of one pound ———	600		0	600
Powder and Powder Barrels	7108	112	3	1220
Implements for the powder	1368	132	0	1500
Crows, Handfpikes, Gunners Utenfils, and Stores	3200	1500	2	700
Mufquets, Cutlaffes, and Pole Axes		900	0	900
	23846	79374	51	1220

Weight of the Men and their Equipage.

	Under water Pounds.	Above water Pounds.	Total in Tons. Tons	Pounds
8 principal Officers and their Effects		4000	2	00
200 Men and their Effects		40000	20	00
Total		44000	22	00
Ballast	200000		100	00

RECAPI-

RECAPITULATION.

	Under water Pounds.	Above water Pounds.	Total in Tons. Tons	Pounds
The Hull	453380	248008	350	1388
The Furniture	40344	75784	58	128
The Provisions	393220	5000	199	220
Officers Stores	7140	1460	4	600
Guns and Ammunition	23846	79374	51	1220
Weight of the Men		44000	22	00
Ballast	200000		100	00
Total	1117930	453626	785	1556

An Estimate of the Weight of a Frigate of Fifty Guns, with Six Months Provisions.

	Under water Pounds.	Above water Pounds.	Total in Tons. Tons	Pounds
The Hull	774270	769134	771	1404
The Furniture	98237	163184	130	1421
Ballast	300000		150	000
Guns and Ammunition	67960	199320	133	1280
Provisions	659400	8000	333	1400
Stores	9800	2800	6	600
Men and their Equipage		77000	38	1000
	1909667	1219438	1564	1105

But as all ſhips of the ſame claſs are pretty near the ſame dimenſions, and have the ſame number of guns, &c. we may have the exact weight of each only by examining the draught of water, and computing the weight of that column of water which is diſplaced by the ſhip.

Now if the *Intrepide* weighs 2718 tuns, ſhe muſt ſink ſo far into the water till ſhe has diſplaced a column of water containing 73459 $\frac{17}{37}$ cubick feet, for a cubick foot of ſalt water being ſuppoſed to weigh 74 lb. the 73459 $\frac{17}{37}$ will weigh 5436000 lb. or 2718 tuns, or if ſhe diſplaces 73459 $\frac{17}{37}$ cubick feet of ſalt water, we may thence conclude that ſhe weighs 2718 tuns.

In like manner, if the weight of the ſhip which is to be laid down in the

the draught be known; as, for instance, that of a ship of 78 guns, is 2350 tuns, we may with certainty know if the water line in the draught be properly placed, only by reducing the bottom into cubick feet.

The antient builders were unacquainted with the manner of performing this, but our moderns make an exact calculation of the contents of the bottom before they begin to build, whereby they will be sure to keep the lower tier of guns well out of the water.

If a ship's body were any regular figure, the solid contents of it could easily be found geometrically, but as the case is quite otherwise, we must be satisfyed with dividing it into several parts, of which we may have a great number, and they will thereby become so small, that they may, without any sensible error, be esteemed as regular figures, limited by streight lines, tho' some of them are actually curves.

In the draught of the 70 gun ship which we have laid down, the bottom is divided on the plain of elevation into several parts, in a vertical way by the lines that represent the frames; and in an horizontal way by the water lines, so that the whole may be said to be divided into so many parallelopipedons, A, B, C, D, or *a*, *b*, *c*, *d*, contained betwixt the two frames 6 and 7, and limited on the side A B by a plain supposed to be erected vertically upon the keel, and on the other side by the round of the outside of the ship, at the height of the breadth water line, or *a c*. Now it is very plain that the area of the surface, which limits the lower part of this solid, is less than the area of the surface, which limits the upper part: But if we increase the water lines, and frames we may find the solid contents to a sufficient exactness for our purpose.

Now, in order to find the area of the upper surface A B D C, let A C be 16 feet 11 inches, and B D 13 feet 6 inches; add these two, the sum is 30 feet 5 inches, the half of which is 15 feet two inches and a half, and this sum multiplyed by A B, which suppose 8 feet, the distance betwit the frames, the product is 121 feet 8 inches, the area of the upper surface of the parallelopipedon.

The area of the lower surface of the parallelopipedon may be found after the same manner, which suppose 97 feet 4 inches. Now, if these two areas be added together their sum will be 219 feet, the half of which is 109 feet 6 inches for the mean area, and this multiplied by *a b*, the distance betwixt the water lines, which suppose 4 feet 4 inches, produces 474 feet 6 inches cubick.

By the same process we may find the solid contents of the other parallopipeds, and adding them together, and doubling that sum we shall have the

the ſolid content of the whole bottom of the ſhip in cubick feet to a ſufficient degree of exactneſs.

I made uſe of this method before Mr *Bouguer's* treatiſe was publiſhed, where there is one which is more convenient and expeditious, for inſtead of finding the area of every ſingle ſurface contained betwixt the frames upon the ſection of a water line, he finds by one operation the area of the whole ſurface formed by the horizontal ſection or water line, except that part intercepted betwixt the aftermoſt frame and the poſt, and the part contained betwixt the the foremoſt frame and the ſtem, which upon account of the rake muſt be meaſured ſeparately, as alſo all that lies betwixt the upperſide of the keel and the firſt water line. His method is as follows:

Take the lengths of all the lines that repreſent the frames on the horizontal plane, add all theſe together, excepting the foremoſt and aftermoſt, of which take only one half of each, ſo if it were required to find the area of the ſurface formed by a horizontal ſection in the plane of the load water line, it will be $\frac{1}{2}$ ZZ+BD+AC+IH+LK, *&c.* + $\frac{1}{2}$ XS+AB, ſuppoſing AB to be the diſtance betwixt the frames equally ſpaced betwixt ZZ and NO.

To demonſtrate this, let it be conſidered by what operation the two trapezia ABDC and HIAC are meaſured. We obſerved in the preceeding article that this was performed by adding the length of the lines BD and AC together, and then taking half that ſum; the length of the lines AC and HI, muſt likewiſe be added together, and the half of that ſum taken; now it is evident that it will be ſame thing to take half the line BD, and half the line HI, and the whole line AC, and add all theſe three together, becauſe the line AC, is common to both the trapezia.

After the areas of all the water lines are thus found, the ſolid content of the ſpace contained betwixt the water lines may be had by multiplying the area by the diſtance between the water lines: But becauſe the areas of the two ſurfaces which limit this part are unequal, a mean area muſt be found; this is half the ſum of the two areas, ſo that all that is now to be done, is to add the areas of the water lines into one ſum, excepting that of the uppermoſt and lowermoſt, of which only one half of each muſt be taken, and if this ſum is multiplied by the diſtance betwixt the water lines, the product will give half the ſolid content of the bottom, obſerving that the water lines in the plain of elevation be equally diſtant from one another.

The application of this method in finding the cubick feet contained in a 70 gun ſhip laid down in the draught.

The

The forepart is divided into eight, and the after into nine equal parts, besides that betwixt the aftermost timber and the post, and that betwixt the foremost timber and the stem.

The bottom is likewise divided into four equal parts by water lines drawn parallel to the load water line, all which are formed upon the horizontal plane, for it will be very useful to know the solid content of each particular part contained betwixt the water lines, also to distinguish that of the fore body from the after body, whereby we may be enabled to know if the weight be duly poised. We shall consider all this in the following calculation.

Note, there must be four inches added to each line that represents the frames in the horizontal plane for the thickness of the plank, that being nearly a mean betwixt the thickness of the plank next the wale, and that next the keel.

The Area of the Upper Water Line abaft.

The breadth of the surface at the load water line, upon the midship frame *a* Q is 21 feet 2 inches,

		feet.	inch.
	one half is	10	7
Breadth at	1st Frame	21	2
	2d Frame	20	11
	3d Frame	20	9
	4th Frame	20	5
	5th Frame	19	11
	6th Frame	18	11
	7th Frame	17	4
	8th Frame	15	7
The 9th Frame X S is 12 feet 9 inches, one half of which is		6	$4\frac{1}{2}$
	Total	171	$11\frac{1}{2}$

which total doubled is 343 feet 11 inches, and multiplied by 8, the distance betwixt the frames, is the whole area of the water line from the midship to the after frame, in cubick feet } 2751 4

To this must be added the area of the trapezium X S L *e*

Now half of the lines X S and L *e* is 10 Feet 0 Inches

Distance betwixt them is 9 9

Product is 97 6

which being doubled is — 195 0

The whole area in cubick feet 2946 4

By using the same process we may find the areas of all the other water lines, and adding all these areas together excepting that of the first and fifth, of which taking only one half, multiply this sum by 4 feet 5 inches, which

which is the distance betwixt them, we shall have the area in cubick feet of that part of the ship abaft the midship frame, contained betwixt the lower water line, and load water line.

	feet	inch.	l.	p.
Half the area of the load water line	1473	2	0	0
Whole area of the 4th water line	2516	1	4	0
Whole area of the 3d water line	2052	0	4	0
Whole area of the 2d water line	1452	10	7	6
Half the area of the 1st water line	144	3	2	0
Total	7638	5	5	6
Multiplied by the distance betwixt the water lines	4	5	0	0
Product in cubick feet betwixt the lower, and load water line	33736	6	1	3	6
Betwixt the lower water line and keel	333	6	3	0	0
Keel and post	101	8	0	0	0
Cubick feet abaft the midship frame under water, when loaded	34171	8	4	3	6
Cubick ft. before the midship frame under water, when loaded	28928	6	1	0	0
Total cubick feet under water	63100	2	5	3	6
Multiply by the weight of a cubick foot of salt water		pounds			74

	pounds	tuns.	lb.
Weight of the whole ship with all her furniture provisions &c.	4661498	2334	1498

We have omitted the operation for the fore part, because it is performed exactly by the same method with the after part.

It must be observed that in finding the cubick feet of that part contained betwixt the lower water line and upper side of the keel, we must take the heights of all the frames intercepted betwixt these two lines, and divide their sum by the number of frames abaft the midship, the quotient will be 1 foot 9 inches 9 lines.

	Feet	In.	Lines
The area of the lower water line is	288	6	4
The area of the upper side of the keel	79	6	0
Total	368	0	4
one half is	184	0	2
Area of that part contained betwixt the lower water line and keel	333	6	3

The use of the preceeding calculation is to know if the load water line upon the draught be properly placed.

It has been found that a ship of 70 guns, with every thing on board, should weigh nearly 2350 tons, which is only 15 tons 1297 pound more than what is found by calculating from the load water line in the draught, this difference would occasion the ship not to draw above one inch more water

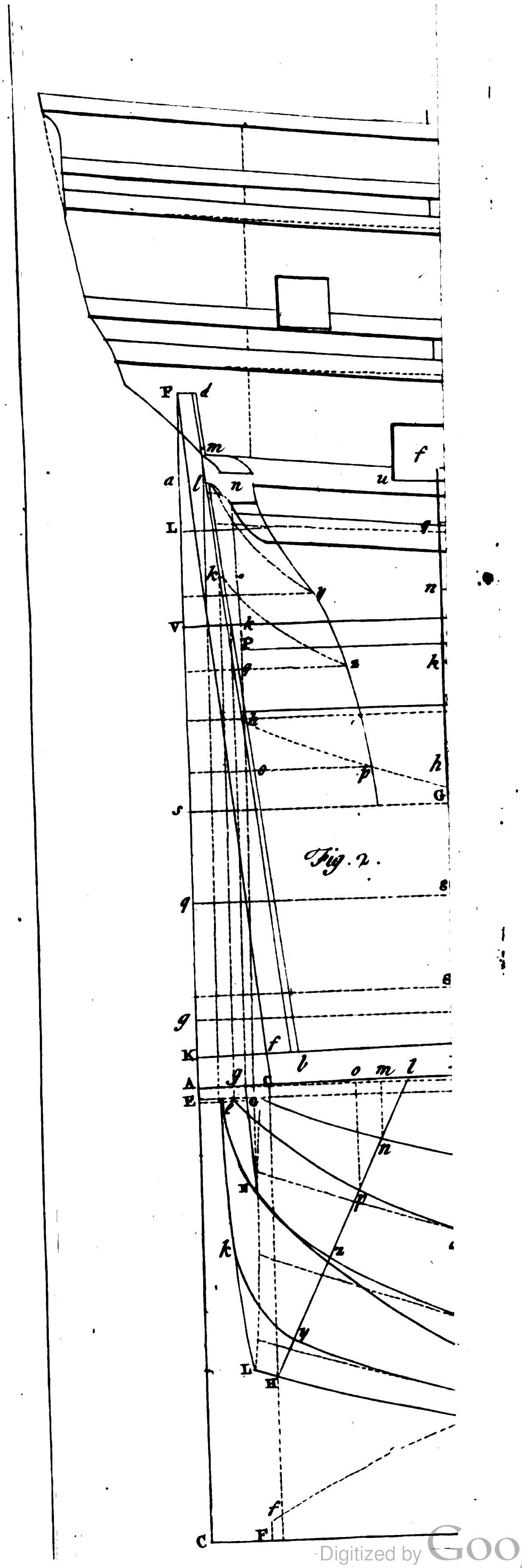
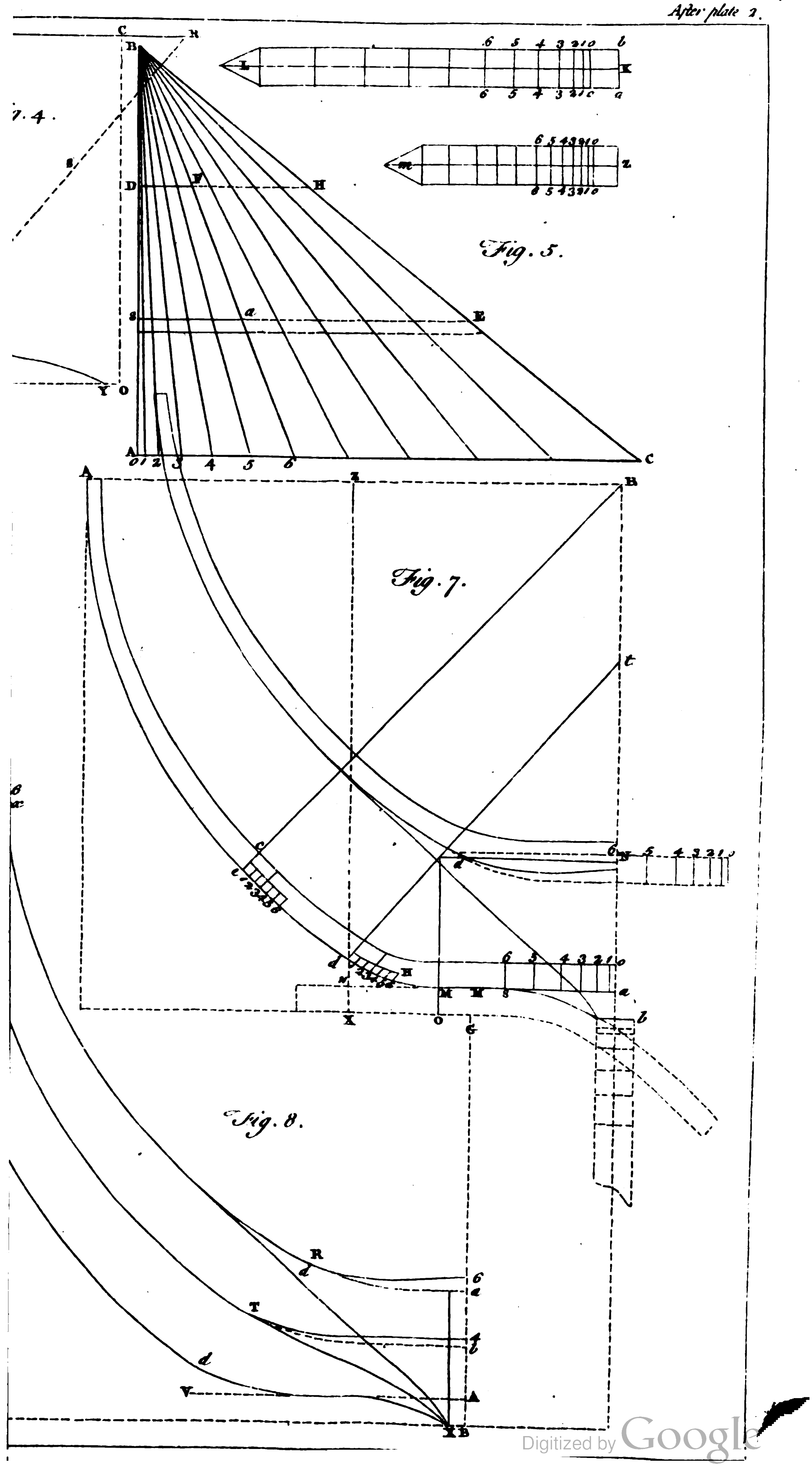


Fig. 2.


7. 4.
Fig. 5.
Fig. 7.
Fig. 8.

water it is not worth the regarding. But then by this calculation we discover that the ship is too lean before; for whereas the fore part should exceed the after part by 30 tuns, we find, by this calculation, that the after part exceeds the fore part by 193 tuns 1995 pounds.

Upon this account we must consider carefully if the midship frame is properly placed; it is here 5 feet before the middle. If the midship frame was exactly in the middle it would augment the weight of the fore part 102 tuns 307 pounds, and diminish that of the after part exactly the same quantity, by which means the fore part would be 1172 tuns 1014 pounds, and the after part 1162 tuns. Now we may fill out the fore part, so as to gain 15 tuns 636 pounds, which was deficient, to make the calculation taken from the draught agree with the real weight proposed for a ship of 70 guns fitted out for sea, with six months provision on board; and the fore-body will weigh 25 tuns 1255 pounds more than the after part, which may be judged sufficient.

CHAP. IX.

A method to calculate the Resistance of the Water upon the fore Part of the Ship.

DAILY Experience sufficiently proves that the fluids, by their motion, attack the solids that oppose them, as bridges, mills, &c. with such violence as to carry all before them; and this is agreeable to the very nature of fluids.

For all fluids are an assemblage of a prodigious number of small solid bodies of a globular form, each of which being easily put in motion will act upon any surface with the same force that any other solid body of the like mass would do. But as these particles have but a very small cohesion with each other, fluids cannot act with the same force as solids which have their parts united.

A mass of water of 20 cubical feet will not act with the same force upon the pier of a bridge which opposes it, as a mass of ice of the same dimensions; because the whole mass of ice having its parts so united together, that one cannot advance without the other, it gives the blow with the united force of all the parts at once, whereas the parts that compose the mass of water, being but slightly united, they cannot act jointly or in concert, and they exert their force one after another; they indeed succeed one another immediately, and are a little united by their reciprocal pressure; but as every part has its own peculiar velocity, so it makes its effort singly by itself, and, being easily put in motion, it will be as easily turned out of its direction, the parts being only retained together by the weight of those that come next them.

Fluids have a continual effort, because when a certain number have produced their effect they are succeeded by others as long as the current lasts.

Hence it will follow, that when a vessel is left to a current of the river, it can receive no more velocity than the current has, and its velocity will be accelerated till it is equal to that of the current.

If, on the contrary, any floating body receives a motion in a contrary direction to that of the current, it will be continually retarded, till it has none, and then it will change its direction to follow that of the current.

We shall here remark, that it is indifferent whether we ascribe the motion to the solid or to the fluid; for the impression of the water upon the ship's stem is the same when under sail, as when at an anchor, provided the motion of the current be equal to that which the ship acquires by sailing.

The effort of fluids is as the square of the velocity of the current.

It is very plain, that the more rapid the current is, the greater will the impression of the fluid be; for the parts will then shock the solid with greater force than when it runs slowly; so that the force is augmented in proportion to the velocity. Again, the number of the parts of the fluid that strike the solid in any space of time, is in proportion to the velocity of the current; for the faster it runs the greater will be the number of the parts that strike the solid in a space of time; so that not only the effort of the fluid, but likewise the number of parts that attack the solid, is augmented in proportion to the velocity of the current, and when these two are united, the effort of the fluid will be in a duplicate ratio of the velocity; so that if the velocity be doubled, the shock will be quadrupled.

Hence, the faster a ship goes through the water, the greater will be the resistance she meets with, and this will be augmented in a duplicate ratio of the velocity with which she sails.

The impression of a fluid increases as the surfaces which oppose its current.

It is very plain, that if one surface is double another it will receive double the number of the parts of the fluid, and of consequence the impression will be double upon a surface, whose area is double the area of another surface. Hence those ships whose midship frames have the greatest capacity meet with most resistance.

The efforts of fluids will be less when the surfaces are in an oblique position to the current, than when in a perpendicular position.

Plate

Plate IV. *Fig*. 7.] Let E A represent the course of the fluid setting perpendicularly on any body A B; it is plain, that it receives the impression of all the parts of the fluid contained between A and B; whereas, if the point B be moved to D, the parts of the water contained betwixt B and G will have no impression upon A D. Hence the quantity of the fluid which attacks A B is to that which attacks A D as A B is to A G; that is, as the radius is to the sine of the angle of incidence E A D. But if there were no other advantage gained by this oblique position, than being exposed to fewer parts of the fluid, it would be of very little service to a ship which must have a sufficient breadth, suppose A B; it is plain, the number of the parts of the fluid which give the impression will be the same, when the fore part of the ship is in the form of A D B, as when it is flat in the form of A B; but the fluid which exerts its force on the surface A D B does not produce the same impression as when it exerts its force on A B, because the direction of each particle of water, which strikes any surface obliquely, may be resolved into two directions, one perpendicular, and the other parallel to the plane.

In order to give us an idea of compound motions, and of the resolution of their forces, let us suppose two rulers A A and B B, (*Plate* IV. *Fig*. 5.) placed upon a plane at right angles to one another, and a small ball C placed at the angle of their meeting, it is plain, if we slide the ruler B B in a parallel position to itself, it will carry the ball C along the edge of the ruler A A; but if both the rulers be made to slide together, so that they still preserve the same angle, in such a manner that when the ruler A A arrives at the line VII, VII, the ruler B B arrives only at the line 3, 3. It is plain, the ball will describe the diagonal of the parallelogram C, VII, D, 3, the sides of which will be proportional to the distance the rulers have moved, that is, D VII is to D 3 as 3 to 7; but if the rulers be supposed to be moved equally, so that when A A arrives at the line VII, VII, B B shall arrive at the line 7 7, the ball will describe the diagonal C F of the square C VII, F 7.

Now, if we substitute any other two agents in the place of the rulers, such as two hammers, and both be supposed to strike the ball with equal force at the same time, it is plain, the ball will go in the direction of the diagonal C F; but if the force with which one hammer strikes the ball be to that by which the other hammer strikes the ball, as 7 to 3, then the ball will move in the direction of the line C D.

The principal Effects of Compound Motions. (*Plate* IV. *Fig.* 6.)

If two powers C and B act with equal force on the body A, but in the contrary directions of the lines C A and B A, the body A will remain at rest; but if one of the powers acts with greater force than the other, the body will follow the direction of that which predominates, diminished by the quantity of the smaller force.

2d, If two powers D and E act upon the body A in the same direction, viz. in the lines D A and E A, the body A will follow the direction of both, and pass through the point F, with this only difference, that it will go with greater velocity when impelled by both powers than with one.

3d. Let the two powers G and H strike the solid A, in the direction of the lines G A and H A, it will thereby receive a compound motion, the force and direction of which may be expressed by the diagonal of a parallelogram, as was before observed.

In order to construct this parallelogram, which is called the *resolution* of the forces, let the two powers G and H be supposed equal and expressed by the lines H A and G A; from the point G draw the line E G equal and parallel to H A, and the diagonal E A (the result of the two powers represented by the sides of the parallelogram H A and G A) shall express the velocity and direction of the compound motion; the effect of which will be, that the body A will be carried to the point F. But supposing the forces unequal, and let that of H, (*Fig.* 9) represented by the line H A, be double that of G, represented by the line R A; then from the point R draw the line R S equal and parallel to H A, which shall express the force and direction of the power H; and from the point H draw the line H S parallel to R A, which will express the force and direction of the power G; the diagonal S A expresses the velocity and direction of the body A, which will pass through the point T, whereas, if the powers were equal, it would pass through the point F.

It may be remarked, that two attractive powers placed at P and Q, would produce the same effect as two impulsive powers at G and H, and that the parallelogram may be constructed on the lines A Q and A P.

CONSEQUENCES.

1st. The acuter the angle of the direction of the power is, the nearer will they approach to one direction, and act with greater force; so the result of G and H is greater than that of K and I, supposing the powers to be equal.

2d,

2d. The greatest effect of two powers is, when they both act in the same direction, and the least when they act in contrary directions.

3d. When two equal powers act in such a direction that they form an angle of 120 degrees, as A K and A I; in this and in no other case, the result will be equal to the single force of A I or A K; it only changes the direction; for when the two powers act jointly, A will be carried to F, whereas if K only acted, it would be carried to T; or if I only acted, A would be carried to V.

4th. If the direction of two powers make an angle less than 120 degrees, as G A and H A, they will assist one another; but if they form an angle greater than 120 degrees, as L A and A M, they will be reciprocally diminished.

The Results of a Motion impressed upon a Body A, *in Relation to a Surface* a b, *which opposes its Motion.* (Plate IV. Fig. 6.)

1st. When a body strikes a surface obliquely it will be with less force than when it strikes it perpendicularly; for it may strike it so obliquely as only to graze along it; between the perpendicular shock, which is the greatest, and the oblique, which approaches nearest to a parallel to the surface, there may be an infinite number of directions, less or more oblique, and the surface will be struck with more or less force.

2d. If the two powers are united in D, they will act, in the direction D F, with great force upon *a b*, because they not only act jointly, but likewise in a perpendicular direction upon the surface *a b*.

3d, If the two powers be equal in force, and act in the direction of the lines G A and H A, the body A will also fall perpendicularly on the surface *a b*, but with less force than in the first case, because of the obliquity of the directions.

4th, If the power H have double the force of the power G, then the direction will be changed into the line S A, (*Fig. 9.*) and the body will strike the surface obliquely in the direction of the line S T, but with less force than in the second case, not only on account of the diminution of the force of the power G, but also on account of the obliquity of the shock.

5th. It will be indifferent whether the body A receives its impulse from one single power, or from two, so that it strikes the surface *a b* in the same direction. Hence we shall have no occasion to consider the powers which give the motion, but only the velocity and the direction in which they strike the surface.

6th. It will produce the same effect, whether we change the line of direction

direction, in which the body A strikes the surface *a b*, or change the position of the surface *a b* in respect of the line of direction.

From what has been said, it will follow, that if the common effect of two powers acting upon the same body be known, and also the direction and force of one of them, then the direction and force of the other may be found; for let the body C (*Fig.* 5.) be carried to the point D by the action of two powers, and one represented by the line C, VII; draw the line D 3 equal and parallel to C, VII, and compleat the parallelogram, so shall C 3 express the force of the other power.

The Application of what has been said to the Shock of Fluids.

We have hitherto considered the shock of a solid body in different directions upon the surface of another solid, but we will readily grant that fluids do not act in the shock in the same manner that solids do. It is very probable, that when a fluid falls perpendicularly upon a surface, there is a mass of water that rests immoveable before the surface, which occupies the place of a solid body, and has nearly the same effect as if the surface was round, so that the fluid does not attack the body that opposes it in a direction perpendicular to its course; besides, the particles of water which attack a surface, whether obliquely or not, may rebound and change their direction, so that the laws of fluids are quite different from the laws of solids in the shock.

The oblique direction of a particle of water may be resolved into one that is perpendicular to the body which opposes its course, and one that is parallel to it.

In order to construct this resolution, (*Plate* IV. *Fig.* II.) upon the line A C inclined to the current, form the parallelogram A H E F (A E representing the velocity and direction of the current) making E F parallel to C A and E H perpendicular to C A. The diagonal E A, which represents a particle of water and its velocity, will be the result of a motion supposed to be produced by two powers, one parallel to A C, whose force and direction is represented by E F, the side of the parallelogram.

Hence it will follow, that when a surface is exposed to the shock of a current, in different oblique directions, the force of the direct shock is to that of the oblique, as the square of the radius is to the square of the sine of the oblique angle of incidence; for the effort of the particle E A, which strikes the body A B, in a perpendicular direction, is to the effort of the same particle of E A, which strikes the body A C in an oblique direction, as E A is to E H; but E A is to E H as A B, the sine of the right angle,

angle, is to A G, the sine of the oblique angle of incidence. But it was before observed, that the sum of the particles that strike A B is to the sum of the particles that strike A C as the radius is to the sine of the angle of incidence. Hence, by multiplying the effort of one particle, by the number of particles that strike A B; (that is, the effort of the whole water upon A B) and multiplying the effort of one particle, by the number of particles that fall upon A C, (that is, the effort of the whole fluid upon C A) we shall have the following proportion: The effort of the whole fluid upon A B is to its effort upon A C as the square of the radius is to the square of the angle of incidence.

When the surfaces A B and A D, which oppose the current A E, are unequal (*Plate* IV. *Fig.* 10.) the quantities of water which strike these surfaces are as the product of the surfaces by the sines of the angles of incidence; from whence we shall have the following proportion: The effort of the fluid upon A D is to that upon A B as the square of A G, the sine of the angle of incidence multiplied by the surface A D, is to the square of A B the radius, multiplied by the surface A B.

CONSEQUENCES.

1st, If two equal surfaces, exposed to the same current, receive its shock in different obliquities, the impressions will be to one another as the squares of the sines of the angle of incidence.

2d. A surface parallel to the current can receive no shock, because there is no angle of incidence.

3d, If two unequal surfaces are exposed to the same current, the impressions they receive by the shock in different obliquities, are to one another as the products of the squares of the sines of the angles of incidence, and of the surfaces that receive the shock.

4th. If two equal surfaces receive the shock of two unequal currents, the impressions will be to one another as the products of the squares of the velocities, and of the squares of the angles of incidence.

5th. If two unequal surfaces are exposed to two unequal currents, which strike them with different obliquities, the impressions will be to one another as the products of the squares of their velocities; of the squares of the sines; of the angles of incidence; and of the surfaces.

All these consequences may be deduced from the preceding principles; it only remains to apply them.

Let A B (*Plate* IV. *Fig.* 4.) represent the extreme breadth of a vessel, and let the fore part be formed according to the angles A C B, or A F B, or A L B. In order to find the efforts of the fluid, suppo-

sing

sing the velocity and direction to be the same, and parallel to the keel, in the three cases; upon the middle of the line A B, erect the perpendicular E L, which will pass through the tops of all the triangles; then to find the sines of the angles of incidence, with the radius A B describe two arches A F and B F to intersect one another in the vertex of the equilateral triangle; the arches will intersect the sides of the angle, that is less than 60 degrees; but not the sides of that which is more than 60 degrees, produce one of the sides from C to M (*Plate* IV, *Fig.* 4.) Lastly, let fall the perpendiculars M D, F E, P K, upon the line A B, from all the points where the arches intersect, either the sides of the triangles, or the sides that are produced. So shall A E, A D, and A K, represent the sines of the angles of incidence upon the different triangles A F B, A C B, and A L B.

It will be easier to observe, that the effort of the fluid upon the intire fore parts A C B, A F B, or A L B, is to the effort upon the extreme breadth, as the effort upon A C, A F, or A L, is to the effort upon A E; but it was proved before, that the impressions received by two unequal surfaces, opposed to the direction of a current, are as the squares of the sines of the angles of incidence multiplied by the surfaces, so, in this case, the impression on A C will be to that on A E, or (which is the same thing) the impression on A C B will be to that on A B, as the square of A D, the sine of the angle of incidence multiplied by A C B, is to the square of the radius multiplied by A B.

We have also the effort on A F B to the effort on A B; as A F B, multiplied by the square of A E, the sine of the angle of incidence, is to A B, multiplied by the square of A B the radius. This proportion would shew the effort of the fluid upon the prow A F B, in a perpendicular direction to the sides A F and B F, which would be very useful, if it were required to determine the dimensions of the timber, that is, to resist that pressure of water; but in the present case, where only the relative effort upon the prow is considered in the direction of the keel, we must form another resolution. Let then C D (*Fig.* 8) represent the effort upon F B, perpendicular to that surface, if from the point F we let fall the perpendicular D H, and compleat the parallelogram G C D H, C G shall represent the relative effort upon the prow in the direction of the keel, so the whole effort upon F B, may be represented by F B multiplied by the square of the angle of incidence, which is to the relative effort as F B is to E B: The relative effort then is equal to the square of the sine of the angle of incidence multiplied by E B, or by the sum of the particles which fall upon F B; so to find the relative effort on F B, we must

multiply

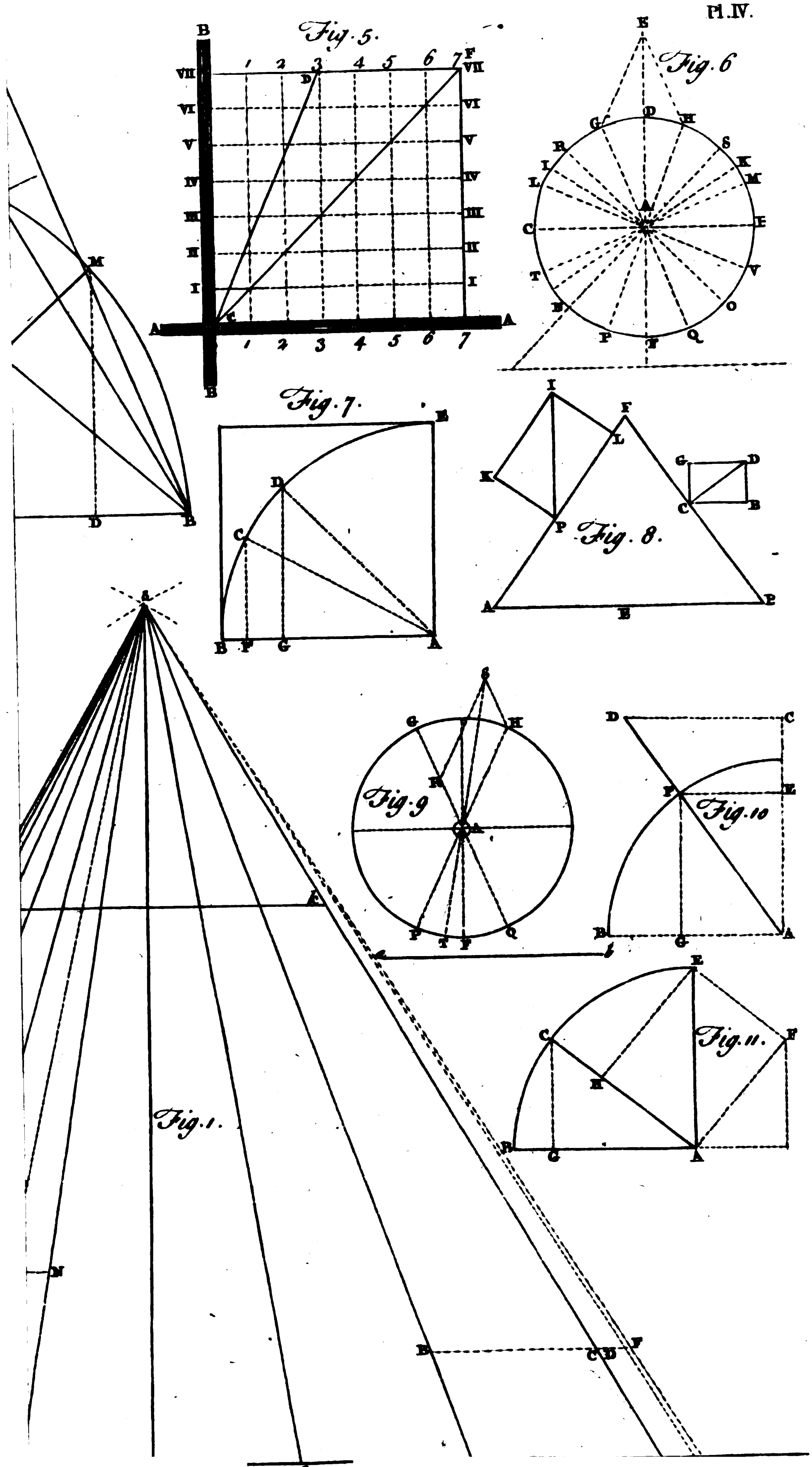


Fig. 5.
Fig. 6
Fig. 7.
Fig. 8.
Fig. 9
Fig. 10
Fig. 11.
Fig. 1.

multiply the square of the angle of incidence by the projection of the plane F B upon the beam B B.

Tho' this method cannot be truly applied but to rectilineal triangles, yet, by dividing curves into a number of small parts, each may be considered, without any sensible error, as a strait line. M. *Bouguer* makes use of this method of approximation to a sufficient degree of exactness. What we have already said upon that head, it is to be hoped, will facilitate this description to such as have only a slight knowledge of the mathematics; so that all that remains now is to apply this to the draught of a ship of 70 guns, which has been already laid down.

A Calculation of the Resistance of a Fluid upon the Prow of the ship of 70 Guns, which we have laid down in a draught compared with the Effort of the same Fluid upon the Area of the Midship Frame.

As the operations are to be performed upon the plane of projection before laid down, all the frames in the fore part must be exactly formed as before in *Plate* II. in order to which it will be necessary to make use of a larger scale, as in *Plate* V.

It will be very convenient to draw the water lines I, II, III, &c. and the frames 1, 2, 3, &c. to the midship frame at equal distances from one another.

It is plain, that the water lines and frames divide the prow into trapezia, such as *r a*, 8 *b*, 7 *c*, &c. corresponding to the trapezium *a b*, and parallelogram *a d* in the plane of elevation *Plate* II.

It will be necessary to observe, that there must be so many water lines and frames that the lines 8 *a*, 7 *b*, 6 *c*, &c. which are curves, may be esteemed strait lines.

We must draw the diagonals *r a*, 8 6, 7 *c*, &c. through the trapezia; but we may take two trapezia at once near to the midships, because the ships sides are there nearly parallel to the current.

It will likewise be proper to observe that these diagonals are the projections of the diagonals of the parallelograms represented upon the plane of elevation, at least, on the surface of the ship; as for instance, the diagonal 8 *b*, on the plane of the projection, is the projection of the diagonal 8 *d* drawn on the plane of elevation.

These diagonals divide the prow into the triangles 1, 2, 3, 4, &c. which strike the fluid with different degrees of obliquity.

We have not the entire areas of these triangles, by reason of the curving of the prow, but only their projection on the midship frame; but this is all we want, for the sum of all the particles of water that strike the triangles are proportioned to the triangle projected on the midship frame, since the water that ranges along each triangle may be considered as a triangular prism, the base of which is equal to the triangle *r a* 8, *Fig.* 1, *Plate* V. and this was the thing in question. Mr. *Bouguer* proposes to calculate the effort of the fluid on each triangle, their sum will give the shock of the water upon the whole prow, and compare this to the shock of the fluid upon a surface parallel to the area of the midship frame.

To attain this Mr. *Bouguer* lets fall perpendiculars to every frame, from the angles formed at the intersection of the water lines and the diagonals that were drawn to form the triangle; for instance, upon the frame 8, the perpendiculars *l b* and *r p*; upon the frame 7 8, the perpendicular 8 *m* on one side, and the perpendicular *n c* on the other side; &c.

This method requires that there be as many right angled triangles formed as there are triangles on the prow, and as there must be a great number of them, it will be necessary to find some method of forming them. The following seems to me to be the most expeditious.

Draw two parallel lines B D and C R; let the distance betwixt them be equal to that betwixt the frames on the plane of elevation, and by this one operation we have the height of all the triangles that are contained betwixt the parallels.

As the base of all the rectangles should be equal to the perpendicular of the corresponding triangle of the prow, we may set off the length of each perpendicular upon the parallel C R; so shall C H *Fig.* 2, be equal to *r p*, *Fig.* 1; H L, *Fig.* 2, equal to *s a*, *Fig.* 1; L E equal to 8 M, &c. to compleat the triangles, draw the perpendiculars H N, L M, and the hypothenuses D H, N L, &c.

If one of these rectangles be considered singly, D H may represent the radius, and C H will be the sine of the angle of incidence.

All these triangles being described, we may begin to find their areas on the plane of the projection, because it is upon this that the relative impulse in the direction of the keel depends, which is the thing now required, as was before observed.

The surface of a triangle is found by multiplying half the base by the perpendicular, so the surface of the triangle *r*, *a*, 8, will be the product of the perpendicular *r p*, (equal to C H) multiplied by half the base *a* 8, and this will be the sum of all the particles which strike the triangle *r*, *a*, 8, which is an element of the prow of the ship.

In order to find the relative force of the fluid in the direction of the keel on that part of the bottom corresponding to the triangle *r*, *a*, 8, it is only

only multiplying the surface of the triangle by the square of the sine of the incidence; in place of multiplying it by the square of the radius, which would give the impulse the triangle would receive from the water in a perpendicular direction; now if we divide this impulse by the oblique one, the quotient will give the quantity that the impulse is diminished by the obliquity of the prow; but there will be no occasion for this last step, for as the sum of all the products of the triangles multiplied by the square of the radius, gives the effort of the fluid upon the midship frame; the direct effort may be found by multiplying the area of the midship frame by the square of the radius, and if this be divided by the sum of the products of all the triangles multiplied by the squares of the sines of the angles of incidence on each triangle, we shall know the diminution of the resistance which the prow meets with in proportion to that of the midship frame.

It would be almost impracticabl to multiply the surface of each triangle by the square of the sine of the angle of incidence, upon which account Mr *Bouguer* substituted proportional lines in place of the squares of the sines, which we shall now explain.

It was before observed, that if D H be the radius, C H will be the sine of the angle of incidence.

If we let fall the perpendicular C O upon the line D H, we shall have the triangle D C H similar to D O C; so taking the equal lines C D, and N H for the radius, C O will be the sine of the angle of incidence.

If we draw O P perpendicular to D C, the triangles D O C and C O P will be similar, therefore the triangles D C H and C O P will be similar, and D H is to C H as O C to C P; but D H is to C H as D C to C O, and by multiplying these two proportions, the square of D H will be to the square C H as D C is to P C; that is, if D C represent the square of the radius, P C will be the square of the sine of the angle of incidence C D H. So the lines C P, H Q, L G, *&c.* give the squares of the sines of the angles of incidence in the triangles 1, 2, 3, *&c*, which are to be multiplied by the surfaces of the triangles; and the parallels D C, N H, *&c.* always represent the radius.

A Calculation of the Resistance of the Fluid upon the Prow of a Ship of 70 Guns, compared with that upon the Midship Frame.

Triangles.	Perpendicular.			Multiplyed by half the base.			Product.				Multiplyed by the square of the sine of the angle of incidence.			Product gives the effort of the water upon the triangle.			
1st.	f.	in.	lin.	f.	in.	lin.	f.	in.	lin.	poi.	f.	in.	lin.	f.	in.	lin.	poi.
1	6	6	0	1	9	6	11	7	9	0	3	2	0	36	10	6	6
2	5	7	6	1	6	3	8	6	7	10	2	8	0	22	9	9	0
3	4	6	6	1	9	6	8	1	7	9	1	11	6	15	11	2	8
4	4	6	6	1	9	6	8	1	7	9	1	11	6	15	11	2	8
5	2	9	6	1	8	6	4	9	2	9	0	11	0	4	4	5	6
6	2	8	0	1	9	6	4	9	4	0	0	9	0	3	7	0	0
7	1	11	6	1	7	6	3	2	2	3	0	5	0	1	3	10	11
8	2	2	0	1	9	0	3	9	6	0	0	6	6	2	0	7	9
9	1	4	0	1	7	6	2	2	0	0	0	2	6	0	5	5	0
10	1	4	0	1	7	6	2	2	0	0	0	2	6	0	5	5	0
11	0	5	0	1	6	6	0	7	8	6	0	1	0	0	0	7	8
12	0	10	0	1	7	6	1	4	3	0	0	2	0	0	2	8	6
13	0	2	6	1	6	6	0	3	10	3	0	0	8	0	0	2	6
14	0	4	0	1	6	6	0	6	2	0	0	1	2	0	0	7	2
15	0	1	0	1	6	6	0	1	6	6	0	0	3	0	0	0	[illegible]
16	0	5	0	1	6	6	0	7	8	6	0	1	6	0	0	[illegible]	6

	f.	in.	lin.	poi.
The total effort of the first piece	194	2	8	8
Of the second	57	19	6	2
Of the third	43	8	6	6
Of the fourth	28	0	8	10
Of the fifth	12	9	8	7
Of the sixth	3	2	8	11
Total	249	10	11	7

We must in the next place find the direct effort of the water upon the area of the midship frame, by multiplying the area by the square of the radius.

OPE-

OPERATION.

Half the 6th water line *r* VI	10	2	0
The whole 5th water line V	20	0	0
The 4th water line	19	5	0
The 3d water line	18	1	0
The 2d water line	15	11	6
The 1st water line	12	0	0
The breadth of the keel	0	0	0
Total	95	9	6
Multiplied by the distance betwixt the water lines, which is	3	2	0
Product is the area of the midship frame	303	4	1
Multiplied by the distance betwixt the frames	8	0	0
Product	2426	8	8

This being divided by 249 : 10 : 11, the sum of the efforts of the fluid upon the triangles of the prow; the quotient is $9\frac{7}{10}$, which shews that the effort of the fluid upon the prow is to that upon the midship frame as 1 is to $9\frac{7}{10}$, which is a sufficient diminution of the resistance for a ship of this force. Hence we may conclude that the water lines in the fore-body are well formed, but a frigate will require more diminution, as will appear by the following examples.

The *Brillant*, as $3\frac{1}{4}$ to 1, a very bad sailer.

The *Tigre*, as 5 to 1, a company keeper.

A ship of 50 guns, designed by M. *Boyer*, but not built, as 8 to 1.

The *Monarque*, of 74 guns, built by M. *Ollivier* in 1745, as $9\frac{1}{2}$ to 1.

The *Palme*, of 12 guns, 4 lb. shot, built by M. *Ollivier* in 1744, as $13\frac{1}{4}$ to 1.

The *Alcid*, of 64 guns, by Mr *Ollivier*, at *Brest* 1741, as $6\frac{1}{4}$ to 1.

The *Renomme*, built at *Brest* by Mr *Desalieurs* 1744, as 10 to 1.—this ship, by the account of the captians, was a very fine sailer.

The *Badine*, 6 guns of 3 lb. shot, as $7\frac{1}{4}$ to 1.

The *Panthere*, of 20 guns, 6 lb. shot, as $10\frac{1}{4}$ to 1.

The *Amazon*, of 44 guns, built by M. *Blaise Pengalot*, as $8\frac{1}{3}$ to 1.

The *Superbe*, built by M. *Helie*, as $5\frac{1}{10}$ to 1.

The *Mutine*, of 24 guns, built by M. *Geffroi*, senior, as $10\frac{1}{2}$ to 1.

We have compared the efforts of the fluid upon the prow of each ship, with that upon a plane, equal to the area of the midship frame.

It will be proper also to examine if the resistance in those be less than in ships which are known to be good sailers; but it may happen that a ship,

ſhip, whoſe midſhip frame has a ſmall area, may meet with little reſiſtance, tho' her prow be not diminiſhed in proportion to that of her midſhip frame; ſo it will not be ſufficient to know this proportion only, to be aſſured whether or not the ſhip will be a fine ſailer. We muſt alſo compare the areas of the midſhip frames, and not reſt ſatisfied with comparing the efforts of the fluid; upon the prow of the ſhip we have laid down, with that upon the prow of a ſhip of the ſame rate, which has gained a good character.

The firſt Example of Compariſon.

We know that the area of the midſhip frame of the 70 gun ſhip, we have laid down, is 606 : 8 : 2, and that the effort of the fluid upon the prow is to that on the midſhip frame as 1 is to $9\frac{7}{10}$. Now if another ſhip of the ſame rate has the area of her midſhip frame 7 or 800 feet, ſuppoſing the ſails, and every thing that may contribute to ſailing, to be alike in both; it is plain this laſt cannot ſail ſo well as that we have laid down, and by this example, it is very plain, that if we would calculate which of two ſhips would ſail beſt, we muſt, after finding how much the reſiſtance of the fluid upon the midſhip frame of each is diminiſhed by the form of their prows; alſo compare the areas of their midſhip frames, that we may know which of the two has the greateſt maſs of water to diſplace; but if it was only required to know, which of two ſhips of the ſame rate would ſail beſt, it would be ſufficient to compare the efforts of fluids upon their prows.

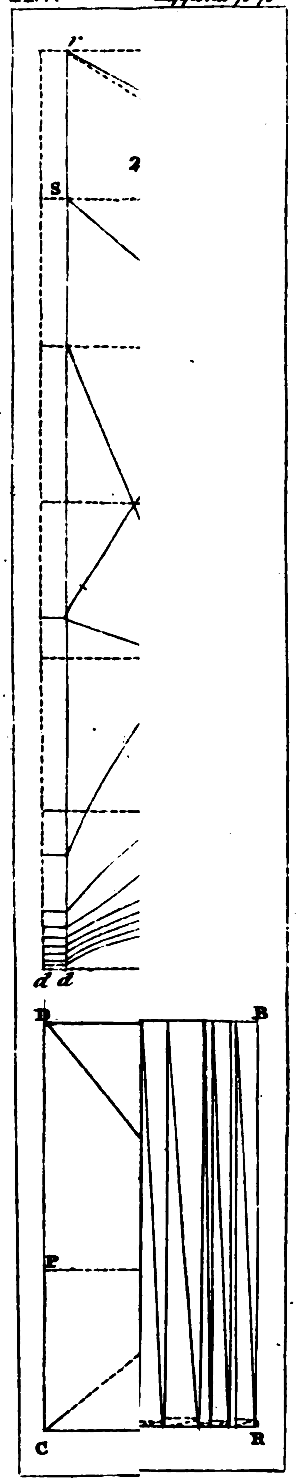
The ſecond Example of Compariſon.

We have found by the calculation, that the effort of the fluid upon the prow of our ſhip of 70 guns, is 249 : 10 : 11 : 7; but if by a like calculation we find the reſiſtance upon the prow of a ſhip of the ſame force, and carrying the ſame quantity of ſail, to be 300 feet, we may thence conclude that ours will ſail beſt.

It will be proper to examine, by the ſame calculation, whether the ſhip we have laid down, can carry a good ſail, drive but litttle to the leeward, and ſteer well; but as this treatiſe has already exceeded the bounds I propoſed, I am forced to confine myſelf to the two preceding conditions, which are the moſt important. The methods to find the other qualities of the ſhips we lay down, may be found in Mr *Bouguer*'s treatiſe.

F I N I S.

Pl. V. Append p. 70.



An Explication *of the* Signs *or* Characters *made uſe of in this* Treatise.

Signs.	Names.	Significations.
+	Plus, or more.	The ſign of addition, as 3 + 5, is 3 more 5, that is, the character + placed between any two or more figures, ſignifies that they are to be added into one ſum.
—	Minus, or leſs	The ſign of ſubtraction, as 5 — 3, is 5 leſs 3, and ſignifies that 3 is to be taken from 5.
×	Multiplied by	The ſign of multiplication, as 5 × 3, ſignifies 5 multiplied by 3.
÷	Divided by	The ſign of diviſion, as 8 ÷ 4, ſignifies 8 divided by 4.
=	Equal to	The ſign of equality, that is when this is placed betwixt numbers or quantities, it ſignifies that they are equal. as 5 + 3 = 8, or 10 — 2 = 5 + 3, that is, 5 more 3, is equal to 8, and 10 leſs 2, is equal to 5 more 3.
: :	So is	The ſign ſimularity of ratio. It is always placed betwixt the two middle terms or numbers in proportion, thus 3 : 9 : : 8 : 24, that is, as 3 is to 9, ſo is 8 to 24.

R. Radius.
S. Sine.
T. Tangent,
Sec. Secant.
S. c. Sine complement.
T. c. tangent complement.
Sec. c. Secant complement.
H. Hypothenuſe.
B. Baſe.
P. Perpendicular.

E R R A T A.

Page	Line	for	read
14	19	a in	in a
25	15	of division	in the divisions
26	31	*f* B	*fb*
30	Margin	Fig. 25	Fig. 25 at Prop. VIII.
30	30	C *fe*	C *f* E
35	13	from the point N, &c.	cancel
38	Margin	Fig. 5.	cancel
40	Margin	Plate II.	Plate I. Fig. 37.
40	Margin	Fig 1.	Plate II. Fig. 1.
41	Margin	Fig 6, 7, 8, 9, 10.	cancel
41	30	Example	Examples
43	Margin	Fig. 30.	cancel
49	4	40	50
49	10	50	40
49	13	base at A . . . *also* for 50	base; at A *read* 40
53	15	T *x* = *x* E	T *x* = *x* F
53	18	*m*, *s*, *n*, *t*,	*m s*, *n t*,
53	34	F D	F *d*
56	17	the	then
56	27	$rt \frac{1}{2} rs$	$rt \times \frac{1}{2} rs$
58	12	1.14592	3.14592
58	31	785	.785
59	5	whose diame-	whose diameter is 1.
59	30	line, as	line
64	11	C *m*	C *s*
64	12	$\frac{ms \times C +}{Cs}$	$\frac{ms \times Ct}{Cs}$
64	21	including the areas	including half the areas
68	Margin		before Case 2d. prefix Fig. 53
82	Margin	Plate I.	Plate II.
82	17	5.9990	5.6990
102	Column 8	Length	Depth
103	22	as in the columns	cancel
104	7	6.25	.625
105	2	height	length
105	16	double line; on the rule	double line on the rule; look
106	21	11,	1, 1,
106	22	11,	1, 1,
106	last line	A C	A D
110	31	*fo* B	*to* B
111	4	Fig. 2.	Fig. 9.
111	5	S S	SS Fig. 2.
114	5	remarked	marked
121	36	1 *s*, 2 *e*, 3 *b*,	1 *r*, 2 *d*, 3 *a*.
122	34	G A E	C A E
124	2 and 3	provided the dimensions and inclination of these planes to one another be given.	cancel
126	32	*s q c b*	*s q* 7

ERRATA.

Page	Line	for	read
130	18	the body	the body plane
131	16	ribband lines	ribbands
133	after line 22	add	See Plate VII. where the curves here defined are diſtinguiſhed by their initial letters.
136	36	M to *y*	M to S
137	37	C	*c*
138	19	BEND	Bend Mould
138	26	H *lw*	Hollow mould
144	20	floor	floor timbers
152	10	*so*	ſo
162	29	*b l*	*b l*
168	40	when this is	cancel
169	1	hewed of	cancel
169	1	of the outſide	(in that direction)
199	8	+.697997	+1.697997
201	17	56° 15′	56° 15′ to 33° 45′
204	4	ſtance	diſtance
214	11	E *o*: E *n*	E *n*: E *o*
217	1	50	15
218	16	C *x*	CX
225	5	Long line	Log line
225	17	A B	A C
225	18	B E	A E
228	31	difference of R	difference of latitude; R.
231	7	by 3 15	cancel
237	4	north	ſouth
244	24	65338	65338
	25	18668	18688
249	6	27°. 2′ R: differenece of latitude	27° 2′: R:: difference of latitude 1063:
249	7	diſtance: 1191	diſtance 1191
249	14	2.738920	2.739030
249	28	75.7	75.5
249	29	18.4	21.1
249	30	546.9	549.6
250	35	ſide	ſine
253	10	30	60
253	11	60	30
253	33	25	15
254	1	25	15
254	2	15	25
254	6	15	25
256	4	P *p*	P P
256	6	P *p*	P P
258	12	tranſcribe	tranſcribes
258	34	make	makes

ERRATA to the APPENDIX.

Page	Line	for	read
18	22	of D	D
22	33	*d* and K	*d* and S
22	35	to *g*	to *l*
22	36	*s* and I	*s* and *l*
23	5	*d* *also* for *d x*	*b* *read* *b x*
27	7	point X	point *x*
27	14	R S	R
27	17	point X	point *x*
35	26	lines	line
37	26	*n* L	N L
38	17	H P	H *p*
40	23	S	*s*
42	10	from N	strike it out
43	27	3d from *l*	3d from *i*
44	20	*z x*	*z y*
54	17	&c. + ½ K S + A B	&c. + X S × A B
62	25	Plate IV. Fig II.	Plate IV. Fig. 11.
62	line 32	add	and the other perpendicular to it whose force and direction is represented by the line E H.

Where the References to the Plates are omitted in Part II. See Plate VII. and in the Appendix, See Plate 2.